The Extraordinary Predictive Power of Holographic QCD

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Based on

- hep-ph/0501128, Erlich et al.
- hep-ph/0501218, Pomarol and Da Rold
- …
- hep-ph/0510388, Katz, Lewandowsky and MDS
- arXiv:0705.0534, Katz and MDS

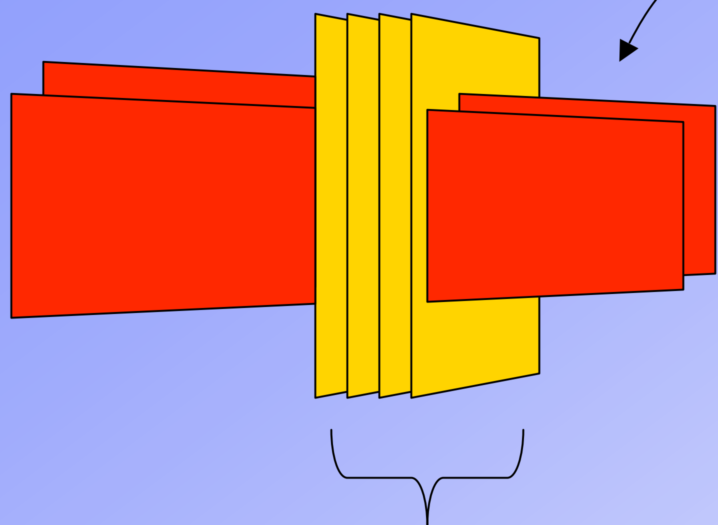
October 11, 2007

Fermilab

Outline

- Introduction
- Holographic QCD
  - basic setup
  - masses and coupling constants
  - vector meson dominance/KSRF
  - tensor mesons
- The U(1) problem
  - The problem
  - AdS/QCD construction
  - Predictions: masses, decay constants, mixing angles
  - Connection to instantons
- Conclusions

AdS/CFT

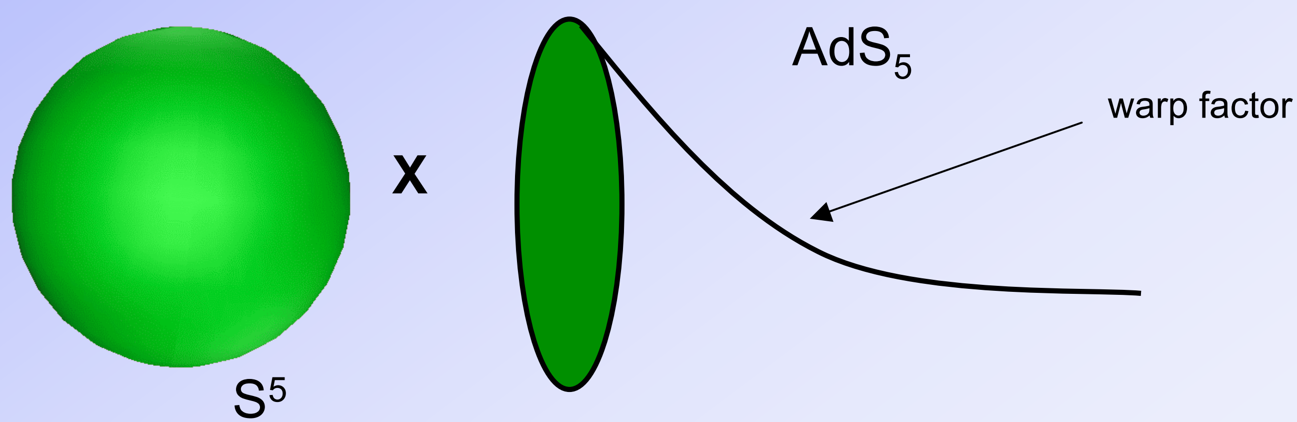


more D-branes, fluxes

(for flavor, confinement, deviations from conformality, finite N, etc.)

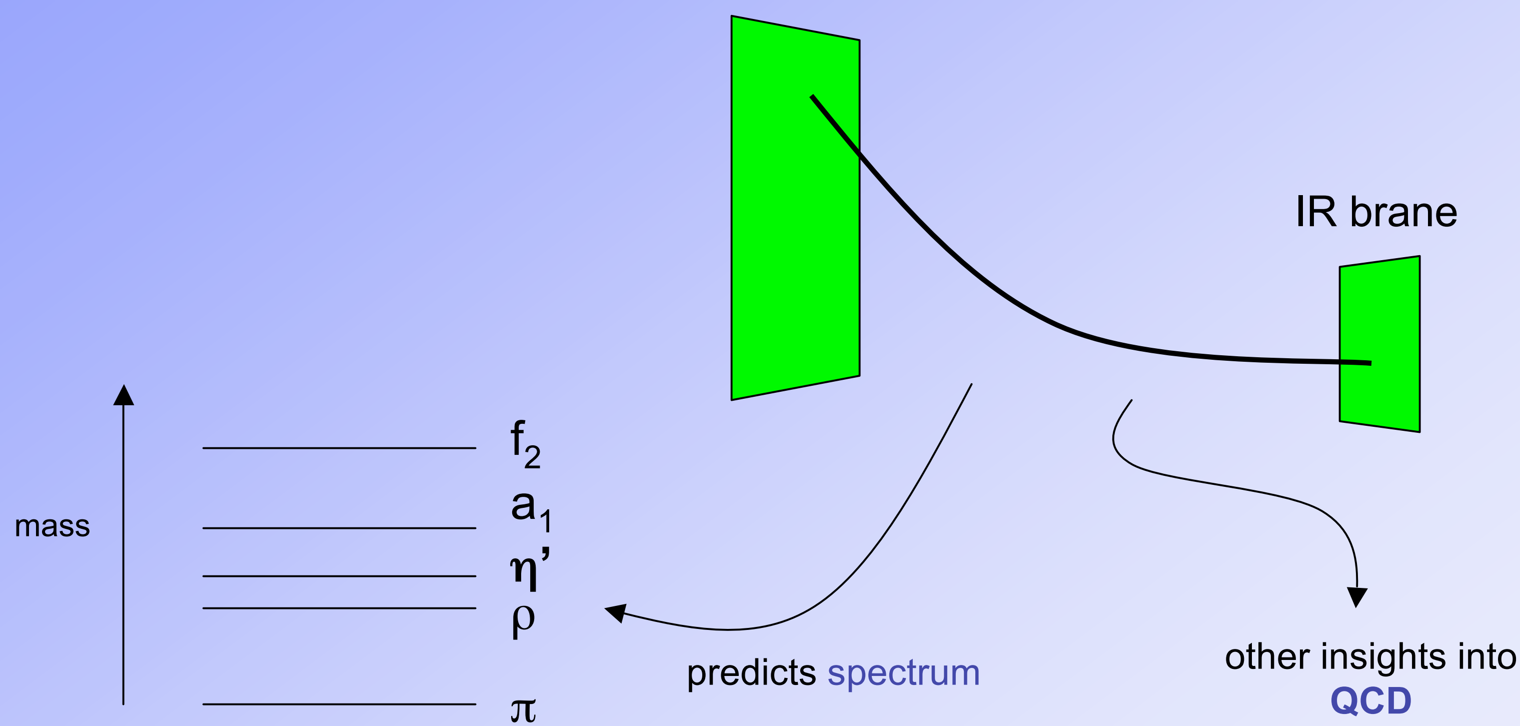
- some qualitative similarities to QCD
- to get QCD, do we need the full string construction?

N D-branes) SU(N) yang-mills symmetry



AdS/QCD

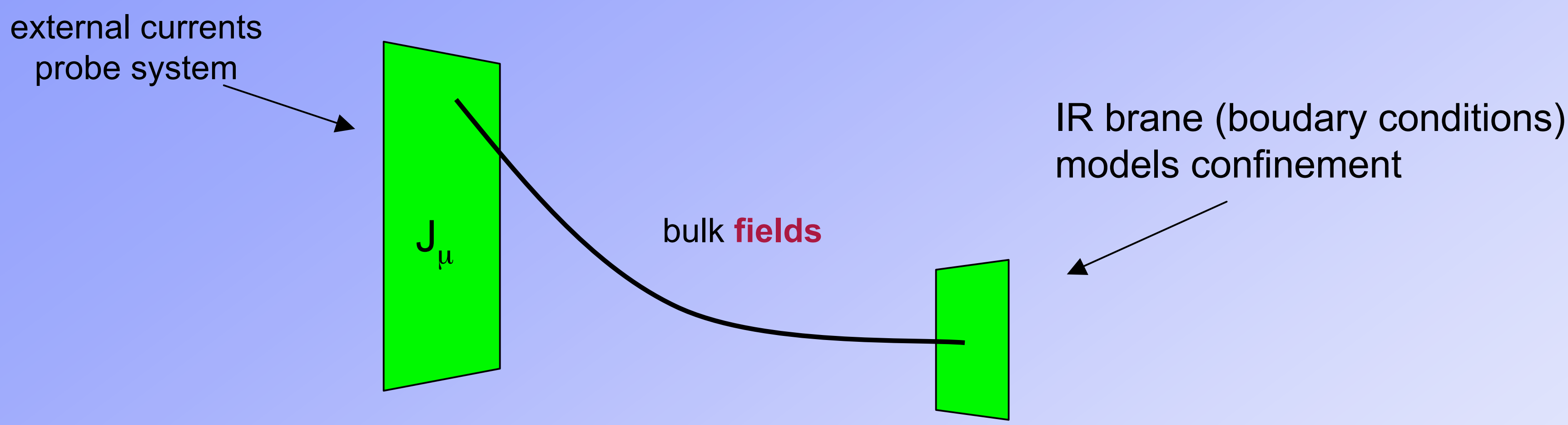
- **Bottom up** approach
- Fit to **QCD**, expanding around the conformal limit
- Check internal consistency as **effective field theory**



Why bottom up?

- **Top down** string-model building approach may be too indirect
  - **Difficult** to find **correct** supergravity background, brane configurations, fluxes, etc…
  - The dual description of **QCD** may not be **simple** to descrbe
- **Bottom up** appoach is directly related to **QCD** data from the start
  - Fit some parameters, **predict** others
  - Has hope of computing **useful** non-perturbative quantites
    - light front wafefunctions, form factors, hadronic matrix elements
  - **Insights** into QCD sum rules, vector meson dominance, quark models, instantons, glueball spectra, etc
  - **Easier** to work with than the lattice
  - **Understanding** dual to **QCD** may be relevant for **LHC**
    - **Randall-Sundrum** models are built the same way
    - May provide insight into more general **strong dynamics**

Basic Idea



QCD | **AdS**

operators $ bulk **fields**

global symmetries $ **local** symmetries

correlation functions $ correlation functions

- effective **low-energy** descriptions are the same
- quantitatively predicts features of dual (confined) theory

hadrons $ **KK modes**

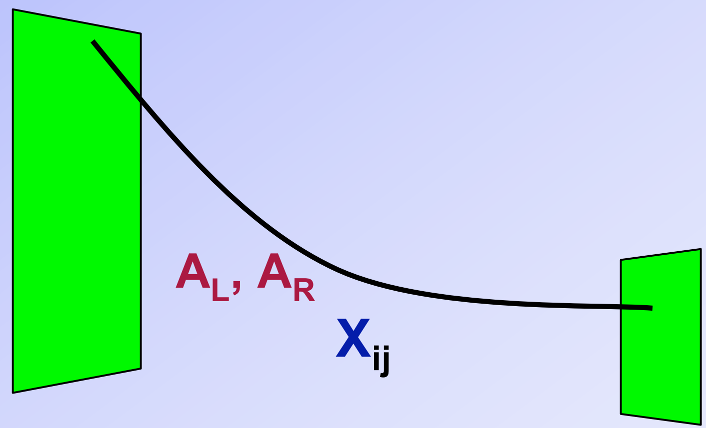
coupling constants $ **overlap** integrals

Set-up

QCD Lagrangian $$\mathcal{L}_{\rm QCD} = F_{\mu\nu}^2 + \bar{q}_i \not{D} q_i + m_q \bar{q}_i q_i + \theta F_{\mu\nu}\tilde{F}_{\mu\nu}$$

has global $SU(3)_L$ x $SU(3)_R$ symmetry.

5D Gauge fields A_L and A_R



Operators $\bar{q}_i q_i$

bulk fields X_{ij}

AdS Lagrangian

$$\mathcal{L}_{\rm AdS} = \sqrt{g}\left\{ |DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right\}$$

mass term determined by scaling dimension

$\bar{q}_i q_i$ ~ X is dimension 3

i.e. $X = \sigma z^3$ solves equations of motion

Gauge coupling from OPE

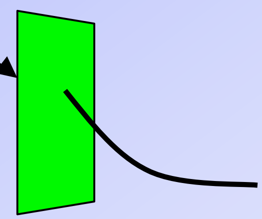
In **QCD**, correlation function of vector **current** $J^a_\mu = \bar{q}\gamma_\mu\tau^a q$ is

$$\int_x e^{iqx}\langle J^a_\mu(x)J^b_\nu(0)\rangle \; = \; \delta^{ab}(q_\mu q_\nu - q^2 g_{\mu\nu})\Pi_V(Q^2)$$

$$\Pi_V(Q^2) = \boxed{-\frac{N_c}{24\pi^2}\ln Q^2}$$ + power corrections

In **AdS**, source fields on **UV brane**

$$g_5 J_\mu V^\mu \delta(z)$$



$$\langle J_\mu J_\nu\rangle = \frac{\delta}{\delta V_\mu}\frac{\delta}{\delta V_\nu}e^{iS} = \frac{1}{g_5^2}V_\mu \frac{1}{z}\partial_z V(z)\Big|_{z=0}$$

$$\Pi_V(Q^2) = \boxed{-\frac{1}{2g_5^2}\ln Q^2}$$

bulk to **boundary** propogator

(solution to eom with V(0)=1)

$$V(Q) = Qz\mathcal{J}_1(Qz) = 1 + \frac{Q^2z^2}{4}\log(Q^2z^2) + \cdots$$

$$\boxed{g_5^2 = \frac{12\pi^2}{N_c}}$$

Chiral symmetry breaking

General solution to bulk equation of motion for **X ~** $\bar{q}_i q_j$

$$\underset{3}{\mathrm{h}}\ \mathbf{X}_{ij}\ \mathrm{i} = \varpi(\zeta) = \mu_\theta \zeta + \sigma\zeta$$

σ ~ $\langle \bar{q}_i q_i \rangle$

quark masses
explicit breaking
relevant in the UV

quark condensate
spontaneous breaking
relevant in the IR

X = v(z) exp(i π)

$|D_M \mathbf{X}|^2$! $\varpi(\zeta)^2(\mathbf{A_M} + d_m \pi + \ldots)^2$

mass term for axial gauge fields
$\mathbf{A_M = (A^L - A^R)_M}$

$$\mathcal{L} = \frac{1}{2 g_5^2 z}\left\{ \left(F^{(L)}_{\mathrm{MN}}\right)^2 + \left(F^{(R)}_{\mathrm{MN}}\right)^2 + \frac{v^2}{2z^3}(A_M - \partial_M \pi)^2 + \cdots \right\}$$

- splits axial from vector
- gives pion a mass

Now just solve equations of motion!

Connect to data

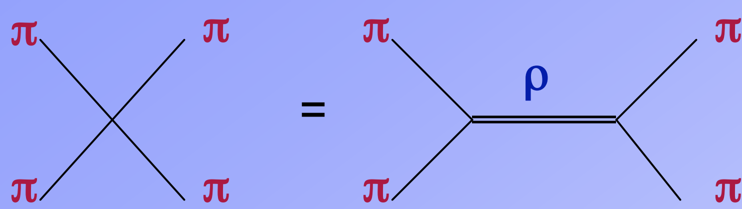
AdS Object		QCD Object
IR cutoff z_m	$\$$	Λ_{QCD}
first vector KK mass	$\$$	m_ρ
first axial KK mass	$\$$	m_{a1}
first A_5 mass	$\$$	m_π
A_5 coupling	$\$$	f_π
	⋮	

Predictions	Data
m_{a1} = 1363 MeV	(1230 MeV)
m_{K^*} = 897 MeV	(892 MeV)
m_ϕ = 994 MeV	(1020 MeV)
m_{K1} = 1290 MeV	(1270 MeV)
$f_\rho^{1/2}$ = 329 MeV	(345 MeV)
$f_{a1}^{1/2}$ = 486 MeV	(433 MeV)
f_K = 117 MeV	(113 MeV)

Observable		AdS Parameter
m_ρ = 770 MeV	$\$$	z_m^{-1}=323 MeV ~ Λ_{QCD}
f_π = 93 MeV	$\$$	σ = (333 MeV)^3 ~ $\langle\bar{q}q\rangle$
m_π = 140 MeV	$\$$	m_q = 2.22 MeV > ½ (m_u+m_d)
m_K = 494 MeV	$\$$	m_s = 40.0 MeV

RMS error ~9%

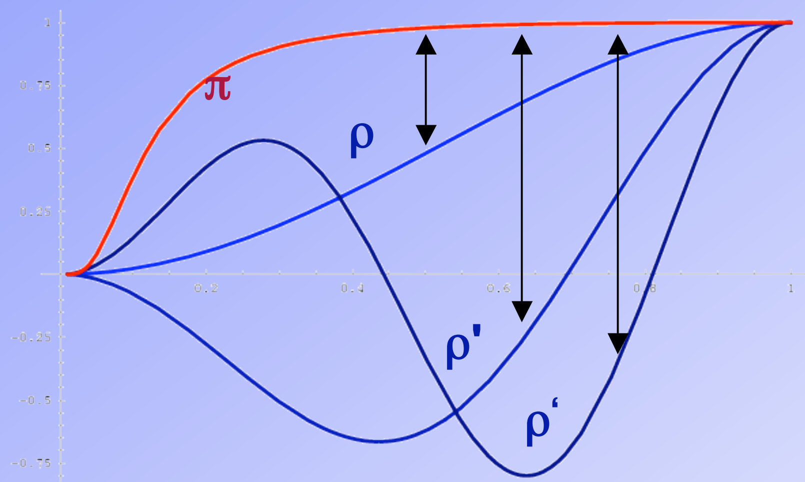
Vector Meson Dominance



KSRF II

VMD + assumptions

$$) \qquad \frac{m_\rho^2}{f_\pi^2 g_{\rho\pi\pi}^2} = 2$$



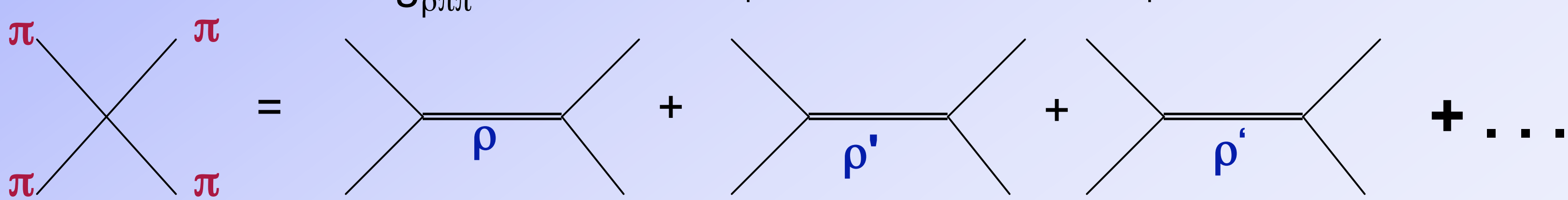
AdS

$$\frac{m_\rho^2}{f_\pi^2 g_{\rho\pi\pi}^2} = 3$$

- **VMD** understood
- **KSRF** II not reproduced
- $g_{\rho\pi\pi}$ is UV sensitive (e.g. F^3 contributes)

In AdS

$g_{\rho\pi\pi} \sim 0.48$ $\quad$ $g_{\rho'\pi\pi} \sim -0.044$ $\quad$ $g_{\rho''\pi\pi} \sim -0.039$



~ 1 $\quad$ ~ 0 $\quad$ ~ 0

Spin 2 mesons

What does chiral perturbation theory say about higher spin mesons, such as the f_2?

Start with free Fierz-Pauli Lagrangian for spin 2

$$\mathcal{L}^{(f)}_{\text{kin}} = \frac{1}{2} h_{\mu\nu} \Box h_{\mu\nu} + h^2_{\mu\alpha,\alpha} - h_{\mu\nu,\mu} h_{,\mu} + \frac{1}{2} h^2_{,\mu} + \frac{1}{2} \boxed{m_f^2} (h^2_{\mu\nu} - h^2)$$

Suppose minimal coupling to energy momentum tensor (like graviton)

$$\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}^{(f)}_{\text{kin}} + \frac{1}{2} \boxed{G_f} h_{\mu\nu} \Theta^{(2)}_{\mu\nu} + \cdots$$

universal coupling to pions and photons $\dfrac{g^2_{f\pi\pi}}{g^2_{f\gamma\gamma}} = 1$

Experimental values are

$$\frac{g^2_{f\pi\pi}}{g^2_{f\gamma\gamma}} = 2.71 \times 10^5 \quad (\text{Exp})$$

Naïve dimensional analysis does better:

$$\frac{g^2_{f\pi\pi}}{g^2_{f\gamma\gamma}} = \frac{1}{c^2_{\text{NDA}}} \left(\frac{16\pi^2}{e^2} \right)^2 = \frac{1}{c^2_{\text{NDA}}} 2.96 \times 10^6 \quad (\text{NDA})$$

2 free parameters (m_f and G_f) and still not predictive

Spin 2 -- AdS

- **Graviton** excitation $ **spin 2** meson (f_2)
- Equation of motion is $\partial_z \frac{1}{z^3}\partial_z h + \frac{m^2}{z^3} h = 0$
- Boundary conditions $h(0) = h'(0) = 0 \quad \Rightarrow m_2$ satisfies $\mathcal{J}_1(m_{f_2} z_m) = 0$

) $\boxed{\mu_{\phi 2} \; = \; 1236 \text{ MeV}}$ (EXP: 1275 MeV) (3% off)

- 5D coupling constant $g_f \sim M_{5D}$ fit from OPE

AdS: $\langle TT \rangle = -\frac{1}{2 g_f} p^4 \log p^2$

QCD: $\langle TT \rangle = \left(-\frac{N_c N_f}{160 \pi^2} + \frac{N_c^2 - 1}{80 \pi^2} \right) p^4 \log p^2$

quark contribution gluon contribution

) solve $g_f = \frac{4\pi}{\sqrt{5}}$

- predict $\boxed{\Gamma(\phi_2 \text{ T } \gamma\gamma) = 2.70 \text{ keV}}$

(EXP: 2.60± 0.24 MeV) (within error!)

completely UV safe

$\boxed{\Gamma(\phi_2 \text{ T } \pi\pi) = 37.4 \text{ MeV}}$

(EXP: 156.9 ± 4 MeV) (off by a lot)

UV sensitive

Higher Spin Fields

- **Higher spin mesons** dual to highr spin **fields** in **AdS**

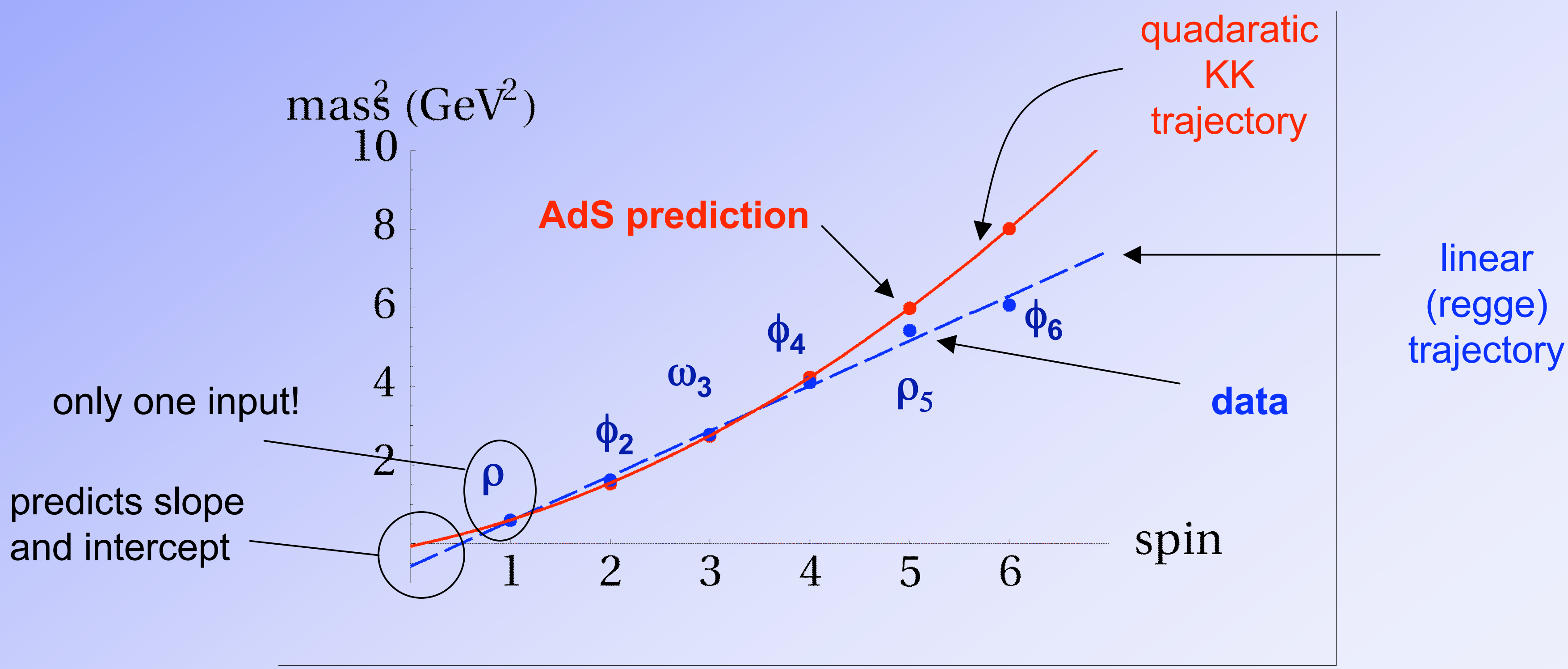
ω_3 $ massless **spin 3** field ϕ_{abc} in AdS_4

f_4 $ massless **spin 4** field ϕ_{abcd} in AdS_5

...

- Linearized higher spin gauge invariance leads to

$$\partial_z \frac{1}{z^{2s-1}} \partial_z \phi_s + \frac{m^2}{z^{2s-1}} \phi_s = 0 \quad) \quad \mu_\sigma \text{ satisfies } \vartheta_{s-1}(\mu_\sigma \zeta_\mu) = 0$$



The U(1) problem

The QCD Lagrangian $\mathcal{L}_{\rm QCD} = F_{\mu\nu}^2 + \bar{q}_i \not{D} q_i + m_q \bar{q}_i q_i + \theta F_{\mu\nu}\tilde{F}_{\mu\nu}$

has a global (classical) U(3) x U(3) symmetry $q_i \to e^{i\gamma_5} q_i$

) expect 3 neutral pseudogoldstone bosons: π (139), η(547), and η'(957)

Chiral Lagrangian

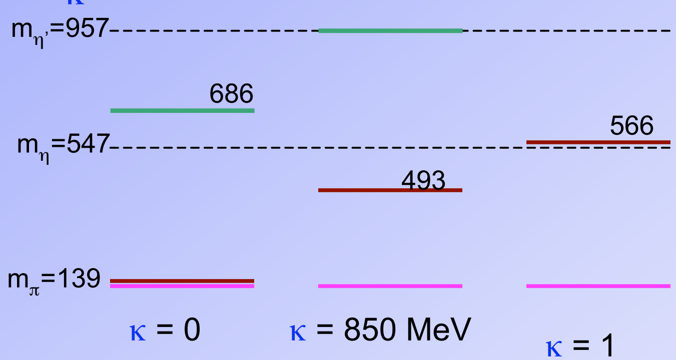
$$\mathcal{L} = f_\pi^2 (D_\mu U)(D_\mu U^\dagger) + m_q \mathrm{Tr}(U + U^\dagger) \Rightarrow m_{\eta'} \leqslant \sqrt{3} m_\pi = 242\,\mathrm{MeV}$$

•Chiral anomaly breaks U(1)

$$\mathcal{L} = f_\pi^2 (D_\mu U)(D_\mu U^\dagger) + m_q \mathrm{Tr}(U + U^\dagger) + \kappa \det(U)$$

new term now **allowed**

with $\phi_\pi = \phi_\eta = \phi_{\eta'}$, we can almost fit with κ



can fit exactly by tuning

$$f_{\eta 0} = \langle \eta | J_{(0)} \rangle$$
$$f_{\eta 8} = \langle \eta | J_{(8)} \rangle$$
$$f_{\eta' 0} = \langle \eta' | J_{(0)} \rangle$$
$$f_{\eta' 8} = \langle \eta' | J_{(8)} \rangle$$

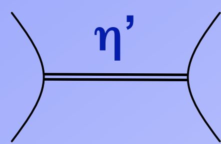
and κ

chiral lagrangian can *accommodate* η'

but does not predict anything about it

How can the Anomaly lift the η' mass?

The U(1) current $J^{\mu}_{(0)} = \bar{q}_i \gamma_5 \gamma^{\mu} q_i$ is anomalous $\partial_{\mu} J^{\mu}_{(0)} = \frac{\alpha}{8\pi^2} F\tilde{F}$

$\langle \partial J_{(0)} \partial J_{(0)} \rangle$ = 

+ . . . = $\frac{f_{\eta}^2}{p^2 - m_{\eta}^2}$ + . . .

extract mass from p T 0

But in perturbative QCD, anomaly vanishes as p T 0

$\langle \partial J_{(0)} \partial J_{(0)} \rangle$ » $\chi_t = \left\langle (F\tilde{F})(F\tilde{F}) \right\rangle$ and $F\tilde{F} = \partial_{\mu}(\epsilon^{\mu\nu\alpha\beta} A_{\nu} F_{\alpha\beta} + \cdots)$

factor of overall momentum) **vanishes** as p T 0 anomaly cannot contribute in perturbation theory

The solution … **instantons**!

$$\left\langle (F\tilde{F})(F\tilde{F}) \right\rangle = \int_0^{\infty} d\rho \frac{D(\rho)}{\rho^5} \left[Q^2 \rho^2 \mathcal{K}_2(Q\rho) \right]^2$$

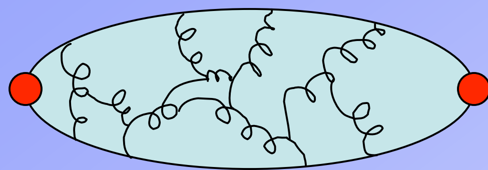
$F\tilde{F}$ evaluated on a one instanton solution

$$A_{\mu} = \frac{x_{\mu}}{\rho^2 + x^2}$$

However, integral is divergent, so it cannot be used quantitatively

Lattice Calculations

For currents with **flavor**, like J^8, masses extracted from



$J_\mu^{(8)} = \bar{q}\gamma_5\gamma_\mu\lambda^8 q$

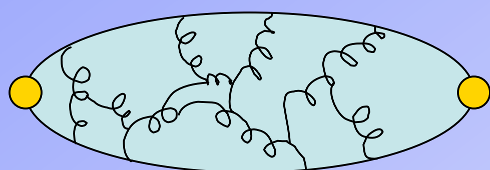
Lattice results

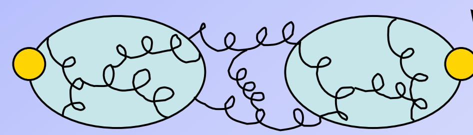
$m_{\eta'} = 871$ MeV

$m_\eta = 545$ MeV

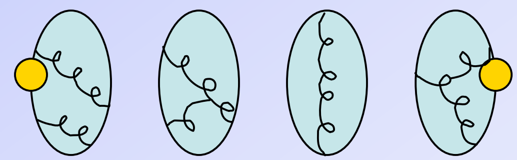
$J_\mu^{(0)} = \bar{q}_i\gamma_5\gamma_\mu q_i$

For U(1) currents, **other diagrams** are relevent





OZI-rule violating



disconnected diagrams

Suppressed for large N_C

(like the anomaly, and the η'-π mass splitting)

- Lattice calculations are **difficult**
  - **quenched** approximation **fails**
  - need strange/up/down quark masses
- In what sense is the η' mass due to **instantons**?
  - is the p T 0 limit **smooth**?
  - what do **OZI** suppressed diagrams do?
  - do we need to calculate in **Euclidean** space?

The η' from AdS

Recall we had a field **X** ~ $q^2 = \bar{q}_i q_j + i\bar{q}_i \gamma_5 q_j$ dual to pions, π_i

Now introduce new field **Y** ~ $F^2 = F_{\mu\nu}^2 + i F_{\mu\nu}\tilde{F}_{\mu\nu}$ dual to “axion”, a

- anomalous global U(1) is now gauged local U(1) in AdS
- both a and π^0 are charged

$$\mathcal{L}_{\rm AdS} = \sqrt{g}\left\{ -\frac{1}{4g_5^2}(F_L^2 + F_R^2) + |DX|^2 + 3|X^2| + |DY|^2 + \kappa Y \det X \right\}$$

Recall $\langle \mathbf{X}_{ij} \rangle = v(z) = m_q z + \langle \bar{q} q \rangle z^3$

Now, $\langle \mathbf{Y} \rangle = C + \Xi z^4$

α_s (keep)

$\langle F_{\mu\nu}^2 \rangle \sim \Lambda_{\rm QCD}^4$ power correction (neglect)

$$\mathcal{L} = \frac{1}{2g_5^2 z}\left\{ \left(F_{\rm MN}^{(L)}\right)^2 + \left(F_{\rm MN}^{(R)}\right)^2 + \frac{v^2}{2z^3}(A_M - \partial_M \pi)^2 \right.$$

$$\left. - \frac{1}{2z^3} C^2 (A_5 - \partial_z a)^2 + \frac{1}{z^5}\kappa \overbrace{C v(z)^{N_f}}^{\text{scales like } z^9 \text{ (strongly localized in IR)}} (a - \pi_0)^2 \right.$$

Match to QCD

In perturbative QCD (for large Euclidean momentum) $\langle \partial J_{(0)} \partial J_{(0)} \rangle = \boxed{-\frac{N_f \alpha_s^2}{16\pi^4} Q^4 \log Q^2}$

In AdS $\langle \partial J_{(0)} \partial J_{(0)} \rangle = -\frac{Q^2}{g_5^2} A_5^{(0)}(z) \frac{1}{z} A_5^{(0)}(z) \Big|_{z=0} = \boxed{-\frac{C^2}{16} Q^4 \log Q^2}$

bulk to boundary solution

$$\boxed{\Rightarrow C = \frac{\alpha_s}{2\pi^2}\sqrt{2N_f}}$$

We take

$$C \sim \alpha_z(z) = \frac{1}{\beta \log(\Lambda_{\rm QCD} z)}$$

$$\beta = \frac{1}{2\pi}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)$$

$\Lambda_{QCD} = \zeta_\mu^{-1}$

That's it – no new free parameters (except for κ**)**

Solve differential equations

5 coupled differential equations

a eom: $$\partial_z \frac{C^2}{z^3}\partial_z a - \frac{C^2}{z^3} m^2 \Big(\sqrt{\frac{2}{3}}\varphi^q - \frac{1}{\sqrt{3}}\varphi^s - a\Big) + \kappa C v_q^2 v_s \Big(\sqrt{\frac{2}{3}}\pi^q - \frac{1}{\sqrt{3}}\pi^s - a\Big) = 0$$

η_q eom: $$\partial_z \frac{1}{z}\partial_z \varphi_q - g_5^2 \frac{v_q^2}{z^3}(\varphi_q - \pi_q) - g_5^2 \sqrt{\frac{2}{3}}\frac{C^2}{z^3}\Big(\sqrt{\frac{2}{3}}\varphi^q - \frac{1}{\sqrt{3}}\varphi^s - a\Big) = 0$$

η_s eom: $$\partial_z \frac{1 + c_4 z^4}{z}\partial_z \varphi_s - g_5^2 \frac{v_s^2}{z^3}(\varphi^s - \pi^s) + g_5^2 \frac{1}{\sqrt{3}}\frac{C^2}{z^3}\Big(\sqrt{\frac{2}{3}}\varphi^q - \frac{1}{\sqrt{3}}\varphi^s - a\Big) = 0$$

$A_5^{(q)}$ eom: $$m^2 z^2 \partial_z \varphi^q - g_5^2 v_q^2 \partial_z \pi_q - \sqrt{\frac{2}{3}} g_5^2 C^2 \partial_z a = 0$$

$A_5^{(s)}$ eom: $$m^2 z^2 (1 + c_4 z^4)\partial_z \varphi^s - g_5^2 v_s^2 \partial_z \pi_s + \frac{1}{\sqrt{3}} g_5^2 C^2 \partial_z a = 0$$

ϕ is stuckleberg field (longitudinal mode of A_μ)

correction to warp factor due to strange quark mass

- choose boundary conditions

  $\pi(0) = \phi(0) = 0$

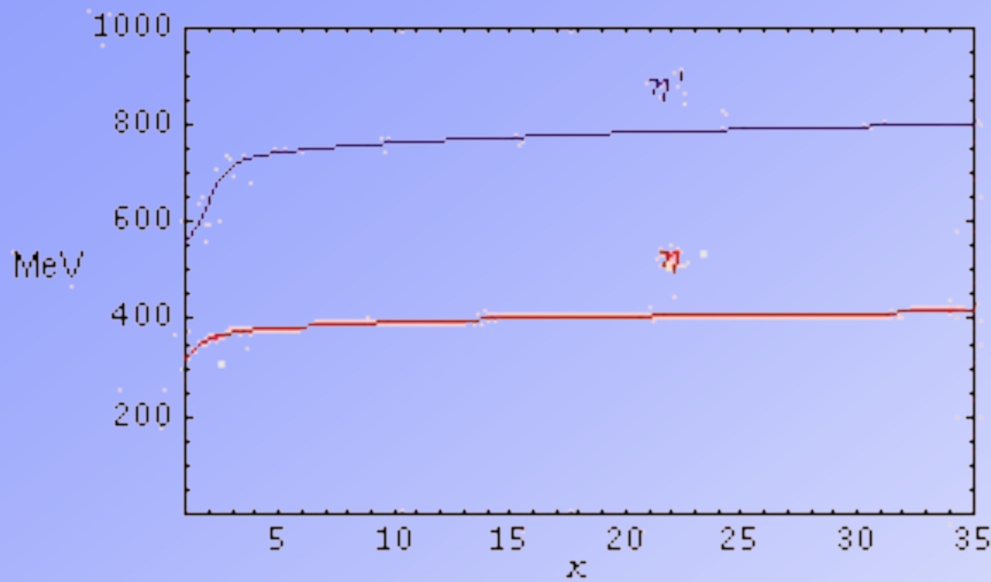
  $\pi'(z_m) = \phi'(z_m) = 0$

- we will scan over κ. But note

$$\kappa C v_q^2 v_s = \kappa \alpha_s(z)(m_q z + \sigma z^3)^2 (m_s z + \sigma z^3) \sim \kappa z^9$$

- forces $\pi^0(\zeta_\mu) = \alpha(\zeta_\mu)$
- as κ T 1 forces $\pi^0(\zeta) = \alpha(\zeta)$

Solution



AdS	EXP	error	lattice
867 MeV	**957**	9%	**871**
520 MeV	**549**	5%	**545**

Also calculate decay constants

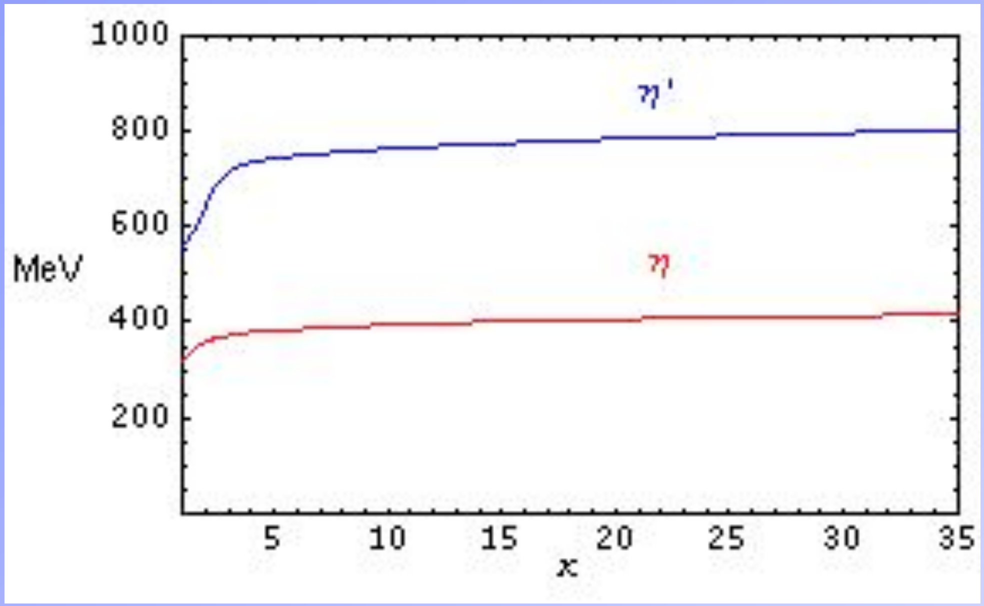
$$f_{\eta 0}^2 = \langle \partial_\mu J_\mu^0 | \eta' \rangle^2 = \langle J_\varphi^0 | \eta \rangle^2 = \frac{1}{g_5^2} \lim_{z \to 0} \frac{\varphi^0 \partial_z \varphi^0}{z} = (46.8\,\mathrm{MeV})^2$$

$$f_{\eta 8}^2 = \langle \partial_\mu J_\mu^8 | \eta' \rangle^2 = \langle J_\varphi^8 | \eta \rangle^2 = \frac{1}{g_5^2} \lim_{z \to 0} \frac{\varphi^8 \partial_z \varphi^8}{z} = (117\,\mathrm{MeV})^2$$

$$f_{\eta' 0} = 118\,\mathrm{MeV}$$

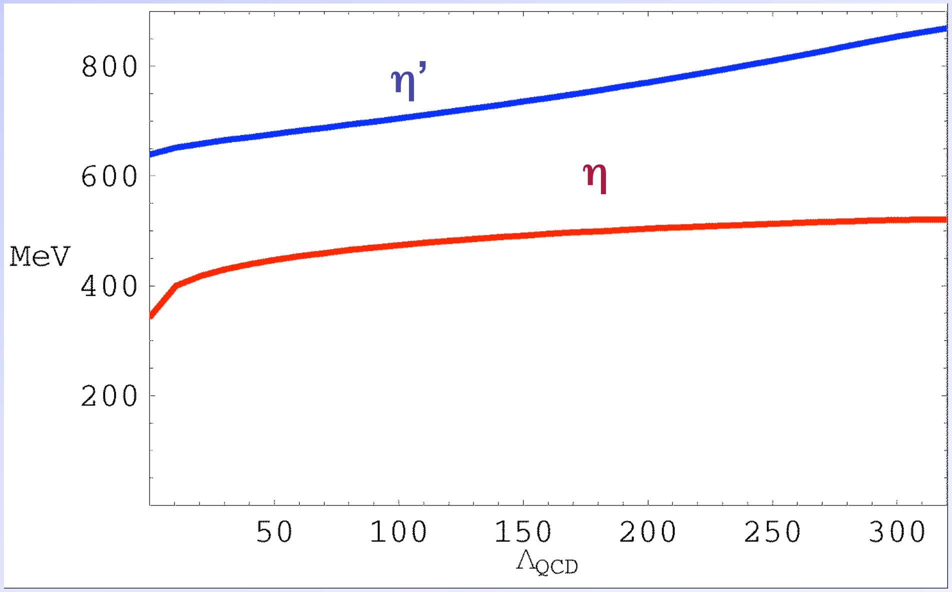
$$f_{\eta' 8} = 61.6\,\mathrm{MeV}$$

Turn off anomaly

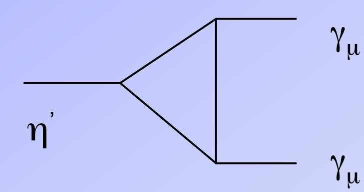


Turn off κ

Turn off Λ_{QCD}



Decay to photons



Effective interaction described by Wess-Zumino-Witten term

$$\mathcal{L}_{\mathrm{WZW}} = \sqrt{g}\frac{\sqrt{2}N_c e^2}{4\pi^2}\varepsilon^{\mathrm{ABCDE}}F_{AB}F_{CD}(A_M^b + \partial_M \pi^b)\mathrm{Tr}[Q^2\tau^b]$$

(in units of TeV^{-1})

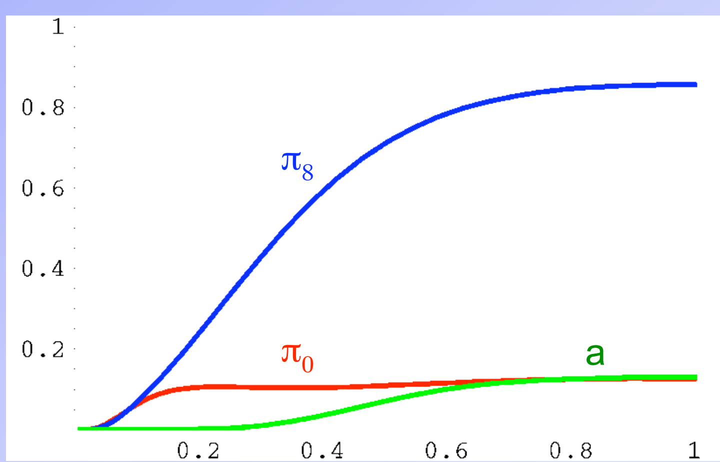
This is a total derivative, so

$$A_{\gamma\gamma} = \frac{e^2}{4\pi}\left[\frac{1}{3\sqrt{3}}\pi_8(z_m) + \frac{2}{3\sqrt{3}}\pi_0(z_m)\right]$$

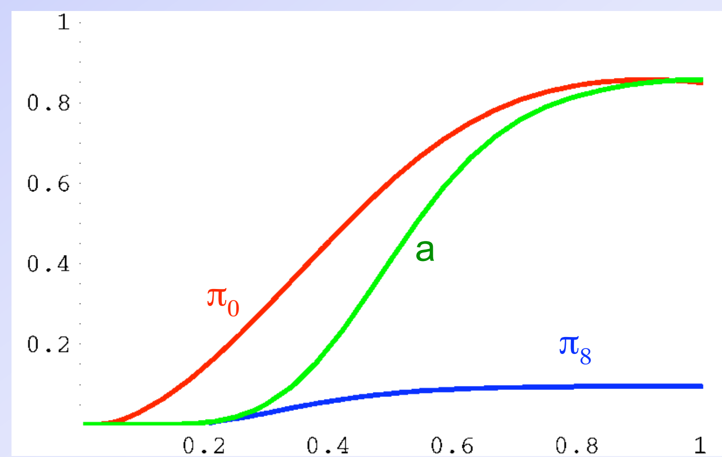
	AdS	EXP
$A_{\eta\gamma\gamma}$	= 24.3	24.9
$A_{\eta'\gamma\gamma}$	= 48.1	31.3

Mixing angle **varies** with z

η

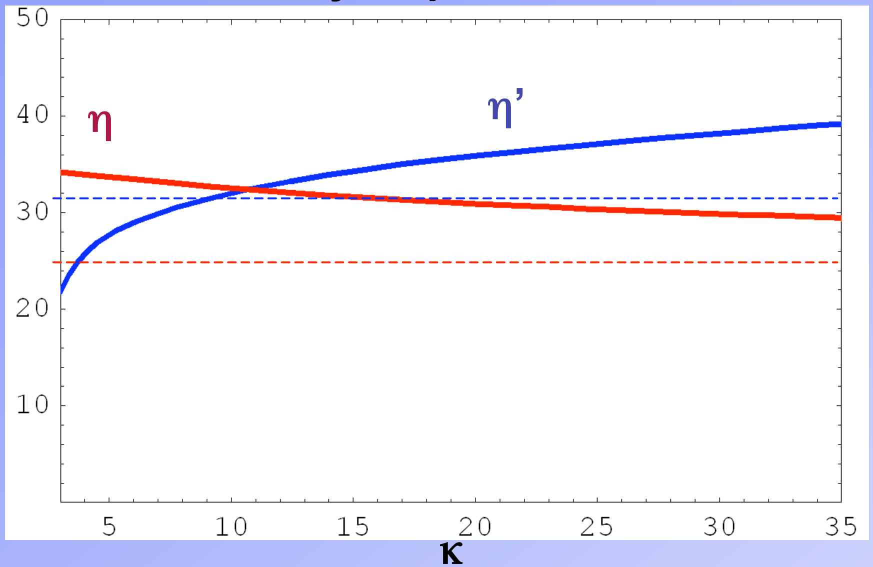


η'



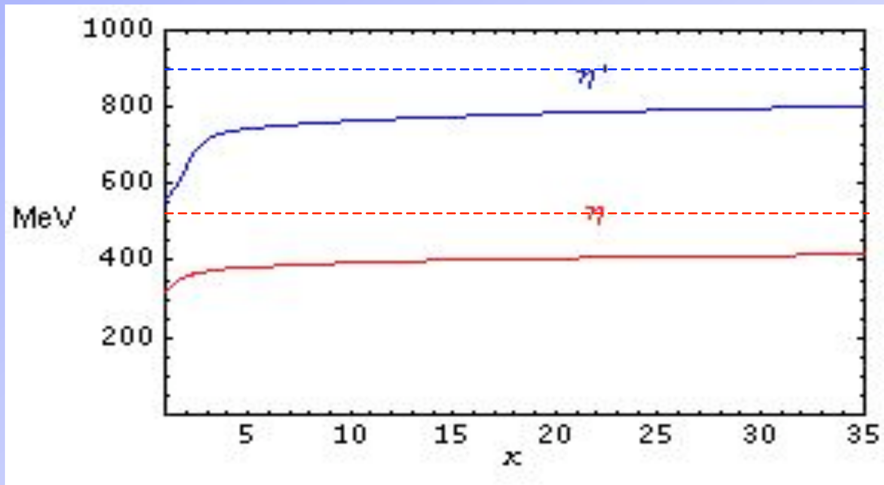
κ dependence

decay amplitudes



AdS	EXP
48.1	
	31.3
24.3	24.9

masses



AdS	EXP
867	957
520	549

Topological Susceptibility

$$\chi_t(Q) \equiv \langle (\frac{1}{8\pi} G\tilde{G})(\frac{1}{8\pi} G\tilde{G}) \rangle$$

In **AdS**

$$\chi_t(Q) = \frac{1}{4\pi^4} \lim_{z\to 0} \frac{a\partial_z a}{z^3}$$

bulk to boundary propogator for a

With **no quarks**

$$a(z) = 1 - \left(\frac{z}{z_m}\right)^4$$

quarkless AdS	Lattice	large N
(109 Mev)^4	(191 Mev)^4	(171 MeV)^4

with quarks

$$\chi_t(0) = \frac{1}{\pi^4} \frac{m_q z_m (m_q z_m + \sigma z_m^3)}{2C^2 + m_q z_m (m_q z_m + \sigma z_m^3)} z_m^{-4}$$

Witten-Veneziano relation

$$\chi_t = \frac{f_\pi^2}{4N_f}(m_\eta^2 + m_{\eta'}^2 - 2m_K^2)$$

- numerical value strongly **depends** on C~α_s
- For **C=0** (no κ term) and $\mathbf{m_q \neq 0}$ $\chi_t(0) \neq 0$
- For **C=0** (no κ term) and $\mathbf{m_q = 0}$ $\chi_t(0) = 0$

arguments about whether θ and χ_t are physical produce the same qualtitative results

$\mu_{\eta'}^2 \sim 1/N_c$ for large N_c

Instantons

In the **conformal limit** (no IR brane, C~α_s const)

$$z^3\partial_z \frac{1}{z^3}\partial_z a^{(0)} + Q^2 a^{(0)} = 0 \quad \Rightarrow \quad a^{(0)}(z) = \frac{1}{2} z^2 Q^2 \mathcal{K}_2(Qz)$$

With **IR brane** and z-dependent **κ = κ(ζ)**

$$z^3\partial_z \frac{1}{z^3}\partial_z a + Q^2 a + \frac{1}{z^5}\kappa(z) a = 0$$

Expanding around the conformal solution, and integrating the action by parts on the bulk-to-boundary equation of motion gives

$$\chi_t(Q) = \frac{1}{16\pi^4}\int_0^{z_m} \mathrm{d}z \frac{\kappa(z)}{z^5}\left[Q^2 z^2 \mathcal{K}_2(Qz)\right]^2$$

This is the same as the **instanton** contribution

$$\chi_t(Q) = \cdots - \frac{1}{2}\int_0^{\infty} \mathrm{d}\rho \frac{\mathcal{D}(\rho)}{\rho^5}\left[Q^2 \rho^2 \mathcal{K}_2(Q\rho)\right]^2$$

with $\Delta(\rho)$ ~ $\kappa(z)$ and z~ρ

Conclusions

- The **holographic** version of **QCD** works **really really well**
  - meson **spectrum**, coupling **constants** predicted
  - insights into **v**ector **meson** **d**ominance, **KRSF**
  - **higher spin** fields can be understood
  - insights into **regge physics**
- The **U(1) problem** is solved **quantitatively** and analytically (i.e. not with a lattice)
  - η and η' masses predicted to **5%** and **9%** respectively
  - decay rate $\Gamma(\eta\ \mathrm{T}\ \gamma\gamma)$ predicted well $\Gamma(\eta'\ \mathrm{T}\ \gamma\gamma)$ is IR-model-dependent
  - simple **mixing angle** interpretation for η η' decays fails
  - new handles to turn off the anomaly
  - **Witten-Veneziano** relation reproduced
  - Direct connection with **instantons**, with z ~ ρ and an interpretation of the **instanton density**
- Many **open questions**
  - **Why** does AdS/QCD work so well?
  - Can the expansion be made **systematic** to all orders?
  - Can we calculate **useful** non-perturbative observables: form factors, hadronic matrix elements, etc
  - Can lessons from AdS/QCD be applied to **other** strongly coupled **gauge theories**

    (e.g. RS/technicolor)