

Precise Predictions for $H \rightarrow WW/ZZ \rightarrow 4$ Fermion Decays with PROPHECY4f

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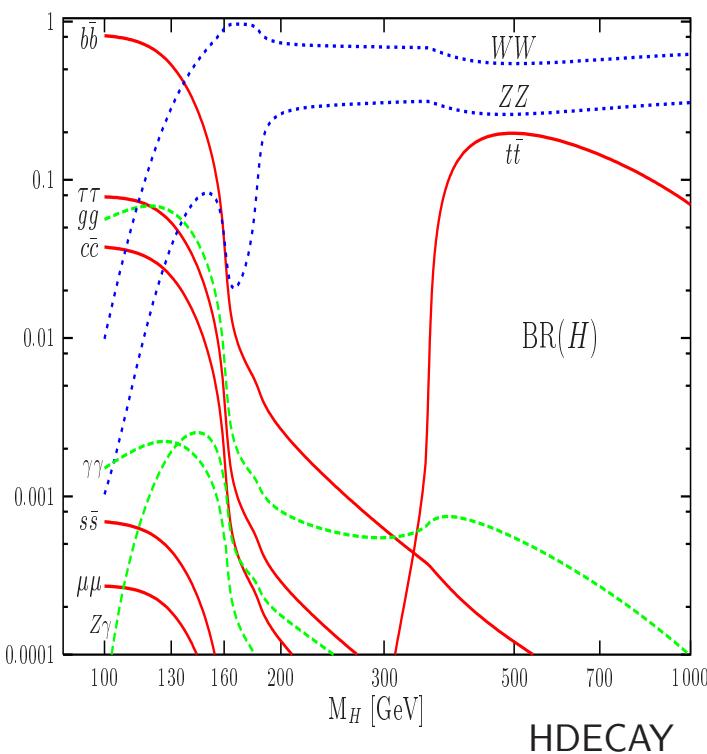
in collaboration with
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see PRD74 (2006) 013004 [hep-ph/0604011] and JHEP 0702 (2007) 080 [hep-ph/0611234]

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- Introduction to $H \rightarrow WW^{(*)}/ZZ^{(*)} \rightarrow 4f$
- Calculation of radiative corrections
- Numerical results
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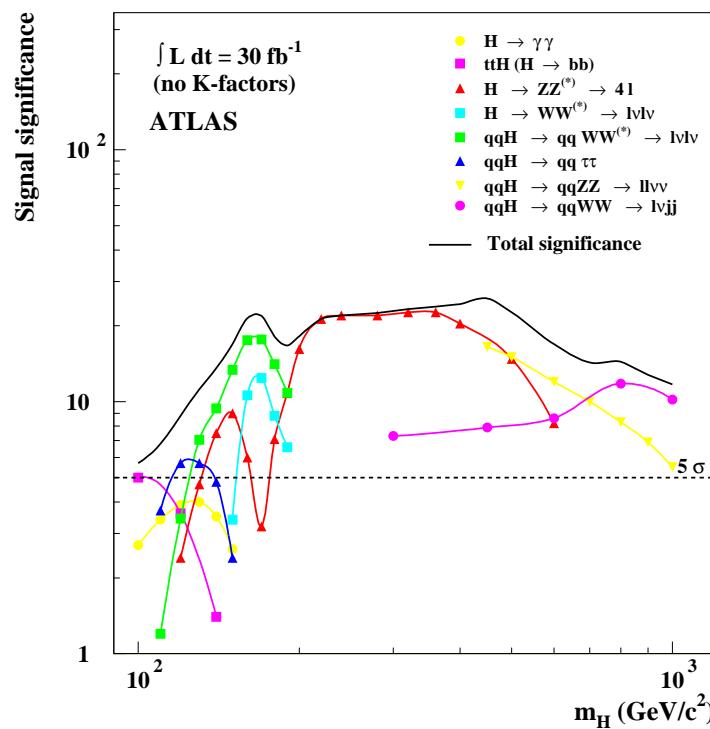
Introduction: $H \rightarrow WW^{(*)}/ZZ^{(*)}$ decays



most important decay channels for $m_H \gtrsim 140$ GeV

LHC

- most important discovery channels for $m_H \gtrsim 130$ GeV
 - most accurate Higgs mass measurement for $m_H \gtrsim 130$ GeV using $H \rightarrow ZZ \rightarrow 4l$
 - linear collider
 - measurement of branching fractions to several percent level
- precise theoretical prediction for $H \rightarrow WW^{(*)}/ZZ^{(*)} \rightarrow 4f$ needed



$H \rightarrow WW^{(*)}/ZZ^{(*)}$ decays

theoretical status

- $m_H > 2m_V$: $H \rightarrow WW/ZZ$ real pair production
 - $\mathcal{O}(\alpha)$ corrections known [Fleischer, Jegerlehner '81, Kniehl '91, Bardin et al '91]
 - some leading higher order [Kniehl, Spira '95; Kniehl, Steinhauser '95] [Ghinculov '95; Frink et al '96]
- $m_H < 2m_V$: $H \rightarrow WW^{(*)}/ZZ^{(*)}$ three-body decay
 - leading order only [e.g. HDECAY:Djouadi, Kalinowski, Spira '98]

distributions important

- kinematical reconstruction of Higgs and W/Z
 - invariant mass distributions \rightarrow real photon radiation important
- verification of Higgs boson spin and CP:
 - \rightarrow uses angular and invariant-mass distributions [Choi et al '02]

$\Rightarrow H \rightarrow WW/ZZ \rightarrow 4f$ Monte Carlo generator with NLO corrections needed

recent work

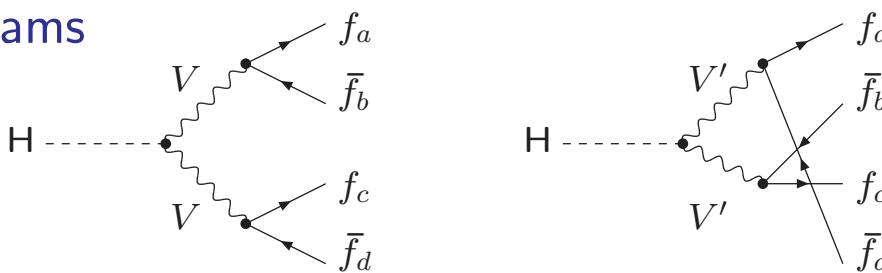
PROPHETY4F: generator for $H \rightarrow WW/ZZ \rightarrow 4f$ with EW and QCD corrections

related: QED corrections to $H \rightarrow WW/ZZ \rightarrow 4l$:

[Carloni-Calame et al '06]

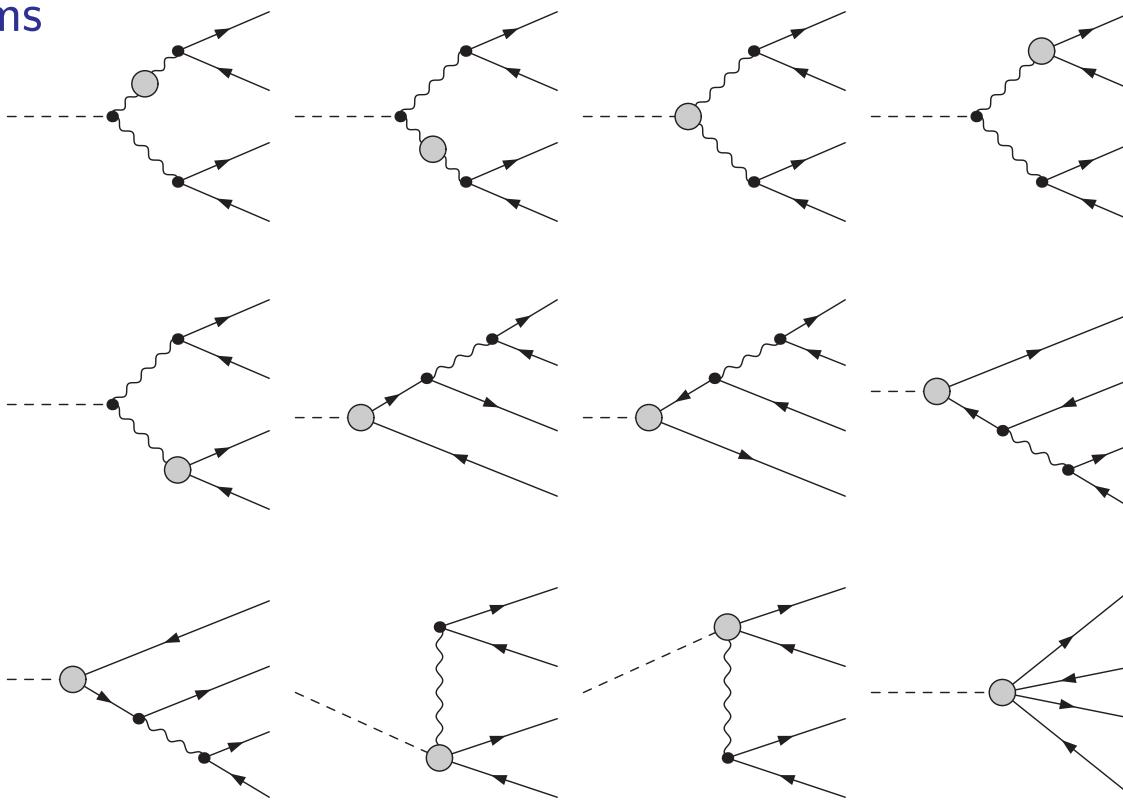
$H \rightarrow 4f$ Feynman diagrams

Born diagrams



crossed diagram only for
 $f\bar{f}f\bar{f}$ and $f\bar{f}f'\bar{f}'$ final
states
(f, f' form isodoublet)

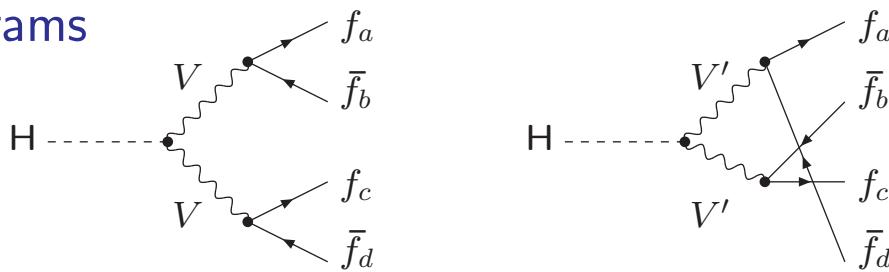
EW virtual diagrams



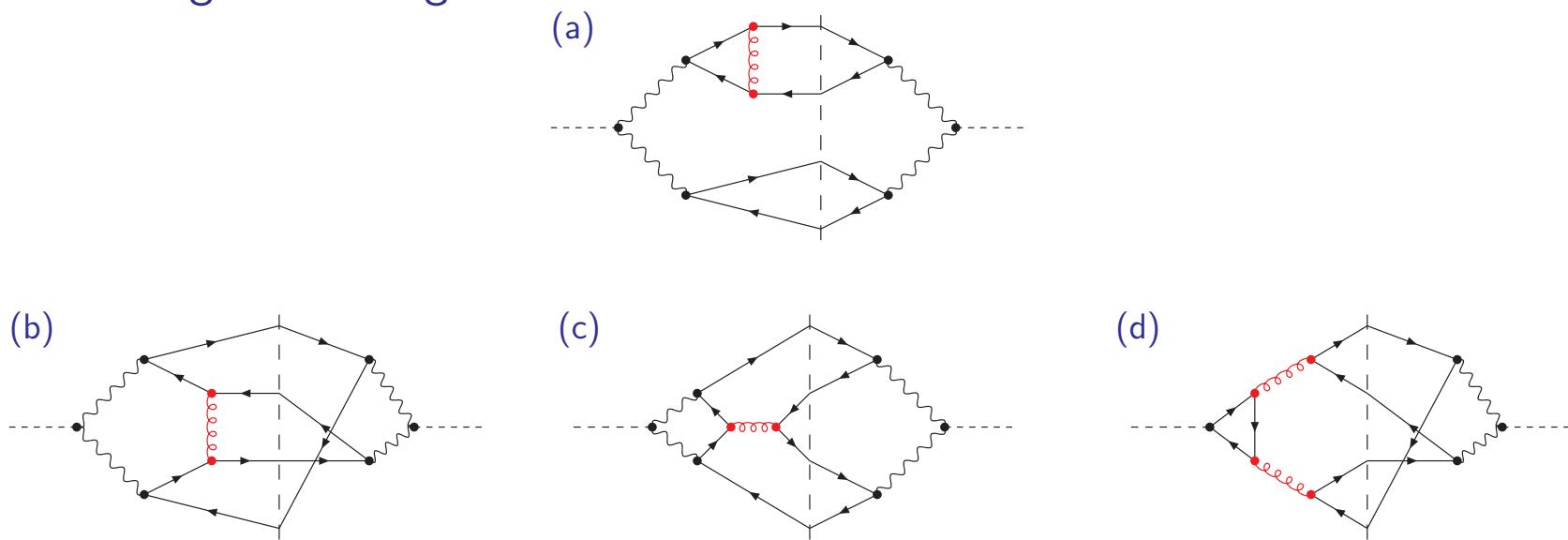
- about 400 Feynman diagrams (Feynman gauge), up to pentagons
- external fermions massless

$H \rightarrow 4f$ Feynman diagrams

Born diagrams



QCD virtual diagrams: categories



(a): corrections to W/Z decays

(b),(c),(d): interference corrections, only for $f\bar{f}f\bar{f}$ and $f\bar{f}f'\bar{f}'$ final states

NLO calculation: general remarks

- external fermions massless where possible
→ keep fermion mass only in mass singular logarithms
- diagram generation: FeynArts
- algebraic evaluation: FormCalc / Mathematica
→ decomposition into standard matrix elements and coefficients
- real amplitudes: evaluation with Weyl–van der Waerden spinor formalism
- renormalization scheme
on-shell scheme generalized to complex masses

derive α from Fermi constant (G_μ -scheme): $\alpha_{G_\mu} = \frac{\sqrt{2}G_\mu m_W^2 s_W^2}{\pi}$

includes

- running of α from $Q^2 = 0$ to EW scale
- universal corrections related to ρ parameter

main technical complications in loop calculation:

- gauge invariant treatment of gauge boson resonances
- numerical instabilities in Passarino-Veltman tensor reduction

Evaluation of 1-loop integrals

Passarino-Veltman reduction of tensor loop integrals

- introduces Gram determinants in denominator
- Gram determinants may vanish (at phase space boundary & inside)
but: **tensor integrals regular**
 - cancellations in numerator
 - numerical instabilities

solution

[Denner, Dittmaier '05]

- **5-point** direct reduction without inverse Gram determinants
- **3- and 4-point**: 2 methods
 - both: Passarino-Veltman reduction where possible
 - reduction to specific tensor coefficient
 - Feynman parameter integral logarithmic
 - numerical integration
 - expansion in small Gram (and other) determinants
- **works with complex masses**

Resonances

Resonances: finite width needed

- Dyson resummation of self energies, mixes perturbative orders
- potential gauge invariance violation

solutions at tree level

fixed-width scheme, complex-mass scheme, effective Lagrangians, fermion-loop scheme

solutions at 1-loop

- naive fixed-width scheme
 - only weak breaking of gauge invariance
 - singularity structure changed

- pole expansion
 - gauge invariant
 - not in threshold region

- effective field theory approach
 - gauge invariant, only in specific phase space regions

- complex mass scheme
 - gauge invariant, all phase space regions, simple
 - see below

[Stuart '91; Aeppli et al '93]

[Beneke et al '04, Hoang, Reisser '04]

[Denner et al '05]

Complex mass scheme at leading order

[Denner et al '05]

idea: $\text{mass}^2 = \text{location of propagator pole in complex } p^2 \text{ plane}$

- decomposition into real mass and width:

$$\mu_V^2 = m_V^2 - i m_V \Gamma_V$$

- μ_V^2 is consistently used everywhere

→ derived quantities complex, e.g. $\cos \theta_W = \frac{\mu_W}{\mu_Z}$

advantages:

- gauge invariant
Ward identities valid, unitarity cancellations hold
- no double counting
- straightforward implementation

disadvantages:

- spurious terms of $\mathcal{O}(\Gamma/M) = \mathcal{O}(\alpha)$
width in spacelike propagators, complex θ_W
- unitarity broken
unitarity cancellations not spoiled since Ward identities remain valid
→ these are due to truncation of the perturbative series

Complex mass scheme at NLO

complex on-shell renormalization scheme:

[Denner et al. '05]

- mass and field splitting:

$$m_{W,0}^2 = \mu_W^2 + \delta\mu_W^2$$

$$W_0^\pm = (1 + \frac{1}{2}\delta Z_W)W^\pm$$

$\mu, \delta Z$ complex

bare Lagrangian unchanged \rightarrow no double counting

- renormalized self energy:

$$\Sigma_R^T(p^2) = \Sigma^T(p^2) - \delta\mu_W^2 + (p^2 - \mu_W^2)\delta Z_W$$

- renormalization conditions

$$\Sigma_R^T(p^2 = \mu_W^2) = 0 \quad \text{mass}^2 \text{ is complex propagator pole}$$

$$\frac{\partial}{\partial p^2} \Sigma_R^{T'}(p^2 = \mu_W^2) = 0 \quad \text{pole residue is 1}$$

solutions:

$$\delta\mu^2 = \Sigma^T(\mu_W^2) \quad \delta Z_W = -\Sigma^{T'}(\mu_W^2)$$

Complex mass scheme at NLO

expansion around real masses

$$\Sigma^T(\mu_W^2) = \Sigma^T(m^2) + (\mu_W^2 - m_W^2)\Sigma^{T'}(m_W^2) + \mathcal{O}(\alpha^3)$$

$$\delta\mu_W^2 = \Sigma^T(m_W^2) + (\mu_W^2 - m_W^2)\Sigma^{T'}(m_W^2) \quad \delta Z_W = -\Sigma_{T'}(m_W^2)$$

→ renormalized self energy

$$\Sigma_R^T(p^2) = \Sigma^T(p^2) - \delta m_W^2 + (p^2 - m_W^2)\delta Z_W$$

with

$$\delta m_W^2 = \Sigma^T(m_W^2) \quad \delta Z_W = -\Sigma^{T'}(m_W^2)$$

→ same form as usual on-shell scheme, but without Re

width calculation

implicitly defined by $m_{W,0}^2 = \mu_W^2 + \delta\mu_W^2$

$$\rightarrow m_W\Gamma_W = \text{Im}(\Sigma^T(m_W^2)) - m_W\Gamma_W \text{Re}(\Sigma^{T'}(m_W^2)) + \mathcal{O}(\alpha^3)$$

→ iterative solution

but: needs 2-loop self energies

equivalent: width from $W \rightarrow f\bar{f}'$ with real W mass

Complex mass scheme at NLO

charge renormalization

$$\frac{\delta e}{e} = \frac{1}{2} \Sigma'^{AA}(0) - \frac{\sin \theta_W}{\cos \theta_W} \frac{\Sigma_T^{AZ}(0)}{\mu_Z^2}$$

complex due to μ^2 , $\theta_W \rightarrow$ renormalized charge complex

but: $\text{Im}(e)$ drops out at 1-loop, contributes starting at 2-loop

top quark

complex mass $\mu_t^2 = m_t^2 - i m_t \Gamma_t$

renormalization constants

$$\begin{aligned}\delta m_t &= \frac{m_t}{2} \left[\Sigma^{t,R}(m_t^2) + \Sigma^{t,L}(m_t^2) + 2\Sigma^{t,S}(m_t^2) \right] \\ \delta Z_{t,\sigma} &= -\Sigma^{t,\sigma}(m_t^2) - m_t^2 \left[\Sigma'^{t,R}(m_t^2) + \Sigma'^{t,L}(m_t^2) + 2\Sigma'^{t,S}(m_t^2) \right]\end{aligned}$$

→ same form as usual on-shell scheme, but without Re

complex mass scheme at NLO

- needs 1-loop integrals with complex masses
- gauge invariant, no double counting, straightforward implementation
- spurious terms, unitarity violation
 - effects of $\mathcal{O}(\alpha^2)$ in NLO calculation

combination of real and virtual corrections

real and virtual corrections: soft and collinear singularities

→ cancel in inclusive quantities (KLN theorem)

inclusive quantities:

- insensitive to emission of soft photons
- insensitive to emission of photons collinear to charged fermions
- energy fraction of fermion emitting photon in collinear limit $z = \frac{p^0}{p^0 + k^0}$
→ independent of z

real/virtual combination: collinear safe case

- phase space slicing

divide phase space using cuts on photon energy E_γ and photon polar angle $\theta_{\gamma f}$

$$\int_5 d\Gamma^R = \left(\int_{5,\text{hard}} + \int_{5,\text{soft}} + \int_{5,\text{coll}} \right) d\Gamma^R = \int_{5,\text{hard}} d\Gamma^R + \int_4 [d\Gamma^{\text{soft}} + d\Gamma^{\text{coll}}]$$

analytical integration over photon phase space (z integrated out)

→ singular terms: $\log m_\gamma$, $\log m_f$

combination of real and virtual corrections

real/virtual combination: collinear safe case

dipole formalism

[Catani,Seymour '96], [Dittmaier '98]

- process independent approach

$$\sigma^{\text{NLO}} = \int_5 [d\sigma^R - d\sigma^{\text{sub}}] + \int_5 d\sigma^{\text{sub}} + \int_4 d\sigma^V$$

- subtraction function $d\sigma^{\text{sub}}$
sum of dipole terms
cancels singularities of $d\sigma^R$
 $\Rightarrow d\sigma^R - d\sigma^{\text{sub}}$ integrable without IR regularization
- phase space factorization

$$\int d\Phi_5 = \int d\tilde{\Phi}_4 \otimes \int d\Phi_\gamma$$

- analytical integration over singular phase space regions

$$\int_5 d\sigma^{\text{sub}} = \int_4 \int_1 d\sigma^{\text{sub}} = \int_4 G_{\text{sub}} d\sigma^0$$

- result for decay process

$$\Gamma^{\text{NLO}} = \int_5 [\text{d}\Gamma^R - \text{d}\Gamma^{\text{sub}}] + \int_4 [\text{d}\Gamma^V + G_{\text{sub}} \text{d}\Gamma^0]$$

non-collinear safe observables

non-collinear safe observables

- need separate identification of collinear f^\pm and γ
- depend on energy fraction $z = \frac{p^0}{p^0 + k^0}$
→ no analytical integration over z possible
- collinear singularities remain → $\log(m_f)$
→ modification of slicing and dipole methods needed

soft and collinear singularities: non-collinear safe case

[Bredenstein et al '05]

- phase space slicing
keep z integration → performed numerically

$$\int_{5,\text{coll}} d\Gamma^R = \int_4 \int dz d\tilde{\Gamma}^{\text{coll}}$$

- dipole formalism
 - keep information on z in subtraction function
 - additional term, z integration numerically

$$\int_4 \int dz d\Gamma^{\text{sub,coll}}$$

- z not integrated out

higher order corrections

corrections beyond $\mathcal{O}(\alpha)$

- higher order final state radiation

collinear photon emission $\rightarrow \alpha \log(m_f^2/q^2)$

structure function approach \rightarrow resummation of leading-logs

\rightarrow only relevant for non-collinear safe observables

- Higgs boson self interaction

leading 2-loop corrections to HVV vertex

[Ghinculov '95; Frink et al '96]

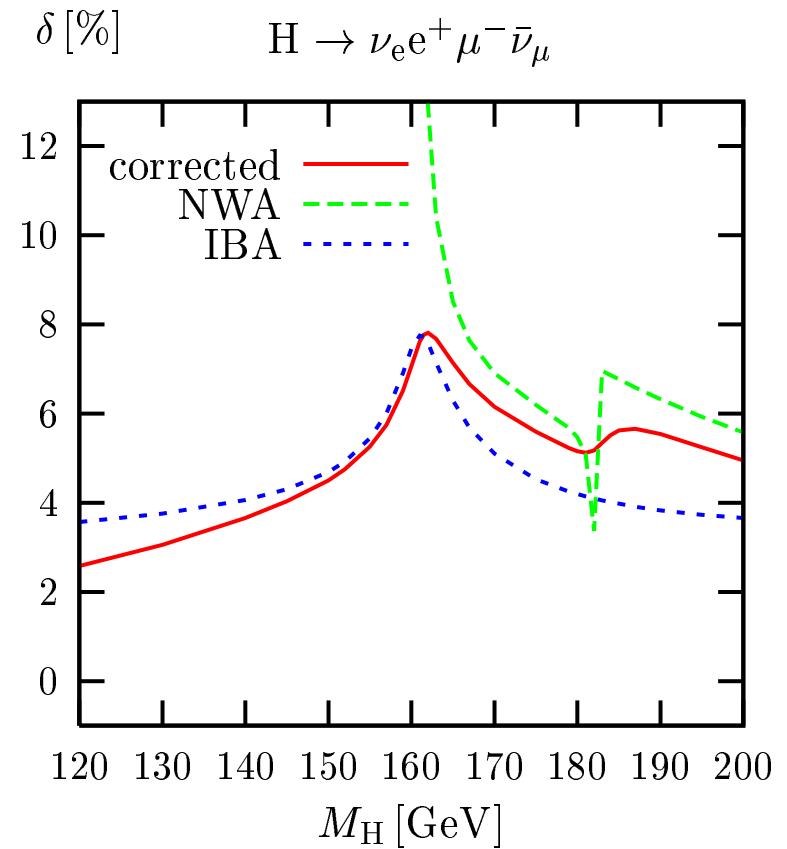
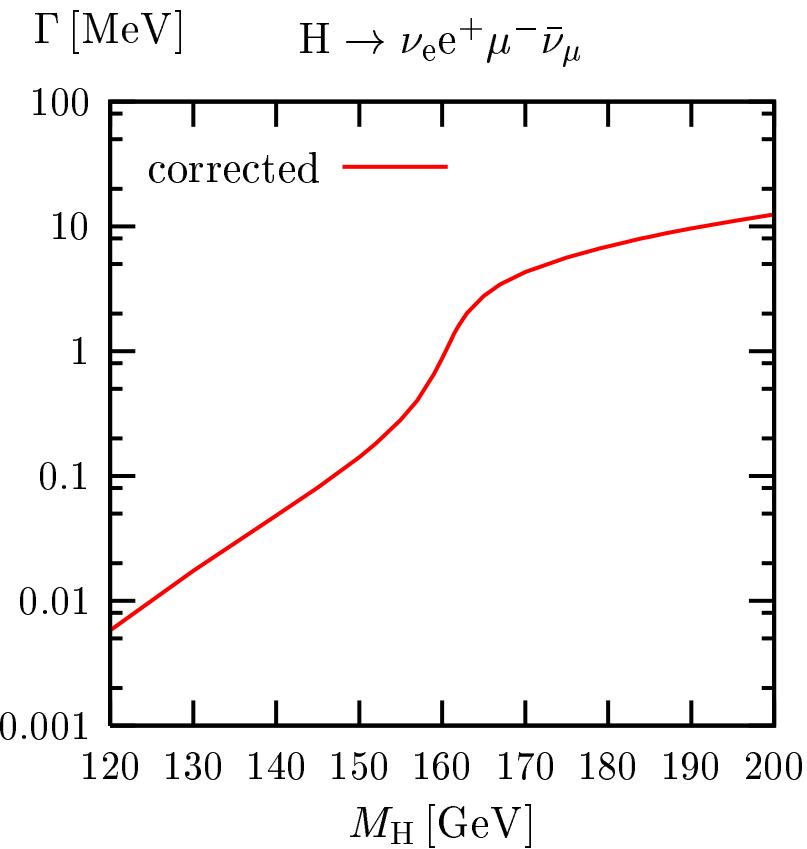
\rightarrow relevant for $m_H \gtrsim 400$ GeV

Checks

- **gauge independence**: 't Hooft-Feynman gauge and background field method
- **UV divergences**: cancel after renormalization
 - no dependence on mass scale μ of dimensional regularization
- **soft singularities**: cancel after real–virtual combination
 - no dependence on $\log m_\gamma$
- **collinear singularities**: drop out in collinear safe observables (e.g. Γ)
 - no dependence on $\log m_f$
- **real corrections**: checked against MADGRAPH
- **combination of real & virtual contributions**: phase space slicing and dipole formalism
- **2 independent calculations**
 - 2 computer codes for numerical evaluation
 - **full numerical agreement (10 digits for $d\Gamma$)**

Partial widths: leptonic

$$H \rightarrow W^{(*)}W^{(*)} \rightarrow \nu_e e^+ \mu^- \bar{\nu}_\mu$$

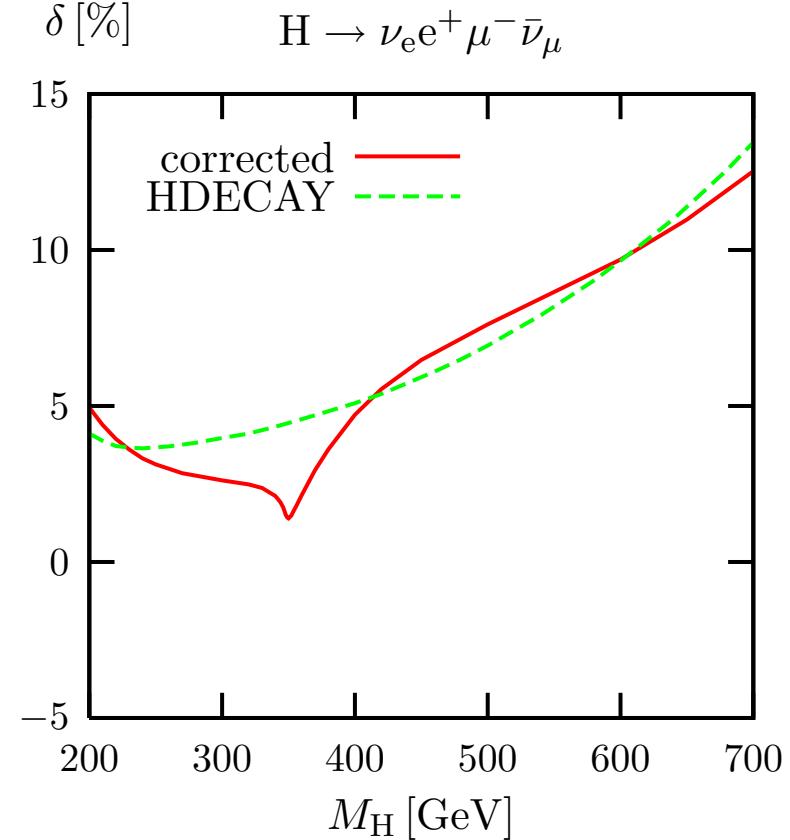
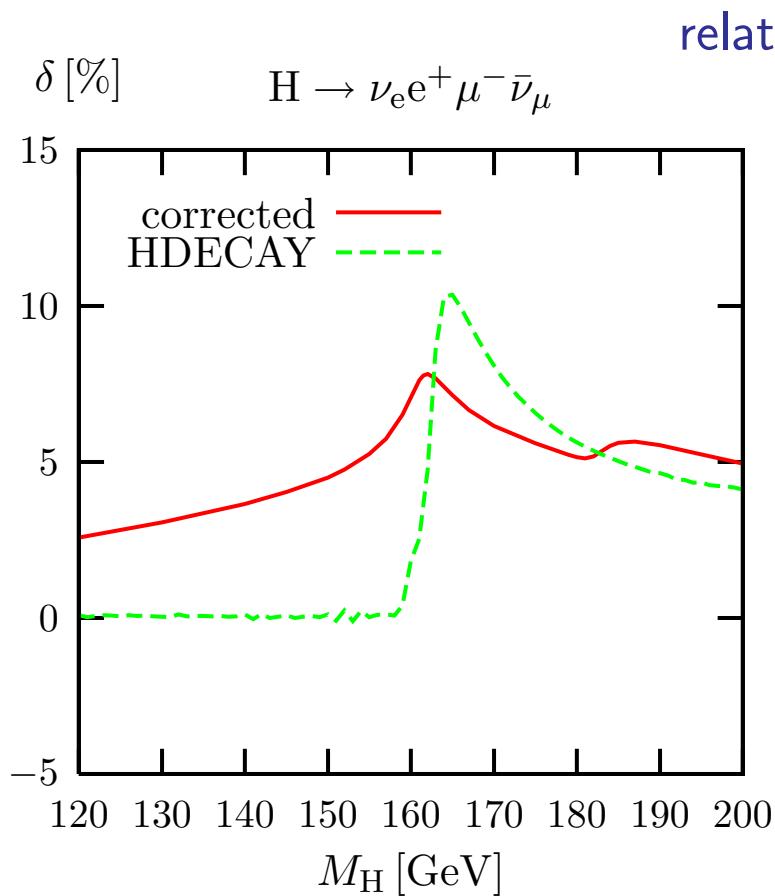


NWA: narrow width approximation = on-shell $H \rightarrow WW$ with $BR(W \rightarrow f\bar{f}')$

IBA: improved Born approximation

Comparison with HDECAY

$$H \rightarrow W^{(*)}W^{(*)} \rightarrow \nu_e e^+ \mu^- \bar{\nu}_\mu$$



HDECAY

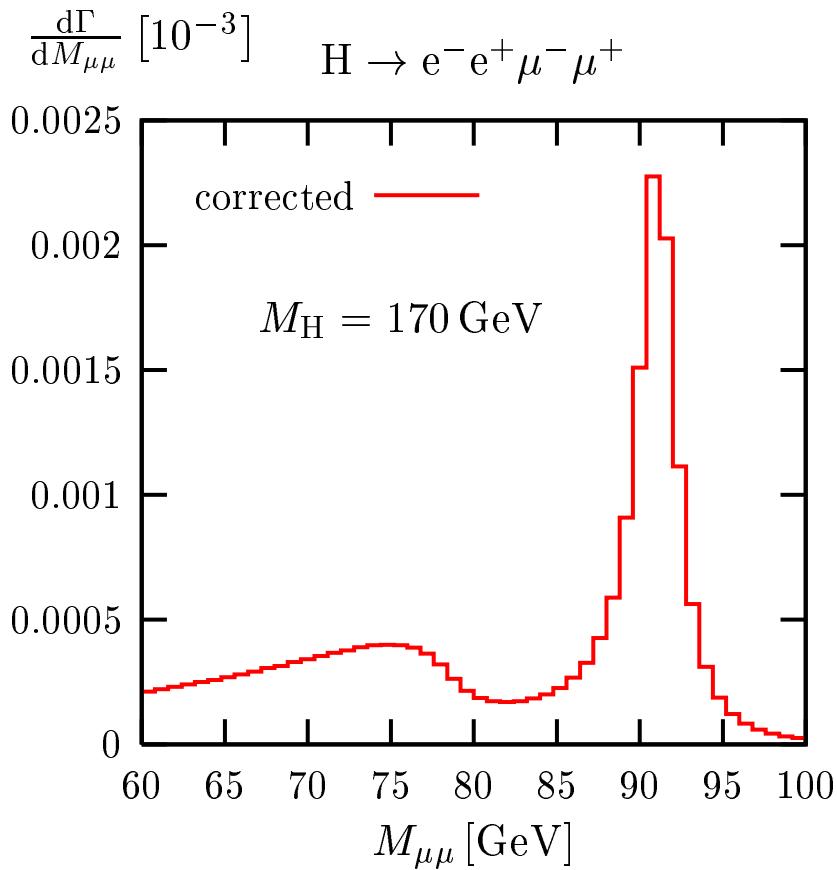
- includes leading 1 and 2-loop corrections for large m_H
- off-shell effects taken into account below threshold

Distributions: invariant mass

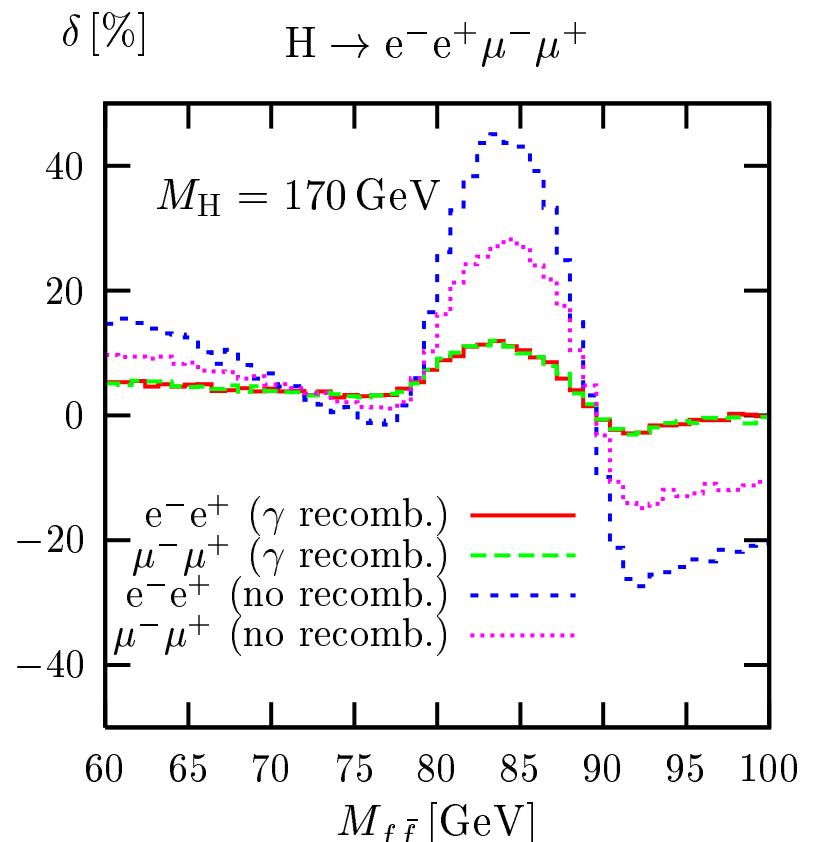
$$H \rightarrow Z^{(*)}Z^{(*)} \rightarrow e^-e^+\mu^-\mu^+$$

$$m_H = 170 \text{ GeV}$$

invariant mass distribution



relative corrections



photon recombination: if $m_{f\gamma} < 5 \text{ GeV}$

→ large corrections from photon recombination in Z reconstruction

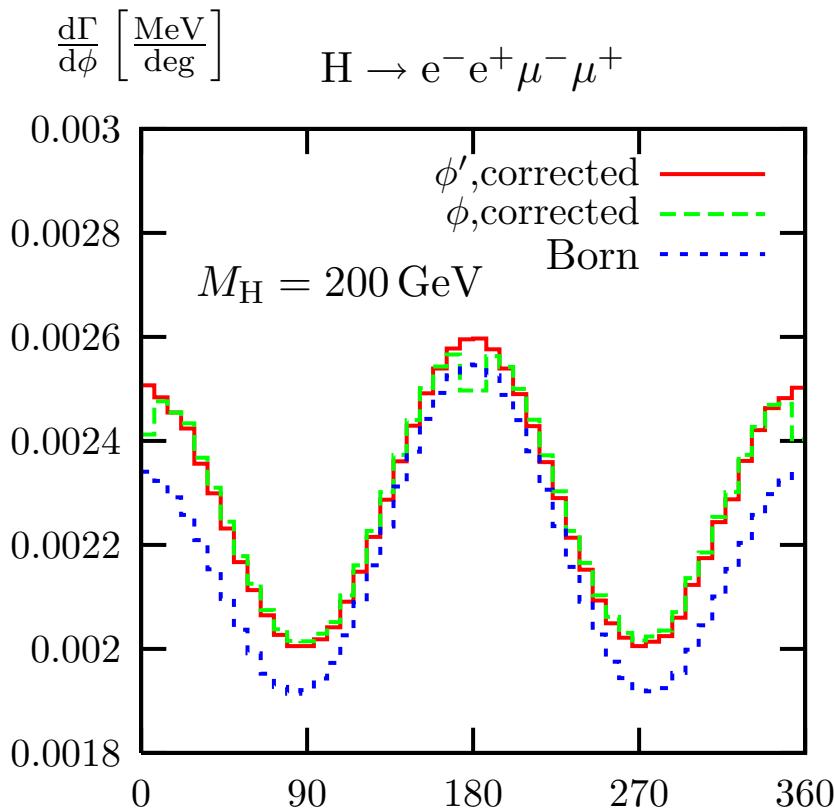
Distributions: angular

$$H \rightarrow Z^{(*)}Z^{(*)} \rightarrow e^-e^+\mu^-\mu^+$$

$$m_H = 200 \text{ GeV}$$

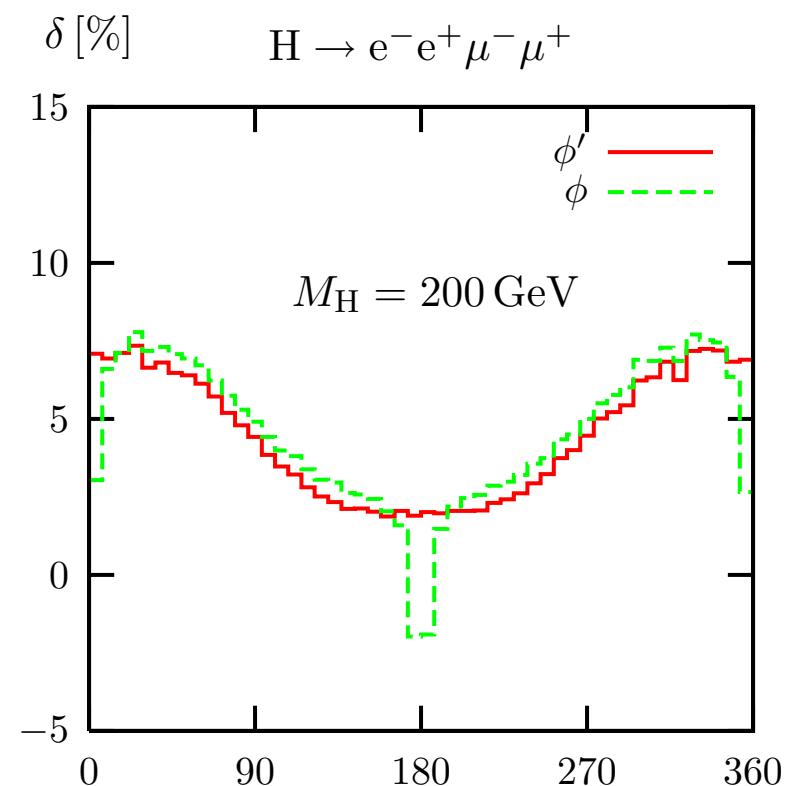
ϕ angle between decay planes of e^+e^- and $\mu^+\mu^-$

angular distribution



$$\cos \phi' = \frac{(\mathbf{p}_+ \times \mathbf{p}_1)(\mathbf{p}_+ \times \mathbf{p}_3)}{|\mathbf{p}_+ \times \mathbf{p}_1| |\mathbf{p}_+ \times \mathbf{p}_3|}$$

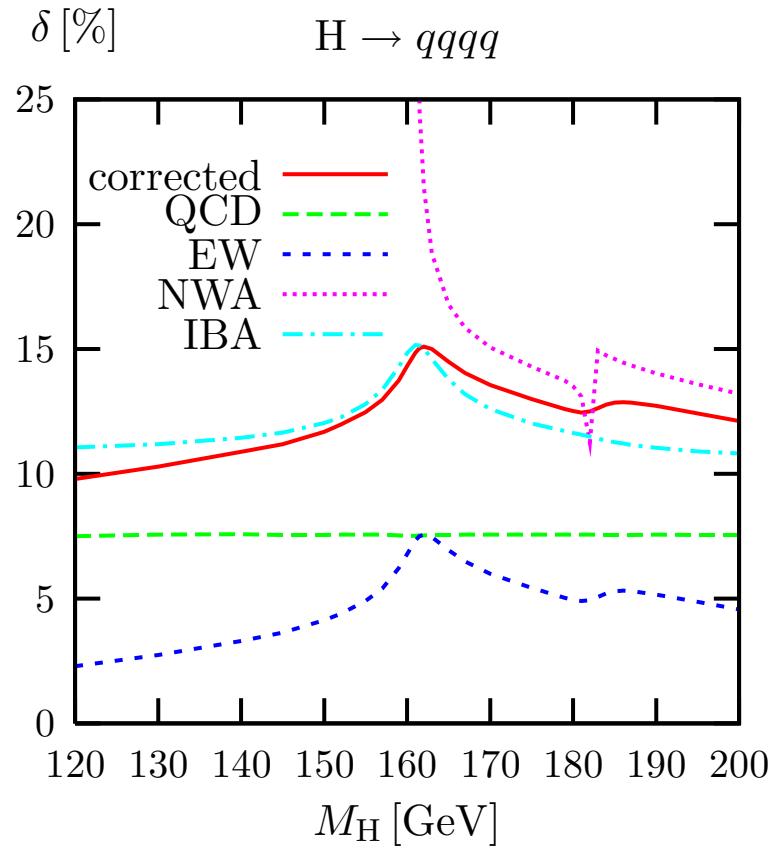
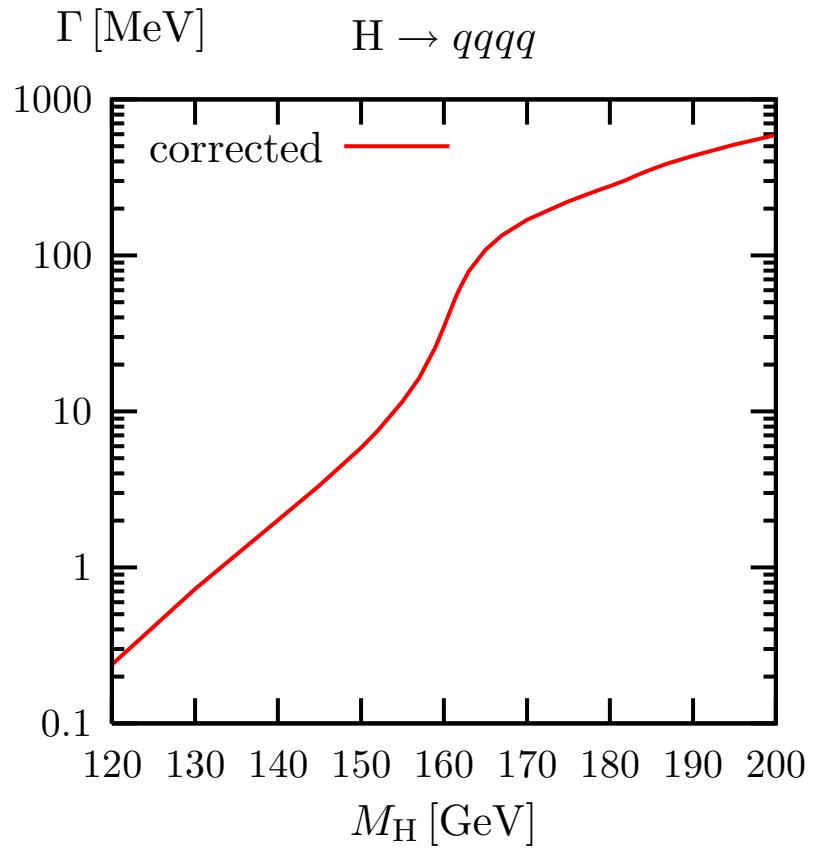
relative corrections



$$\cos \phi = \frac{(\mathbf{p}_+ \times \mathbf{p}_1)(-\mathbf{p}_- \times \mathbf{p}_3)}{|\mathbf{p}_+ \times \mathbf{p}_1| |-\mathbf{p}_- \times \mathbf{p}_3|}$$

Partial widths: hadronic

$H \rightarrow qqqq$

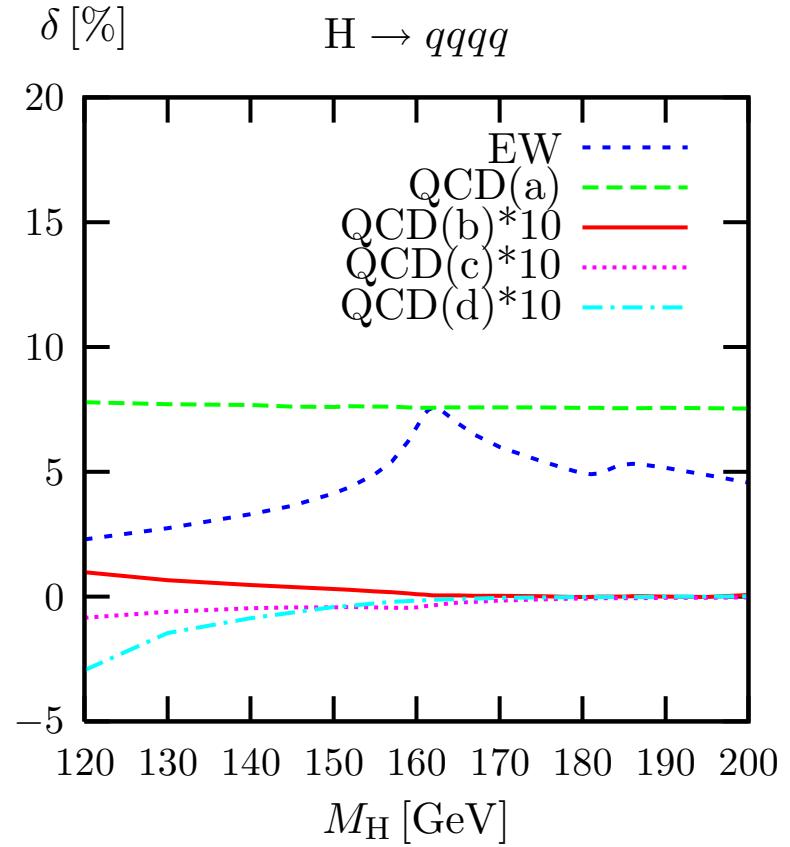
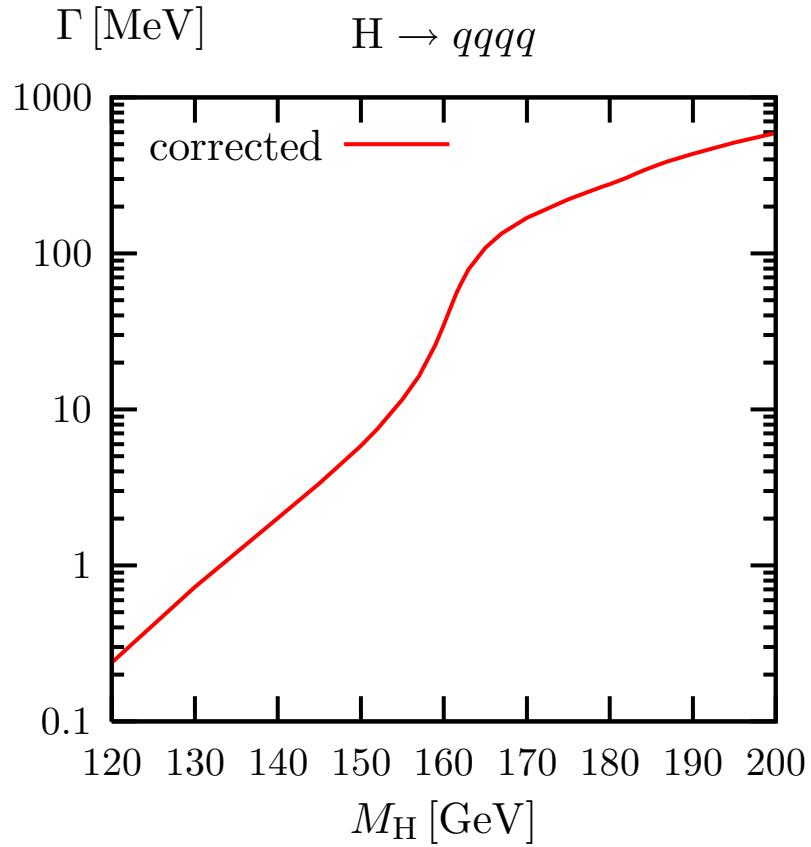


EW corrections: similar for leptonic, semileptonic and hadronic final states

QCD corrections: $\delta_{\text{QCD}}^{\text{semileptonic}} \approx \frac{\alpha_s}{\pi} = 3.8\%$, $\delta_{\text{QCD}}^{\text{hadronic}} \approx \frac{2\alpha_s}{\pi} = 7.6\%$

Partial widths: hadronic

$H \rightarrow qqqq$



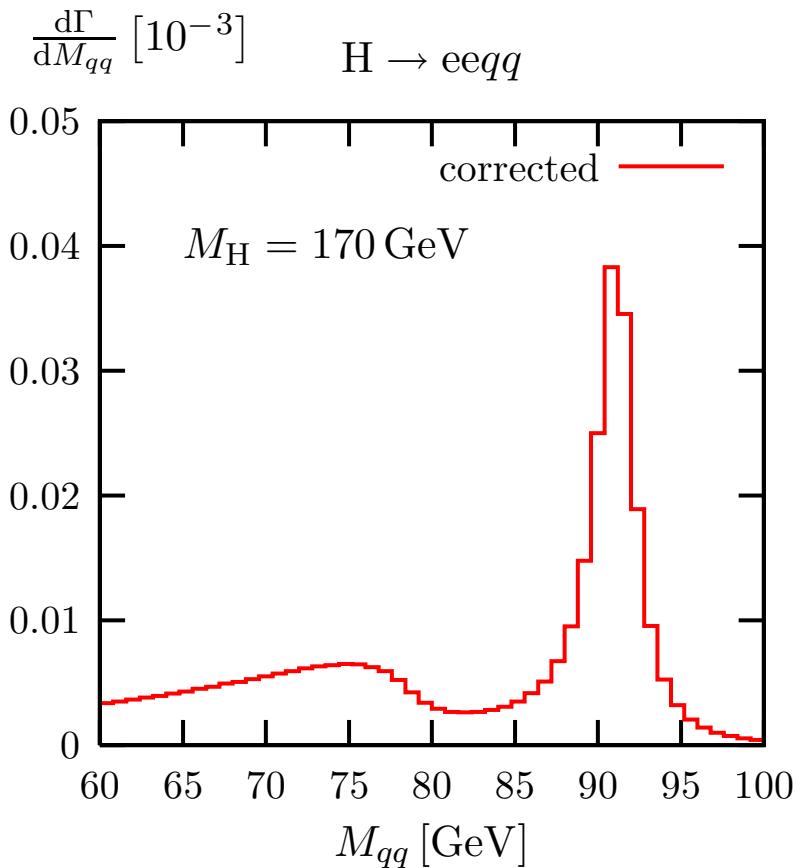
QCD corrections:

- only type (a) relevant = corrections to W/Z decays
- b),c),d): interference corrections $< 0.5\%$

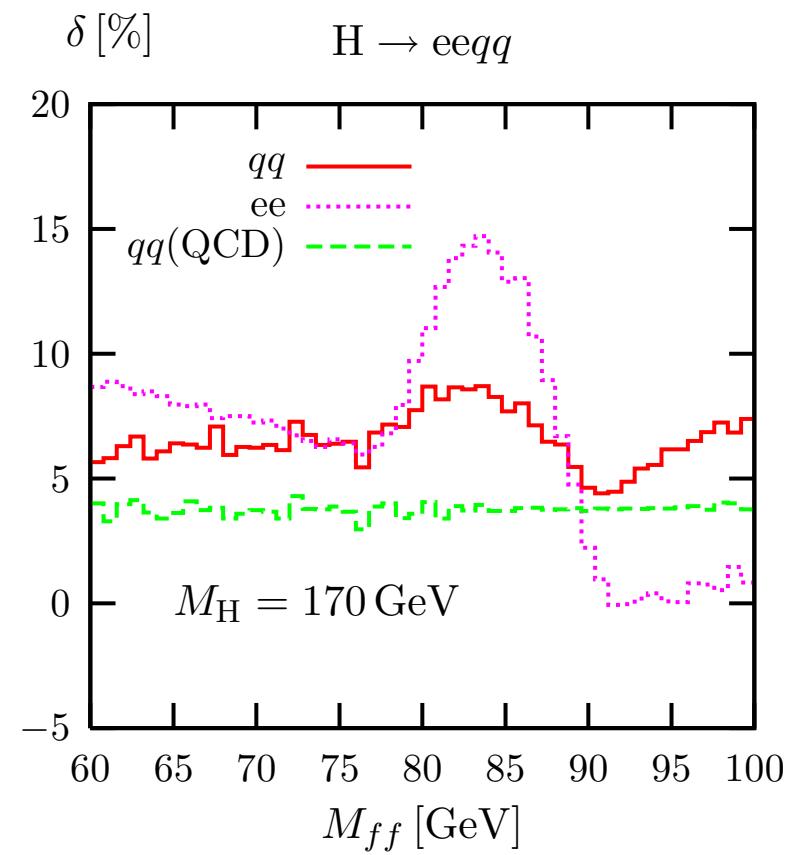
Distributions: invariant mass

$H \rightarrow ee\bar{q}\bar{q}$

invariant mass distribution



relative corrections



gluon recombination: force 2-jet event, recombine 2 partons with smallest m_{jj}^2

Monte Carlo generator: PROPHECY4F

PROPHECY4F = a PROPer description for the Higgs dECaYs into 4 Fermions

Features of Monte Carlo generator PROPHECY4F

- $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha_s)$ calculation of $H \rightarrow WW/ZZ \rightarrow 4f$
 - all final states
 - partial widths and distributions
- non-collinear-safe observables possible
- corrections beyond $\mathcal{O}(\alpha)$
 - higher order final state radiation, large m_H effects
- improved Born approximation for partial widths
 - includes: Coulomb singularity, leading effects for $m_H, m_t \gg m_W$, fitting constant
- phase space integration
 - multi channel Monte Carlo integration [Berends,Kleiss,Pittau '94]
 - up to 10 channels for real corrections
 - adaptive weight optimization [Kleiss, Pittau '94]
 - uses Cuba integration library [Hahn '05]

Unweighted event generation

generation of unweighted events

- using hit-and-miss algorithm
- only with phase space slicing
- currently only for leptonic final states
- no final state radiation
 - higher order FSR: up to 4% (10%) for muons (electrons) in distributions
but only where leading order is small
- fills Les Houches Accord COMMON block for each event
 - user can read out events
- negative weight events: can not be completely avoided
 - are unweighted too
 - but: only few negative events: 0.1% ... 1%

efficiency of unweighted event generation

- 1 unweighted event = $\mathcal{O}(100 - 1000)$ weighted events
- runtime: 5×10^5 unweighted events = 1-2 days (single PC)

Example input file

```
outputfile=out.eemumu.300          ! name of outputfile
nevents=10000000                   ! nevents: number of weighted events
nunwevents=50000                  ! nunwevents: number of unweighted events

contrib=1                          ! contrib: 1=best 2=IBA 3=Born
qqcd=1                            ! qqcd: 0=EW 1=EW+QCD 2=QCD corrections incl.
qsoftcoll=2                        ! qsoftcoll: 1=subtraction 2=slicing

fname(1)=e                         ! fermion 1
fname(2)=anti-e                    ! (anti-)fermion 2
fname(3)=mu                         ! fermion 3
fname(4)=anti-mu                   ! (anti-)fermion 4

qrecomb=0                          ! qrecomb: 0=no recomb., 1=photon recomb.
invrecomb=5d0                      ! recombination condition (for qrecomb=1)

mh      = 300d0                     ! Higgs boson mass
1/alpha0 = 137.03599911d0           ! 1/alpha(0)
alphas   = 0.118d0                  ! strong coupling constant
gf      = 1.16639d-5                ! Fermi constant
mz      = 91.187d0                  ! Z boson mass
mw      = 80.41d0                   ! W boson mass
me      = 0.51099892d-3             ! electron mass
mmu     = 105.658369d-3            ! muon mass
mtau    = 1.77699d0                 ! tau mass
md      = 0.190d0                   ! d-quark mass
mu      = 0.190d0                   ! u-quark mass
ms      = 0.190d0                   ! s-quark mass
mc      = 1.4d0                     ! c-quark mass
mb      = 4.6d0                     ! b-quark mass
mt      = 172d0                     ! t-quark mass
```

Example output file

Input parameters:

```
-----  
1/alpha(0) = 1/137.03599911           alphas = 0.1180  
GF = .1166390E-04 GeV^-2  
MW = 80.41000 GeV        MZ = 91.18700 GeV        MH = 300.00000 GeV  
me = 0.51100 MeV        mmu = 0.10566 GeV        mtau = 1.77699 GeV  
mu = 0.19000 GeV        mc = 1.40000 GeV        mt = 172.00000 GeV  
md = 0.19000 GeV        ms = 0.19000 GeV        mb = 4.60000 GeV
```

Derived parameters:

```
-----  
1/alpha = 1/132.44307185  
GW = 2.09130 GeV        GZ = 2.49866 GeV        gt = 1.46541 GeV
```

Options:

```
-----  
Process: H -> e anti-e mu anti-mu (+Photon/Gluon)
```

```
number of weighted events = 10000000  
number of unweighted events = 50000
```

```
contrib = 1 : Born cross section + radiative corrections  
qqcd = 1 : EW+QCD corrections  
qsoftcoll = 2 : slicing method
```

```
qrecomb = 0 : no photon recombination applied
```

```
qcuts = 0 : no separation cuts
```

```
DeltaE = 0.4E-02 : Slicing energy cut  
DeltaTh = 0.3E-01 : Slicing angular cut
```

Example output file

Narrow width approximation (incl EW+QCD):

Partial width:	5.97660	MeV
Born:	5.82024	MeV (NLO gauge-boson width)
Correction:	0.156355	MeV

Results:

Average	=	5.8390875572 MeV
Standard deviation	=	0.0034318569 MeV

Subcontributions:

Born cross section (NLO gauge-boson width):

Average	=	5.6660697095 MeV
Standard deviation	=	0.0015014548 MeV
largest event	=	0.0000048078 MeV
event number		3054, channel 4

EW Virtual corrections:

Average	=	-1.2061697571 MeV
Standard deviation	=	0.0018532689 MeV
largest event	=	0.0000215364 MeV
event number		3091, channel 4

Photon Bremsstrahlung:

Average	=	1.3791876048 MeV
Standard deviation	=	0.0026299759 MeV
largest event	=	0.0000864870 MeV
event number		4455, channel 8

Example output file

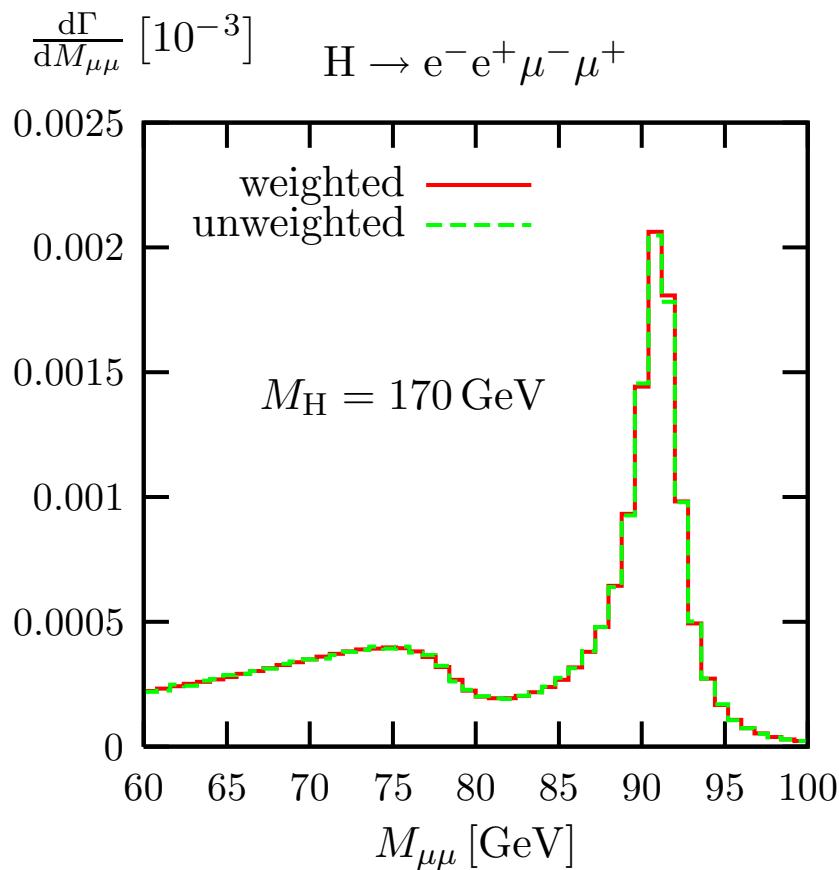
Unweighted events:

```
-----  
generated 50090 unweighted events (including negative events)  
by generating weighted events 11048670 times  
number of weights larger than largest weight from 1st cycle: 0  
number of negative unweighted events: 45  
maximal weight used for generating unweighted events: 1297.3
```

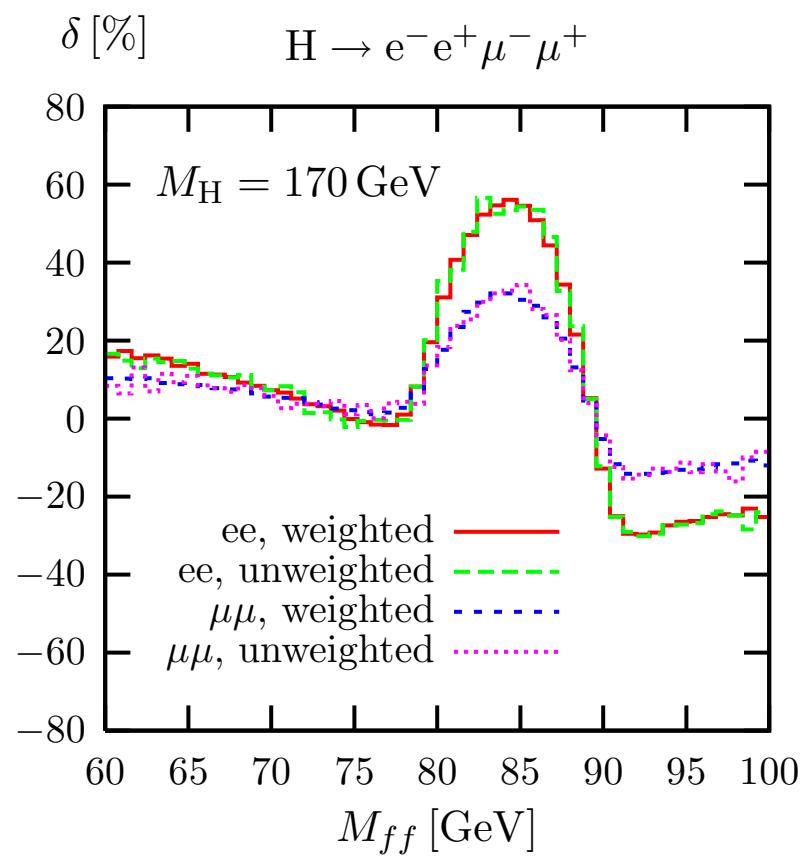
Unweighted events

$$H \rightarrow Z^{(*)}Z^{(*)} \rightarrow e^-e^+\mu^-\mu^+$$

invariant mass distribution



relative corrections



no photon recombination

5×10^7 weighted events

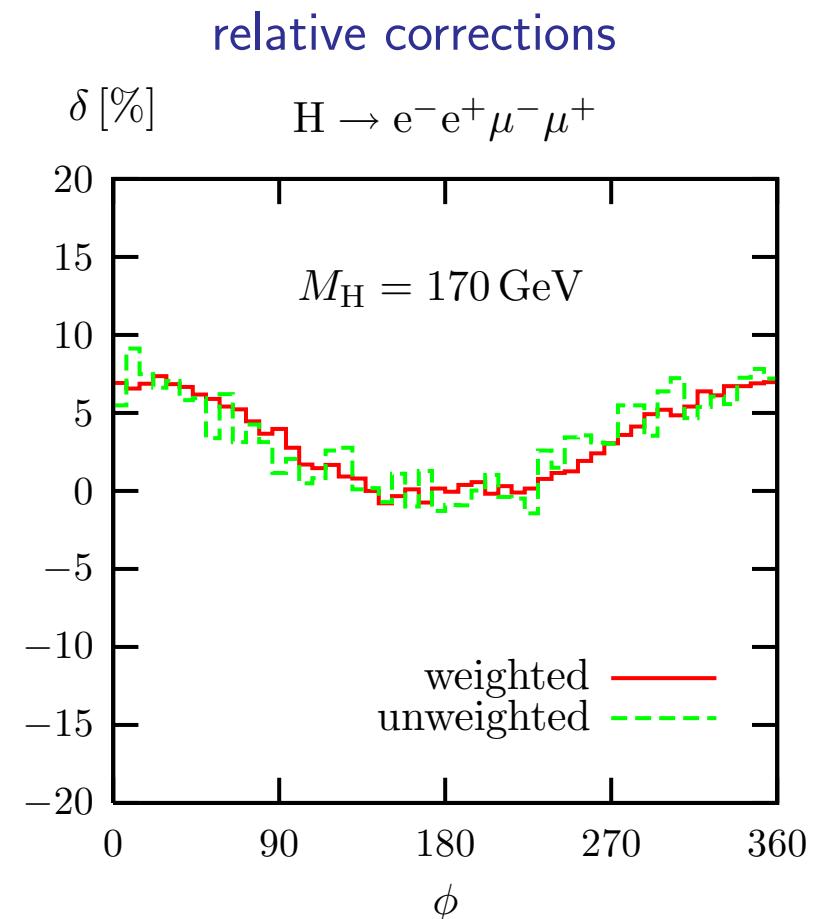
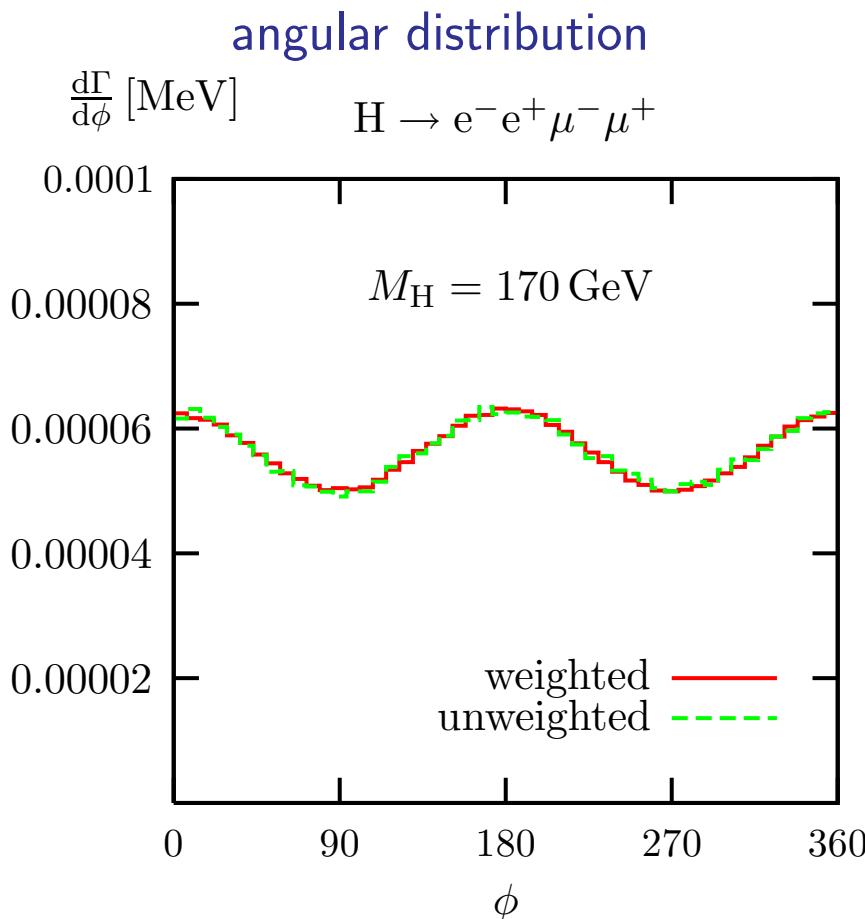
5×10^5 unweighted events

runtime: 1-2 days (single PC)

Unweighted events

$$H \rightarrow Z^{(*)}Z^{(*)} \rightarrow e^-e^+\mu^-\mu^+$$

ϕ angle between decay planes of e^+e^- and $\mu^+\mu^-$



5×10^7 weighted events
 5×10^5 unweighted events
runtime: 1-2 days (single PC)

$$\cos \phi = \frac{(p_+ \times p_1)(p_+ \times p_3)}{|p_+ \times p_1| |p_+ \times p_3|}$$

Conclusions

$H \rightarrow WW/ZZ \rightarrow 4f$ important decay channel

- discovery and mass measurement at LHC
- distributions important for verification of Higgs properties (spin,CP)

PROPHECY4F: Monte Carlo generator for $H \rightarrow WW/ZZ \rightarrow 4f$

- complete $\mathcal{O}(\alpha)$ electroweak and $\mathcal{O}(\alpha_s)$ QCD corrections
gauge boson resonances: complex mass scheme
- universal beyond $\mathcal{O}(\alpha)$ corrections:
heavy-Higgs effects and final state radiation
- non-collinear safe observables possible
- weighted and unweighted event generation
- (almost) publically available

results

- **partial width** EW corrections up to $\simeq 8\%$ for $m_H \lesssim 500 \text{ GeV}$
improved born approximation: accurate to within $\lesssim 2\%$ for $m_H \lesssim 500 \text{ GeV}$
- **distributions**
EW corrections $\mathcal{O}(10\%)$ with γ -recombination (depending on γ recombination)
- **QCD corrections**
associated with W/Z decay