Reheating Metastable O'Raifeartaigh Models

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Layout

- Ancient History
- Metastability and ISS
- Finite temperature ISS
- Classic O’Raifeartaigh models
- Conclusions
Usual SUSY breaking scenario

DSB → Messengers → MSSM
Usual SUSY breaking scenario

Concentrate on this
What do we know about DSBs?

• Affleck, Dine, Seiberg: Spontaneously broken non-anomalous global symmetry *but no flat direction* $\Rightarrow$ SUSY *probably* broken

• Nelson, Seiberg: If symmetry is $U(1)_R$ and general superpotential SUSY *is* broken

• Witten: $\mathrm{Tr}((-1)^F) \neq 0 \Rightarrow$ SUSY *not* broken

Witten index of SU(N) is N, one is lead to chiral theories for DSBs.
A typical example: (Dine, Nelson, Shirman)

$$SU(2N + 5) : (2N + 1) \Box,$$

$$W = \lambda_{ab} \overline{F}^a A \overline{F}^b + W_{ADS}$$

Examples:

3,2 model; 4,1 model; SU(5) model
ISS ask...

Do theories with (global) SUSY vacua also have (local) non-SUSY vacua?

Are they long lived?

If such theories exist are they common?

(Intriligator, Seiberg and Shih; JHEP 0604:021, 2006)
ISS ask...

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YES!
ISS ask...

Do theories with (global) SUSY vacua also have (local) non-SUSY vacua?

Are they long lived?

If such theories exist are they common?

YES!  SQCD!
$N=1$ SQCD, a review

$SU(N_C)$ with $N_F$ flavours ($N_C < N_F < \frac{3}{2}N_C$)

$Q \sim (\Box_{N_C}, \Box_{N_F})$ \quad $Q^c \sim (\Box_{N_C}, \Box_{N_F})$

$W_{el} = m \; \text{tr} Q^c Q$

Seiberg duality

IR, magnetic theory \quad UV, electric theory
Magnetic theory

$SU(N)$ with $N_F$ flavours ($N = N_F - N_C$)

$q \sim (\square_N, \overline{\square}_{N_F}) \quad q^c \sim (\overline{\square}_N, \square_{N_F}) \quad M = (1, \square_{N_F}, \overline{\square}_{N_F})$

\[
W_{mag} = y \ tr \ qMq^c - \mu^2 \ trM
\]

\[
K = q^\dagger q + q^{c\dagger}q^c + M^\dagger M + \mathcal{O}\left(\frac{(q^\dagger q)^2}{\Lambda^2}\right)
\]
Magnetic theory

$SU(N)$ with $N_F$ flavours ($N = N_F - N_C$)

$q \sim (\square_N, \Box_{N_F}) \quad q^c \sim (\Box_N, \square_{N_F}) \quad M = (1, \square_{N_F}, \Box_{N_F})$

$$W_{mag} = y \tr q M q^c - \mu^2 \tr M$$

$$\mu^2 = m \Lambda$$

$$K = q^\dagger q + q^{c\dagger} q^c + M^\dagger M + \mathcal{O} \left( \frac{(q^\dagger q)^2}{\Lambda^2} \right)$$
Magnetic theory

$SU(N)$ with $N_F$ flavours ($N = N_F - N_C$)

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$W_{mag} = y \ tr qMq^c - \mu^2 \ tr M$

$\mu^2 = m \ \Lambda$

$K = q^\dagger q + q^{c\dagger}q^c + M^\dagger M + \mathcal{O}\left(\frac{(q^\dagger q)^2}{\Lambda^2}\right)$

Kahler corrections will be small if $\mu \ll \Lambda \Rightarrow m \ll \Lambda$
SUSY breaking by rank condition

\[ F_{M_i^j} = y q_i^a q_a^c j - \mu^2 \delta_i^j \]

\[ F_{q_i} = y M_i^j q_c^j \]

SUSY is broken by rank condition

\[ \langle M \rangle = 0 \quad \langle q \rangle \sim \mu \mathbb{1}_N \sim \langle q^c \rangle \]

\[ F_M \sim \mu^2 \]

What about the Witten index?
SUSY breaking by rank condition

\[ F_{M^j_i} = y q_i^a q_a^{c_j} - \mu^2 \delta^j_i \]

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SUSY is broken by rank condition

\[ \langle M \rangle = 0 \quad \langle q \rangle \sim \mu \mathbb{1}_N \sim \langle q^c \rangle \]

\[ F_M \sim \mu^2 \quad \text{e.g.} \quad \mu \sim 100 - 1000 \text{TeV} \]

What about the Witten index?
Below scales $\langle M \rangle$ the magnetic quarks decouple

Strong dynamics at $\Lambda_{mag}$ generates non-perturbative contribution to the potential

$$W_{IR} = \mu^2 \, \text{tr} M + y \, \text{tr} q M q^c + (\det M)^{1/N} \, \Lambda^{-a}$$

$$a = \frac{N_F}{N} - 3 \quad (> 0)$$

Affleck, Dine, Seiberg
Below scales $\langle M \rangle$ the magnetic quarks decouple

$\Lambda_{mag}$

Strong dynamics at $\Lambda_{mag}$ generates non-perturbative contribution to the potential

$W_{IR} = \mu^2 \text{tr}M + y \text{tr}qMq^c + (\det M)^{1/N}\Lambda^{-a}$

Affleck, Dine, Seiberg

Non-renormalizable op. restores SUSY

$a = \frac{N_F}{N} - 3 \ (> 0)$
Non-perturbative restoration of SUSY

\[ M \sim \eta \mathbb{1} \Rightarrow W \sim -\mu^2 + \eta^{3+a} \Lambda^{-a} \]

- F-term equations now have a solution, SUSY is unbroken
- SUSY preserving minimum is far out in field space,
  \[ \langle M \rangle = \mu \left( \frac{\Lambda}{\mu} \right)^{\frac{a}{2+a}} \mathbb{1}_{N_F} \]

\[ (\mu^2 \Lambda^a)^{\frac{1}{2+a}} \]
Cartoon of the potential

\[ V_0 = N \Lambda^4 \left| \left( \frac{\eta}{\sqrt{N_F} \Lambda} \right)^{2+a} - \frac{\mu^2}{\Lambda^2} \right|^2 \]
Cartoon of the potential

\[ V_0 = N \Lambda^4 \left| \left( \frac{\eta}{\sqrt{N_F \Lambda}} \right)^{2+a} - \frac{\mu^2}{\Lambda^2} \right|^2 \]

Analytic form known for square potential

\[ \Gamma \sim \mu^4 e^{-S_4} \]
Tunneling rate

\[ \Gamma \sim \mu^4 e^{-S_4} \] with \[ S_4 \sim \frac{\Delta \eta^4}{\Delta V} \sim \left( \frac{\Lambda}{\mu} \right)^{\frac{4a}{2+a}} \]

Would like \( 1/\Gamma > 14 \) Gyr!

\[
\frac{a}{a+2} \log \frac{\Lambda}{\mu} > 0.73 + 0.003 \log \frac{\mu}{\text{TeV}} + 0.25 \log N
\]

i.e. \( \left( \frac{\Lambda}{\mu} \right)^{\frac{a}{2+a}} \geq 2 \)

Conclusion I:
- **SQCD** is a viable, **simple** DSB; very generic
- **If** we start in the SUSY breaking vacuum **and** \( \frac{\mu}{\Lambda} \ll 1 \)
  we stay there for a long time.
PART II

• After inflation the visible universe was hot
• The DSB may well have been hot too
• \( T \gg \mu \) barrier becomes unimportant
• Minimum of potential is at neither SUSY preserving nor SUSY breaking minimum
• Do we end up in the right place?
Finite temperature field theory

Calculate one loop free energy: \( F = E - T S \)

\[
F \sim \begin{cases} V_0(\eta) & T \ll y\eta \\ V_0(\eta) - T^4 + y^2\eta^2T^2 & T \gg y\eta \end{cases}
\]

Entropy of light states \( \rightarrow \) Thermal mass term

In ISS as \( T \rightarrow \Lambda \) origin becomes minimum

As universe cools what are the order and temperature of the phase transitions in each direction?
1: Quark direction \( q = q^c = \frac{1}{\sqrt{2N}} (\xi 1_N 0) \)

\[
V_0 = N \left( \frac{y}{N^2} \xi^2 - \mu^2 \right)^2 \rightarrow V_0 - c_0 N F T^4 + (c_1 g^2 + c_2 y^2) N \xi^2 T^2
\]

Second order phase transition

\[
T_c \sim \frac{\mu}{\sqrt{yN}}
\]
2: Meson direction \( M = \frac{\eta}{\sqrt{N_F}} \mathbb{I} \)

\[
V_0 = N \Lambda^4 \left| \left( \frac{\eta}{\sqrt{N_F} \Lambda} \right)^{2+a} - \frac{\mu^2}{\Lambda^2} \right|^2
\]

\[
V \sim \begin{cases} 
\mu^4 + c_1 y^2 N \eta^2 T^2 - c_0 (N N_F + N^2) T^4 & T \geq \Lambda_m(\eta) \\
V_0 + c_1 y^2 N \eta^2 T^2 - c_0 N N_F T^4 & T \geq y \eta \\
V_0 & T < y \eta
\end{cases}
\]

First order phase transition

\[
T_c \sim \frac{\mu}{(N_F+N)^{1/4}}
\]
Tunneling time

- Use square approximation, now at finite temperature, $\Gamma \sim T^4 e^{-S_3/T}$

$$S_3 \sim \left( \frac{\Lambda}{\mu} \right)^{\frac{3a}{2+a}}$$

- Want universe still in false vacuum, $\Gamma(T)a^3(T)V\Delta t \approx 0$

$$\frac{a}{a+2} \log \frac{\Lambda}{\mu} \gtrsim 0.64 - 0.001 \log \frac{\mu}{\text{TeV}} + 0.17 \log N$$
Second order phase transition, $T_c \sim \mu$

First order phase transition, $T_c \sim \mu$ lifetime controlled by $\frac{\Lambda}{\mu}$

1: $q \neq 0, M = 0$
2: $q = 0, M \neq 0$

$\left(\mu^2 \Lambda^a\right)^{\frac{1}{2+a}}$
Second order phase transition, $T_c \sim \mu$

First order phase transition, $T_c \sim \mu$, lifetime controlled by $\frac{\Lambda}{\mu}$

If you satisfy the zero-temperature lifetime requirement of ISS then you automatically will get the correct cosmology.

$1: q \neq 0, M = 0$

$2: q = 0, M \neq 0$

$\left(\mu^2 \Lambda^a\right)^{\frac{1}{2+a}}$
“Classic” O’Raifeartaigh Models

\[ W = m \psi \psi^c + \lambda Z (\psi^2 - \mu^2) \quad (m \gg \mu) \]

Origin is minimum and SUSY is broken, \( F_Z = \lambda \mu^2 \)

Using “retro-fitting” to explain the small scale \( \mu \) leads to

\[ W = m \psi^c \psi + \lambda Z (\psi^2 - \mu^2) + \frac{1}{2} \epsilon \mu Z^2 \]

A new SUSY preserving minimum develops at large field value, \( Z_{susy} = \epsilon^{-1} \mu \)
“Classic” O’Raifeartaigh Models

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Origin is minimum and SUSY is broken, \( F_Z = \lambda \mu^2 \)

Using “retro-fitting” to explain the small scale \( \mu \) leads to

\[ W = m \psi^c \psi + \lambda Z (\psi^2 - \mu^2) + \frac{1}{2} \epsilon \mu Z^2 \]

\[ \Delta V \sim \frac{\lambda}{16\pi^2} \mu^4 \]

\[ S_4 \sim \left( \frac{\lambda^2}{16\pi^2} \right)^3 \epsilon^{-4} \]
\( T \gg m : \) Origin is stable

\( \mu < T < m : \) Origin no longer stable, barrier very small

\[ T \neq 0 \]

\[ S_3/T \sim \left( \frac{\lambda}{4\pi} \right)^5 \epsilon^{-3} \frac{\mu}{m} \]

No extra light states at the origin: additional finite temperature constraint
$T \gg m$ : Origin is stable

$\mu < T < m$ : Origin no longer stable, barrier very small

$T \neq 0$

$\Delta V \sim \frac{\lambda}{16\pi^2} \mu^4$

$S_3 / T \sim \left( \frac{\lambda}{4\pi} \right)^5 \epsilon^{-3} \frac{\mu}{m}$

No extra light states at the origin: additional finite temperature constraint
Thermalization

ISS like models quickly thermalize

• e.g. Imagine after inflation DSB is cold, $T \ll H$
• Oscillations of $Q \sim H$ dominate energy density
• Quickly dampened by particle production at origin

L. Kofman et al.
JHEP5 (2004) 030

• True also in magnetic theory
Conclusions

• SQCD provides a simple, generic DSB sector

• Provided there is a hierarchy ($m \ll \Lambda$) of scales our vacuum is long lived, at zero temperature

• Starting at the origin there are two phase transitions to the two minima. Because of the extra light states at the origin it evolves to the correct minimum

• No (parametrically) different constraints, from thermal history

• Retro-fitted O’Raifeartaigh models do not satisfy these conditions, there is no entropic cost to moving away