Choosing the Factorization Scale in Perturbative QCD

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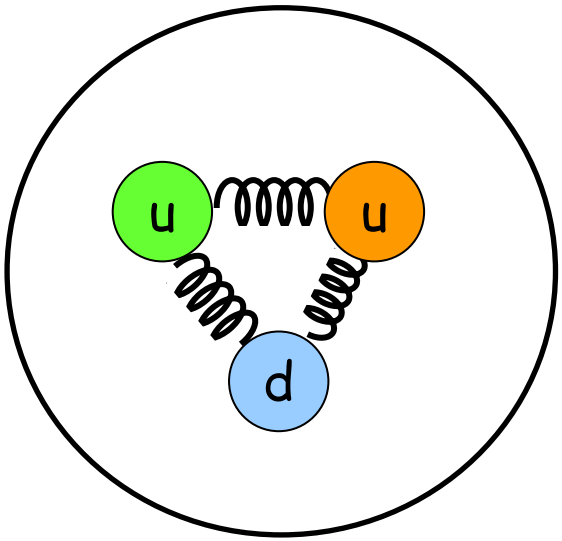
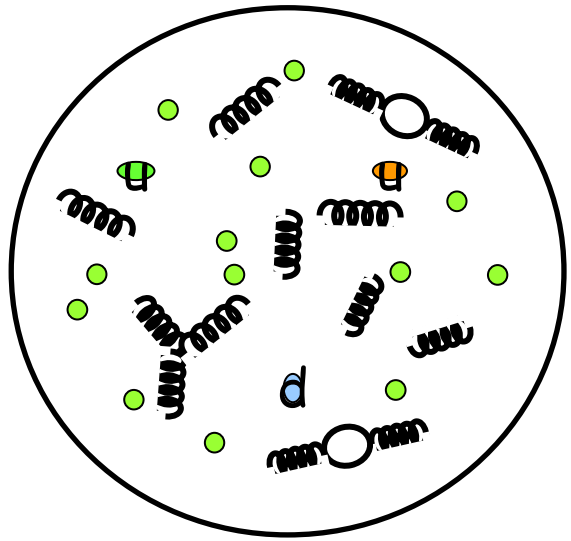
Inside the proton…

Quark Model view

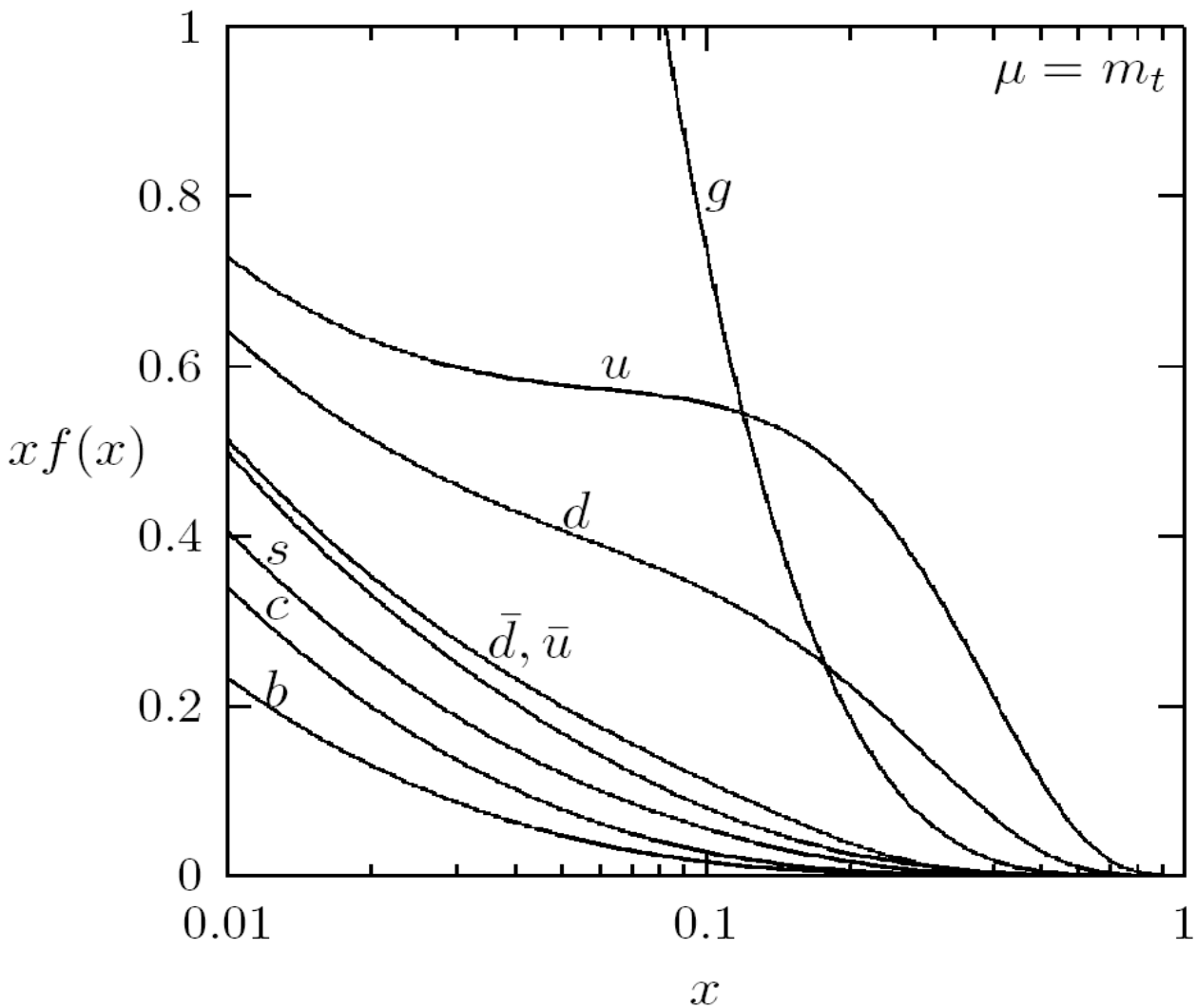


QCD view

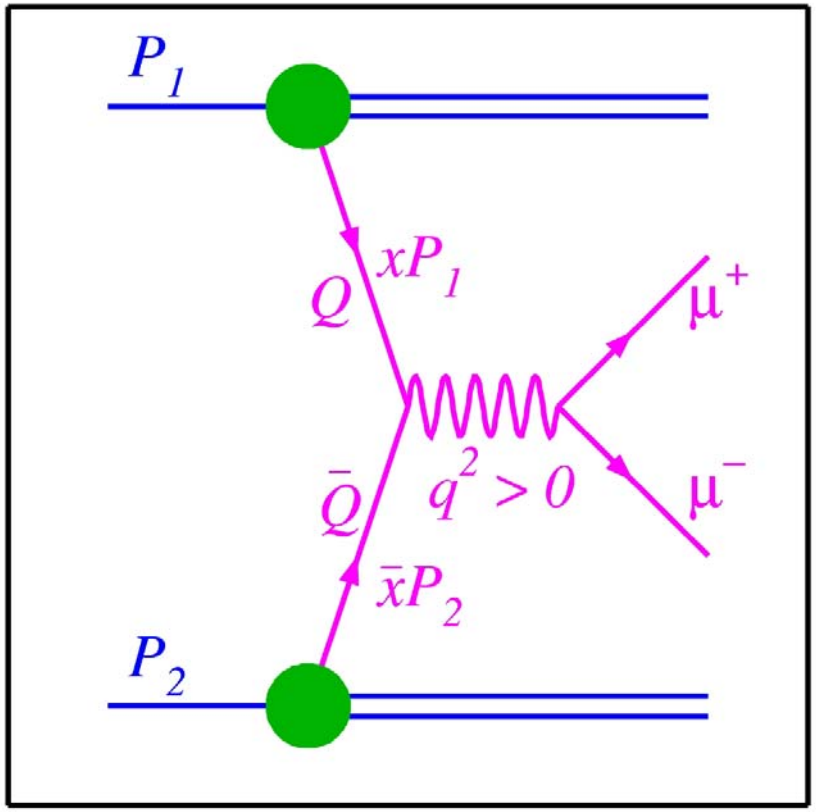


a strongly interacting
many-body bound state

Parton Distribution Functions (PDF)



Drell-Yan Process



$x = x_1$

$\bar{x} = x_2$

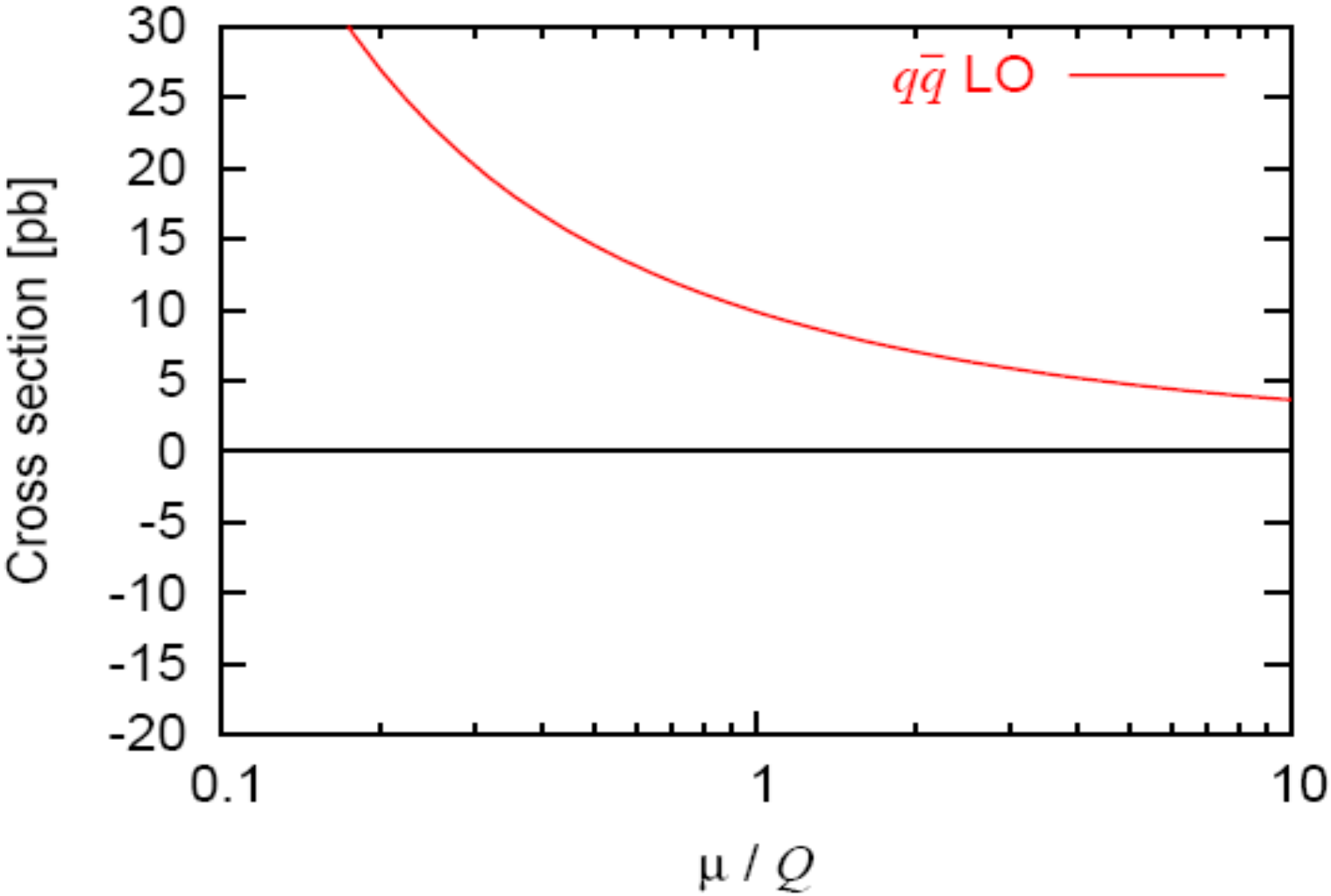
$$\sigma = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 \, f_i(x_1, \mu) f_j(x_2, \mu) \hat{\sigma}_{ij}(x_1, x_2, \mu)$$

μ is the factorization scale $\mu \sim Q = \sqrt{q^2}$

The factorization scale

- The hadronic cross section σ, calculated to all orders in pQCD, is independent of μ,
- but fixed-order results depend on μ.
- This dependence is often significant at LO and NLO, and is reduced at higher orders.
- When higher-order calculations are not available, we are left with a theoretical prediction that depends strongly on μ.

$$pp \to \gamma^*, Q = 20 \text{ GeV}, \sqrt{S} = 38.8 \text{ GeV}$$



How to deal with μ-dependence?

- When possible, calculate higher-order corrections!
- Otherwise we must accept a large theoretical uncertainty due to scale dependence
  - (and this uncertainty may be difficult to estimate)
- or try to choose μ in such a way that we can trust our results.
  - Approaches such as FAC and PMS focus on the mathematical properties of the perturbation series.

    G. Grunberg (1980)
    P. M. Stevenson, H. D. Politzer (1986)
  - Let's try to think about the collinear physics.

Collinear singularities

- Collinear divergences from radiative corrections must be absorbed into the PDFs.
- Choosing a factorization scheme/scale amounts to defining how to separate the infinite and finite pieces.
- Let's try to make our counterterms correspond as closely as possible to the collinear physics.
- Sources of inspiration:

  J. Collins (1990)

  T. Plehn (2003)

  E. Boos, T. Plehn (2004)

  F. Maltoni, Z. Sullivan, S. Willenbrock (2003)

  J. Alwall, J. Rathsman (2004)

DGLAP

The evolution of the PDFs from one factorization scale to another is governed by an analogue of a Renormalization Group equation which are called the DGLAP equations:

$$\frac{d}{d\log\mu^2}q_i(x,\mu) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[g\left(\frac{x}{z},\mu\right)P_{qg}(z) + q_i\left(\frac{x}{z},\mu\right)P_{qq}(z)\right]$$

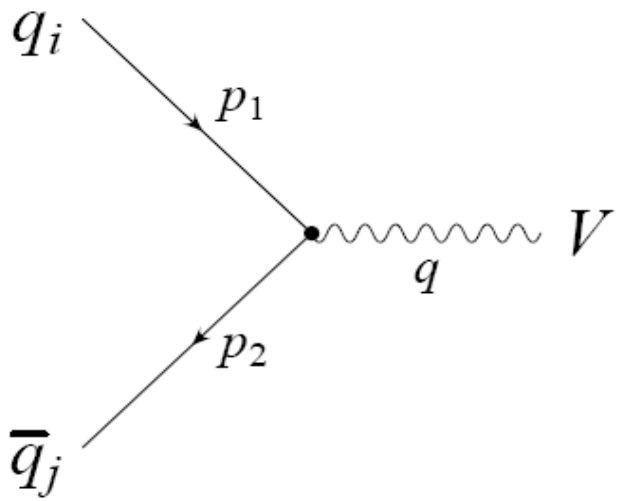
$$\frac{d}{d\log\mu^2}g(x,\mu) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[g\left(\frac{x}{z},\mu\right)P_{gg}(z) + \sum_j q_j\left(\frac{x}{z},\mu\right)P_{gq}(z)\right]$$

PDF

Splitting Function

The DGLAP equations are universal: the collinear behavior is Independent of the scattering process.

Drell-Yan: LO



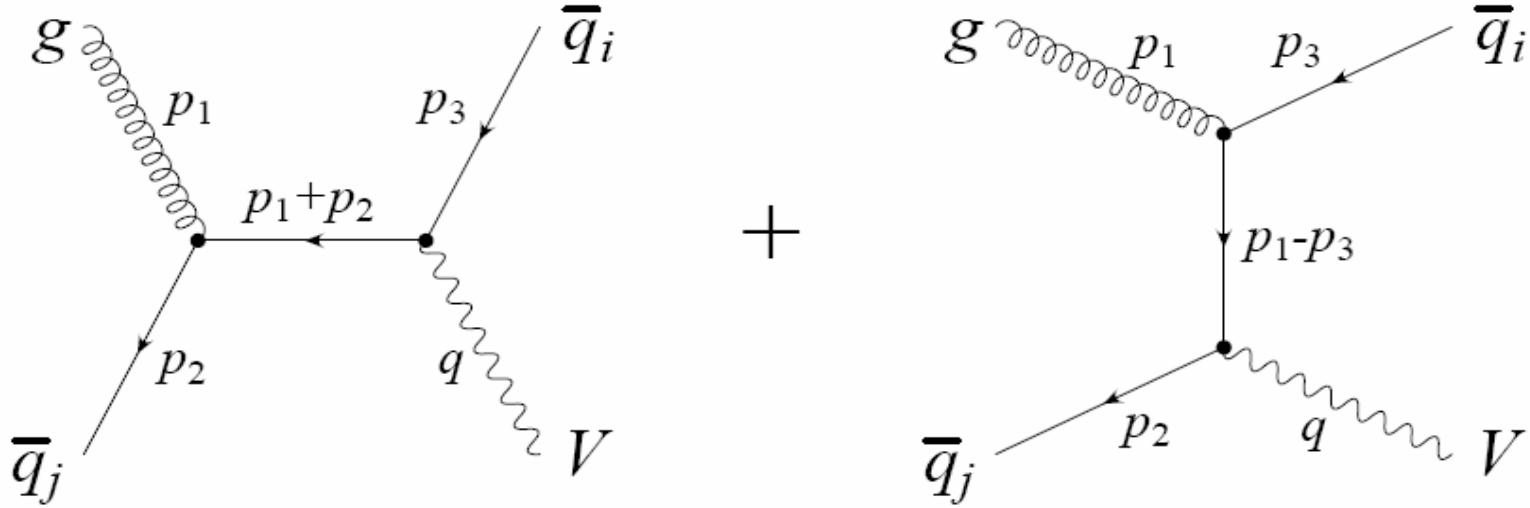
$$V = \gamma^*, W^\pm, Z$$

$$q^2 = Q^2$$

$$\sigma^{(0)}_{q\bar{q}} = \frac{4\pi^2\alpha}{3S} \sum_{i,j} C_{ij} \int_{z_0}^{1} dz \left(q_{0i} \otimes \bar{q}_{0j} + \bar{q}_{0j} \otimes q_{0i}\right) \left(\frac{z_0}{z}\right) \delta(1-z)$$

where $\sqrt{S}$ is the hadronic cm energy, $z_0 = Q^2/S$, and C_{ij} specifies EW couplings.

Initial gluon



- Collinear singularity at $t \equiv (p_1 - p_3)^2 = 0$ due to the splitting $g \to \bar{q}_i q_i$.
- We want to absorb the collinear physics into the PDF $q_i(x)$.

Initial gluon: Cross section

$$\sigma^{(1)}_{g\bar{q}} = \frac{\pi\alpha\alpha_s}{3S}\sum_{i,j} C_{ij}\int_{Q^2}^{S}\frac{ds}{s^2}\left(g\otimes\bar{q}_j+\bar{q}_j\otimes g\right)\left(\frac{s}{S}\right)$$
$$\times\int_{-\infty}^{0}dt\int_{-\infty}^{0}du\,\delta(s+t+u-Q^2)\left[\frac{s}{-t}+\frac{-t}{s}-\frac{2uQ^2}{st}\right]$$

- $d\sigma^{(1)}_{g\bar{q}}/dt \sim 1/t$ as $t\to 0$: a collinear divergence.
- We want to absorb this collinear behavior into the quark distribution function.

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- Integrate over u ...

Initial gluon: Cross section

$$\sigma_{g\bar{q}}^{(1)} = \frac{\pi\alpha\alpha_s}{3S}\sum_{i,j} C_{ij}\int_{Q^2}^{S}\frac{ds}{s^2}\left(g\otimes\bar{q}_j+\bar{q}_j\otimes g\right)\left(\frac{s}{S}\right)$$
$$\times\int_{-s+Q^2}^{0} dt\left[\frac{s}{-t}+\frac{-t}{s}-\frac{2(s-Q^2+t)Q^2}{s(-t)}\right]$$

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- . . . and separate collinear and noncollinear terms.

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$$\times\int_{-s+Q^2}^{0} dt\left[\frac{1}{-t}\left(s-\frac{2Q^2(s-Q^2)}{s}\right)+\frac{2Q^2-t}{s}\right]$$

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$$\times\int_{-s+Q^2}^{0} dt\left[\frac{2s}{-t}P_{qg}\left(\frac{Q^2}{s}\right)+\frac{2Q^2-t}{s}\right]$$

- $d\sigma^{(1)}_{g\bar{q}}/dt \sim 1/t$ as $t\to 0$: a collinear divergence.
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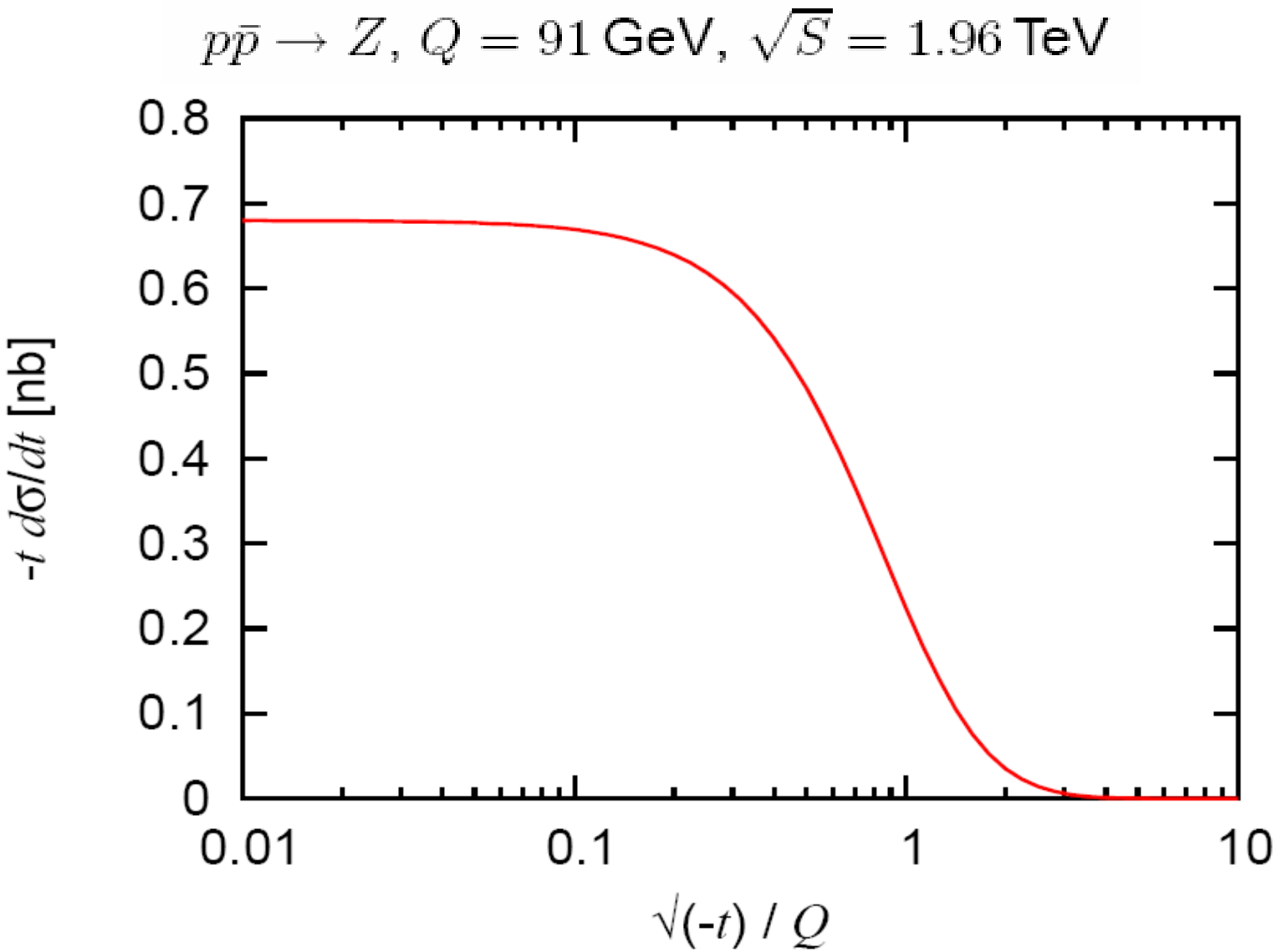
Initial gluon: Collinear plateau



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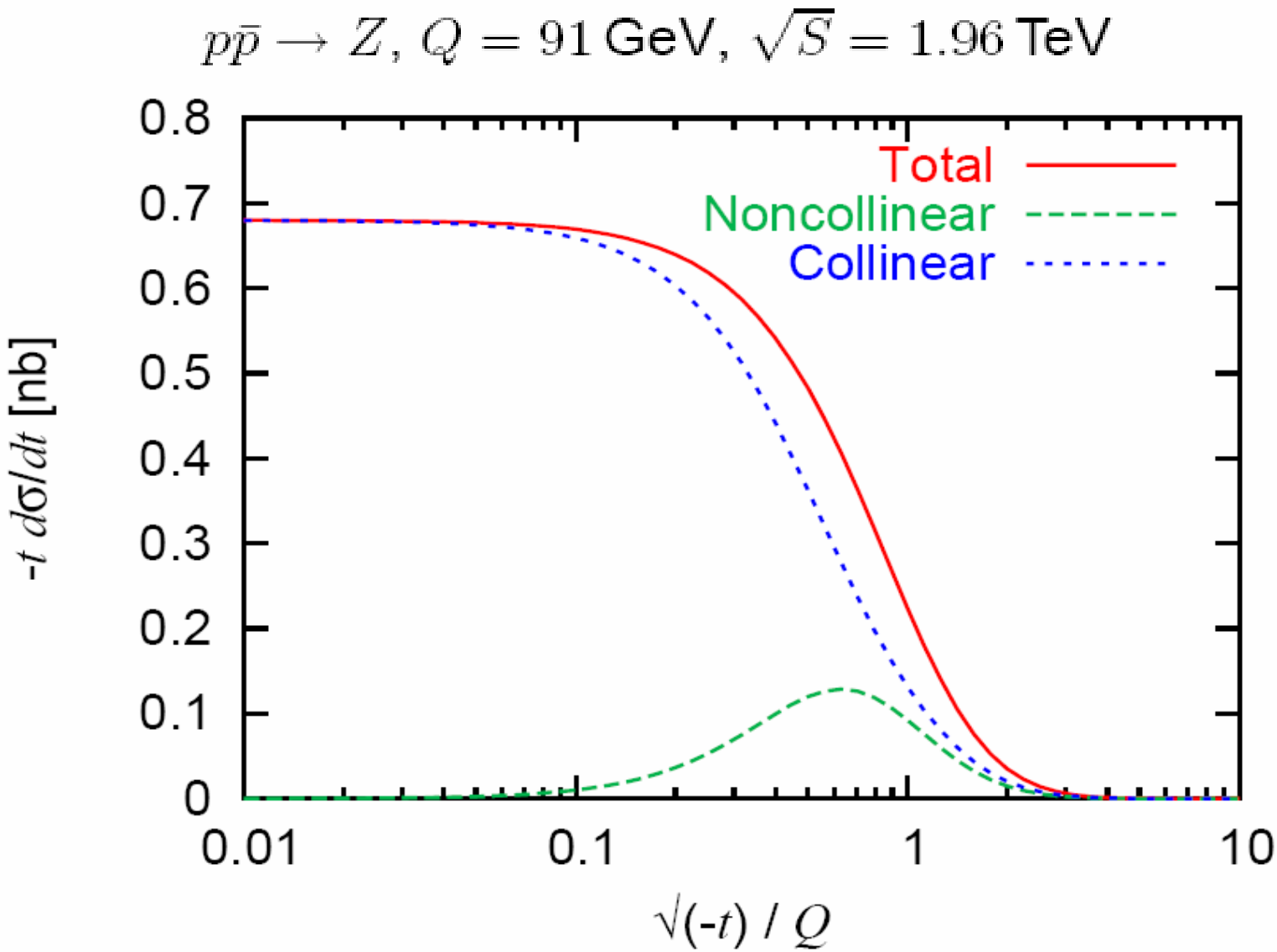

The collinear factorization scheme

- Integrate the leading behavior $1/t$ from 0 to $-\mu^2$.
- We regard μ as the factorization scale in this scheme.

$$\bar{\sigma}^{(1)}_{g\bar{q}} \equiv \left[\lim_{t\to 0}\left(-t\frac{d\sigma^{(1)}_{g\bar{q}}}{dt}\right)\right]\int_{-\mu^2}^{0}\frac{dt}{-t}$$

The collinear factorization scheme

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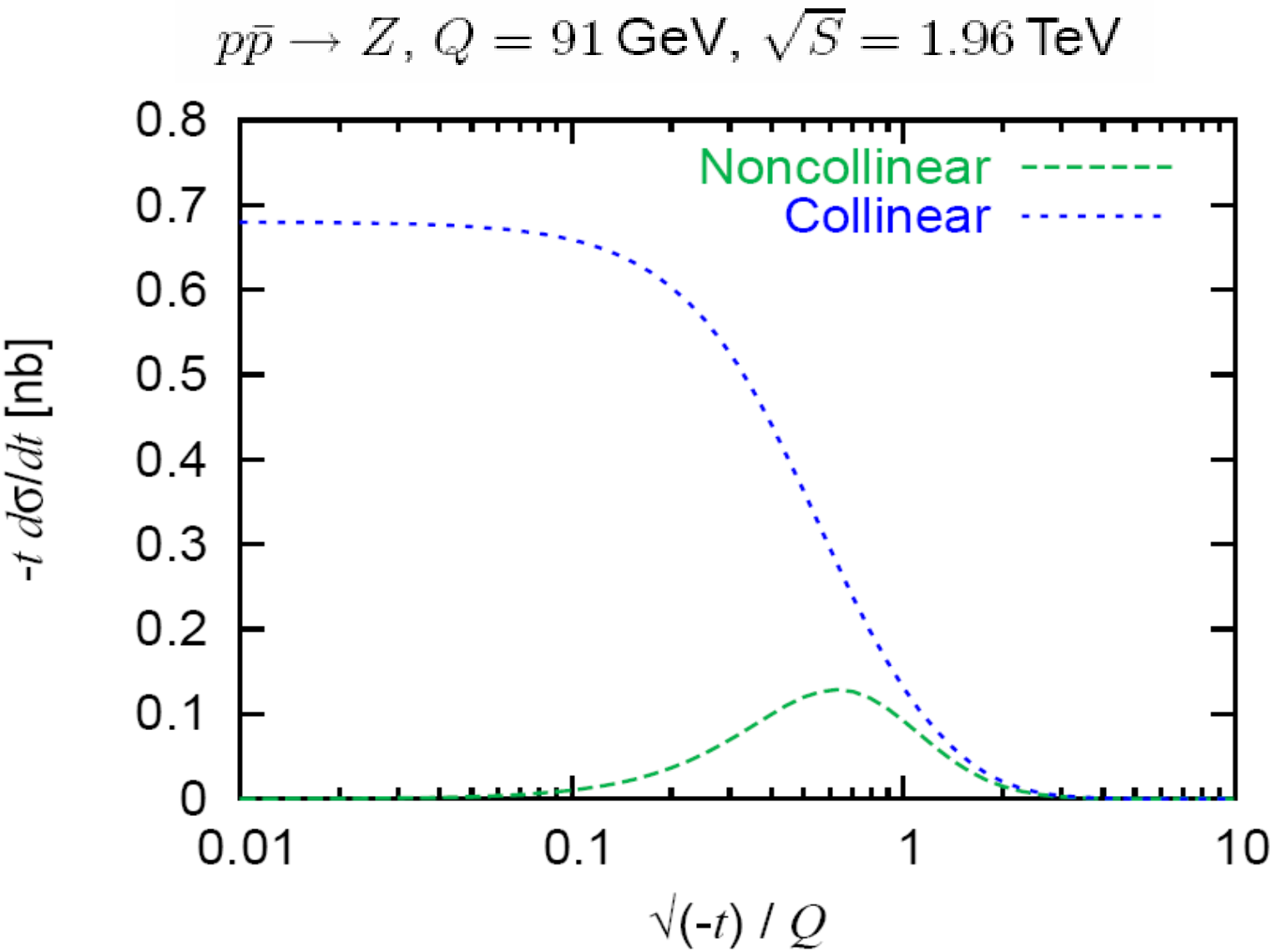
$$\bar{\sigma}^{(1)}_{g\bar{q}} = \frac{2\pi\alpha\alpha_s}{3S} \sum_{i,j} C_{ij} \int_{-\mu^2}^{0} \frac{dt}{-t} \times \int_{Q^2}^{S} \frac{ds}{s} \left(g \otimes \bar{q}_j + \bar{q}_j \otimes g\right) \left(\frac{s}{S}\right) P_{qg}\left(\frac{Q^2}{s}\right),$$

where $P_{qg}(z) = \frac{1}{2}\left[z^2 + (1-z)^2\right]$ is the DGLAP splitting function.

Initial gluon: Collinear plateau



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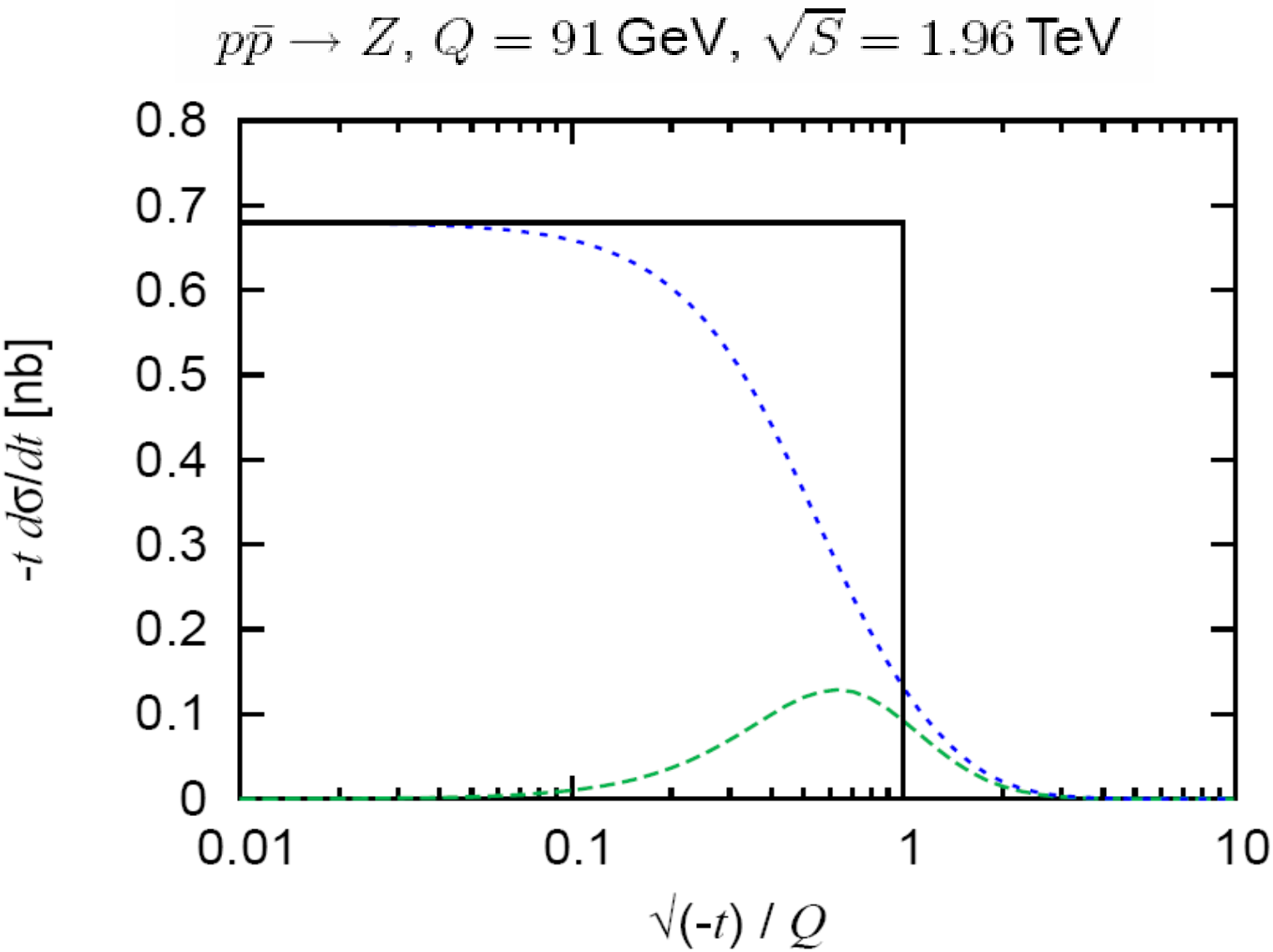
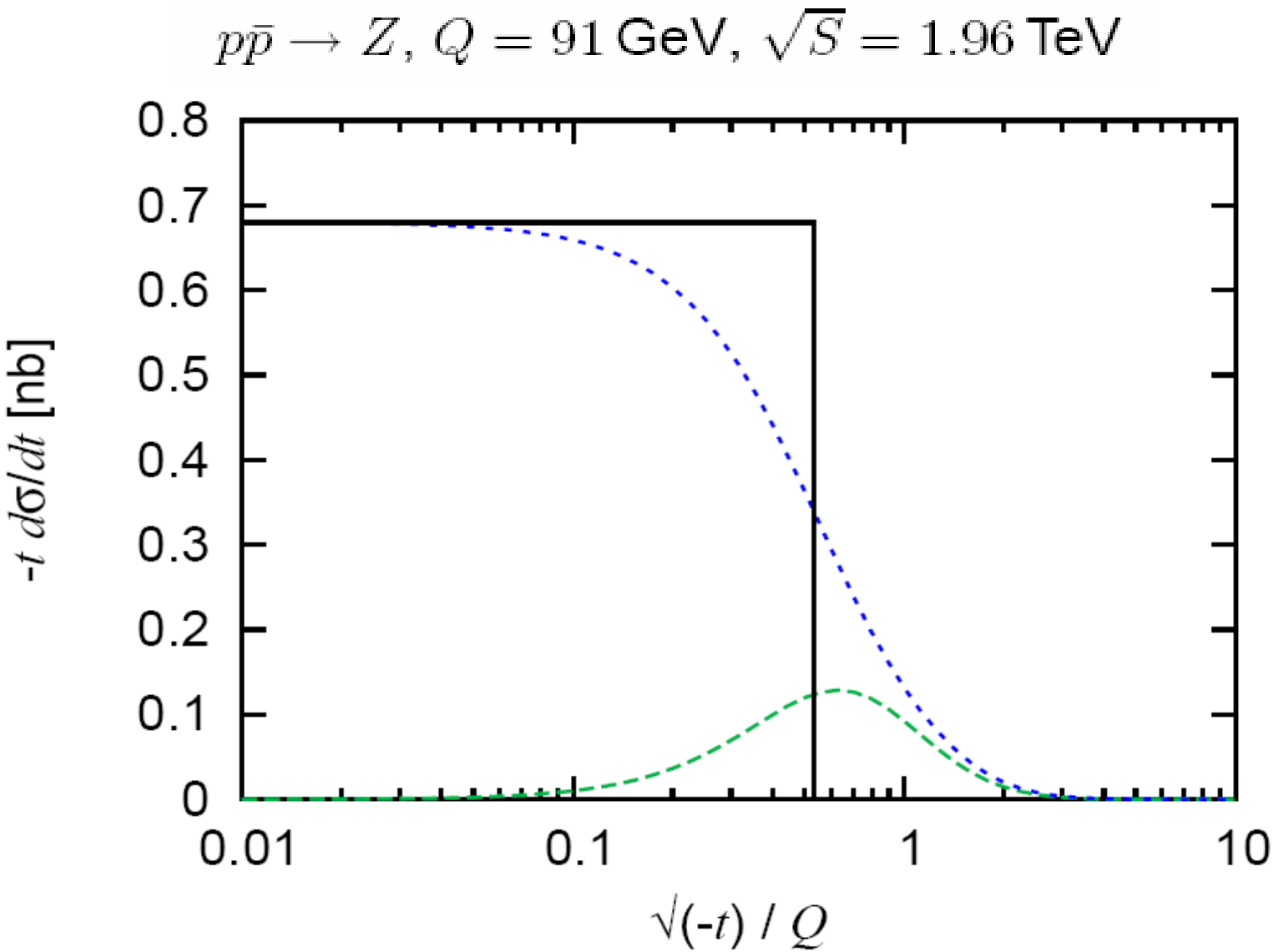


Initial gluon: Collinear plateau



Initial gluon: Explicit correction

The explicit correction to the cross section in the collinear scheme is

$$\sigma^{(1)}_{g\bar{q}} - \bar{\sigma}^{(1)}_{g\bar{q}} = \frac{2\pi\alpha\alpha_s}{3S} \sum_{i,j} C_{ij} \int_{Q^2}^{S} \frac{ds}{s} \left(g \otimes \bar{q}_j + \bar{q}_j \otimes g\right) \left(\frac{s}{S}\right)$$

$$\times \left[P_{qg}\left(\frac{Q^2}{s}\right) \int_{-s+Q^2}^{0} \frac{dt}{-t} + \int_{-s+Q^2}^{0} dt \, \frac{2Q^2 - t}{2s^2} - P_{qg}\left(\frac{Q^2}{s}\right) \int_{-\mu^2}^{0} \frac{dt}{-t} \right]$$

Initial gluon: Explicit correction

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$$\sigma^{(1)}_{g\bar{q}} - \bar{\sigma}^{(1)}_{g\bar{q}} = \frac{2\pi\alpha\alpha_s}{3S} \sum_{i,j} C_{ij} \int_{Q^2}^{S} \frac{ds}{s} \left(g \otimes \bar{q}_j + \bar{q}_j \otimes g\right) \left(\frac{s}{S}\right)$$

$$\times \left[P_{qg}\left(\frac{Q^2}{s}\right) \int_{-s+Q^2}^{-\mu^2} \frac{dt}{-t} + \int_{-s+Q^2}^{0} dt \, \frac{2Q^2 - t}{2s^2} \right]$$

Initial gluon: Explicit correction

The explicit correction to the cross section in the collinear scheme is

$$\sigma^{(1)}_{g\bar{q}} - \bar{\sigma}^{(1)}_{g\bar{q}} = \frac{2\pi\alpha\alpha_s}{3S} \sum_{i,j} C_{ij} \int_{z_0}^{1} \frac{dz}{z} \left(g \otimes \bar{q}_j + \bar{q}_j \otimes g\right) \left(\frac{z_0}{z}\right)$$

$$\times \left[P_{qg}(z) \ln \frac{Q^2(1-z)}{\mu^2 z} + \frac{1}{4}(1 + 2z - 3z^2) \right]$$

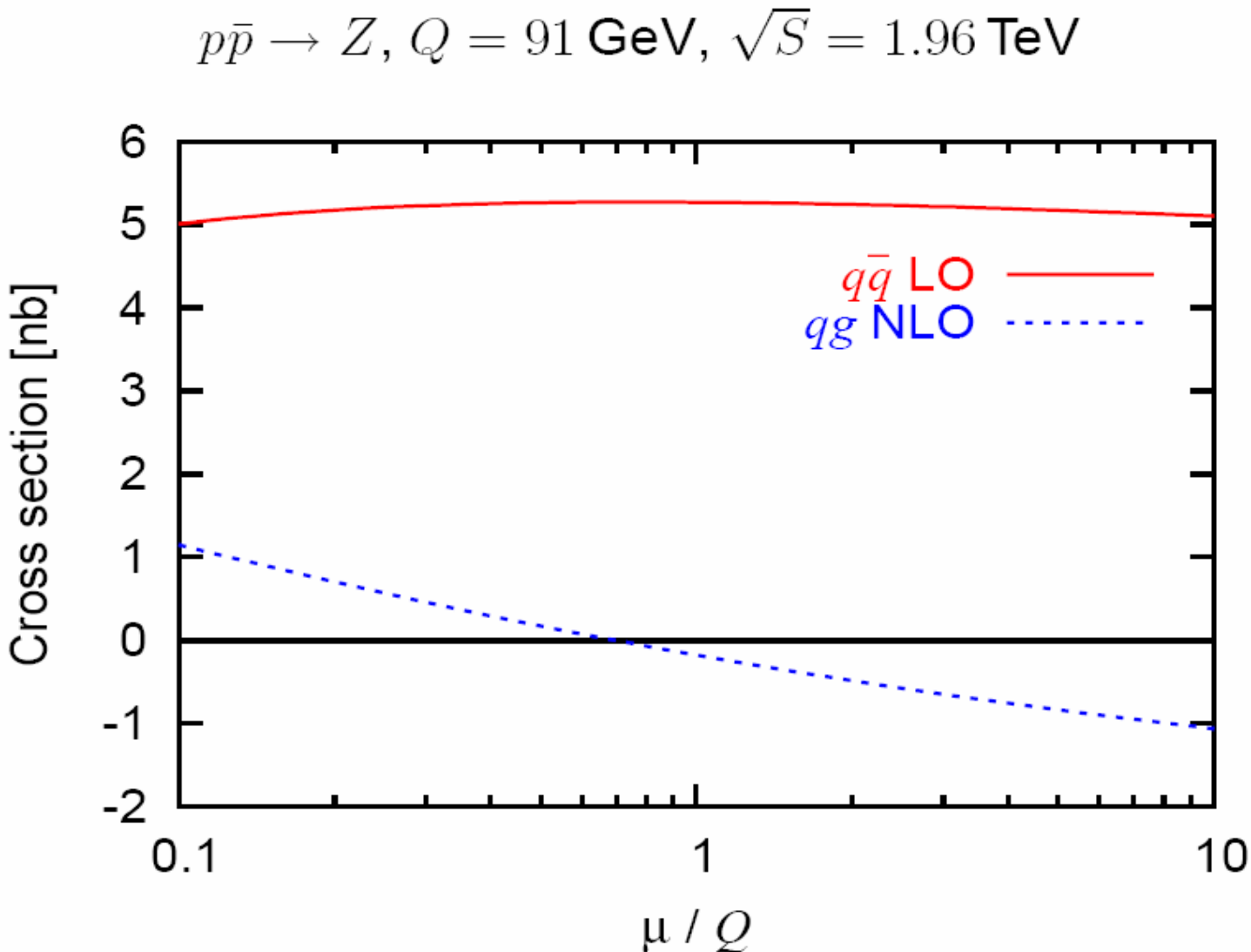
Initial gluon: Correction to the PDF

The correction to the quark PDF can be written in dimensional regularization as

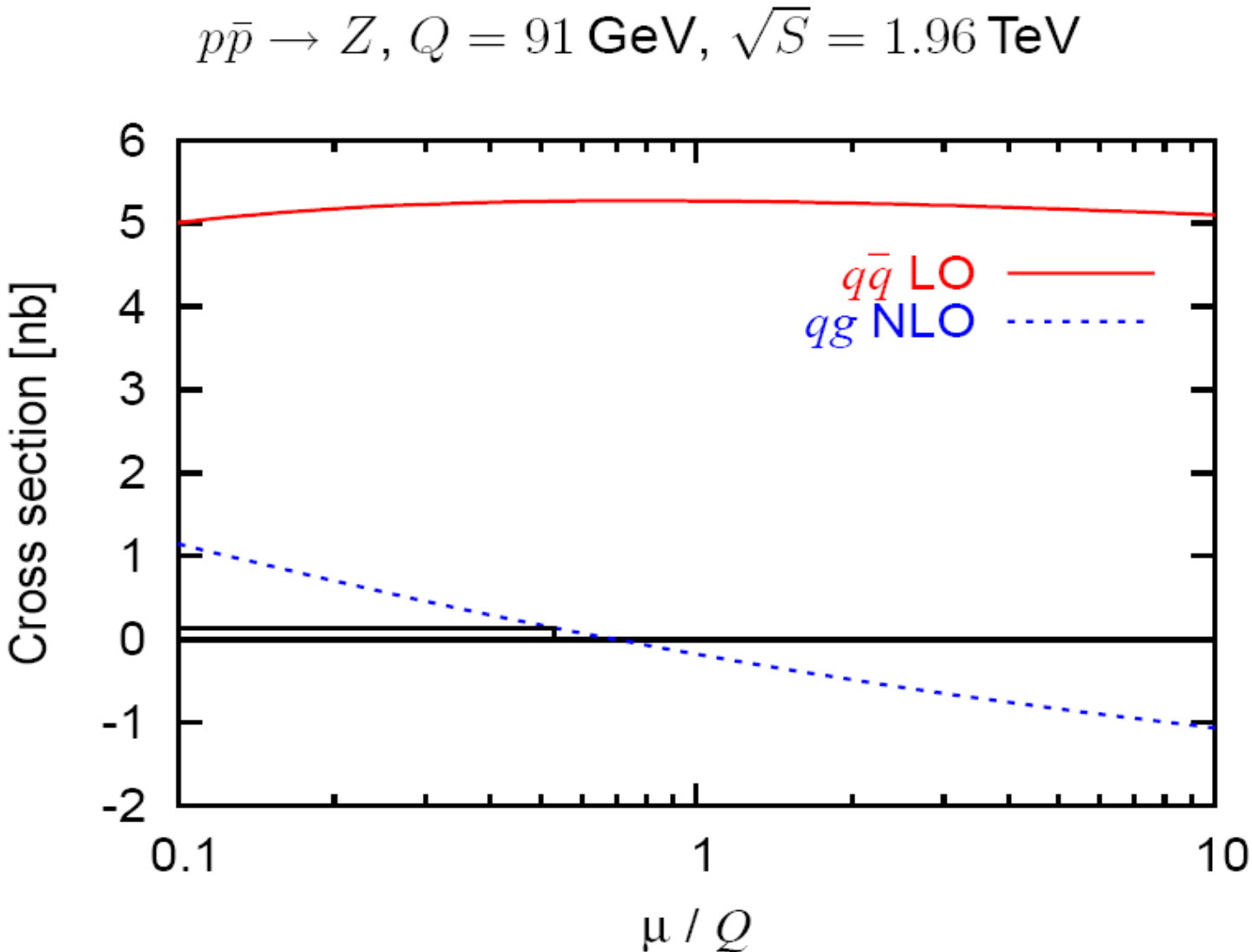
$$\delta q_i(x) = -\frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\, g\left(\frac{x}{z}\right) \times \left[\left(\frac{1}{\epsilon} - \gamma + \ln\frac{4\pi\mu_D^2}{\mu^2(1-z)}\right) P_{qg}(z) - z(1-z)\right],$$

where μ_D is the 't Hooft mass.

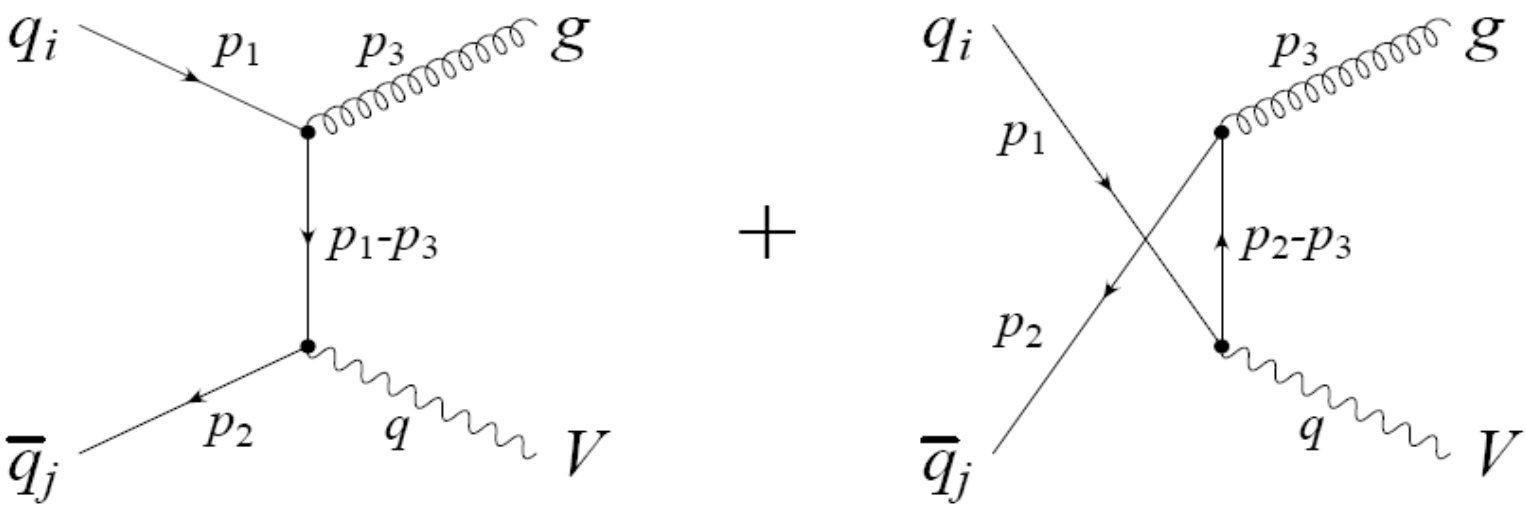
Initial-gluon correction: μ-dependence



Initial-gluon correction: μ-dependence



Real-gluon emission



- This time there are two collinear singularities, at $t \equiv (p_1 - p_3)^2 = 0$ and at $u \equiv (p_2 - p_3)^2 = 0$,
- and an infrared singularity at $t = u = 0 \Leftrightarrow s = Q^2$.

Real+virtual gluons: Counterterm

$$\bar{\sigma}_{q\bar{q}}^{(1,t)} = \frac{8\pi\alpha\alpha_s}{9S} \sum_{i,j} C_{ij} \int_{-\mu^2}^{0} \frac{dt}{-t}$$
$$\times \int_{Q^2}^{S} \frac{ds}{s} \left(q_i \otimes \bar{q}_j + \bar{q}_j + \bar{q}_j \otimes q_i\right) \left(\frac{s}{S}\right) P_{qq}\left(\frac{Q^2}{s}\right),$$

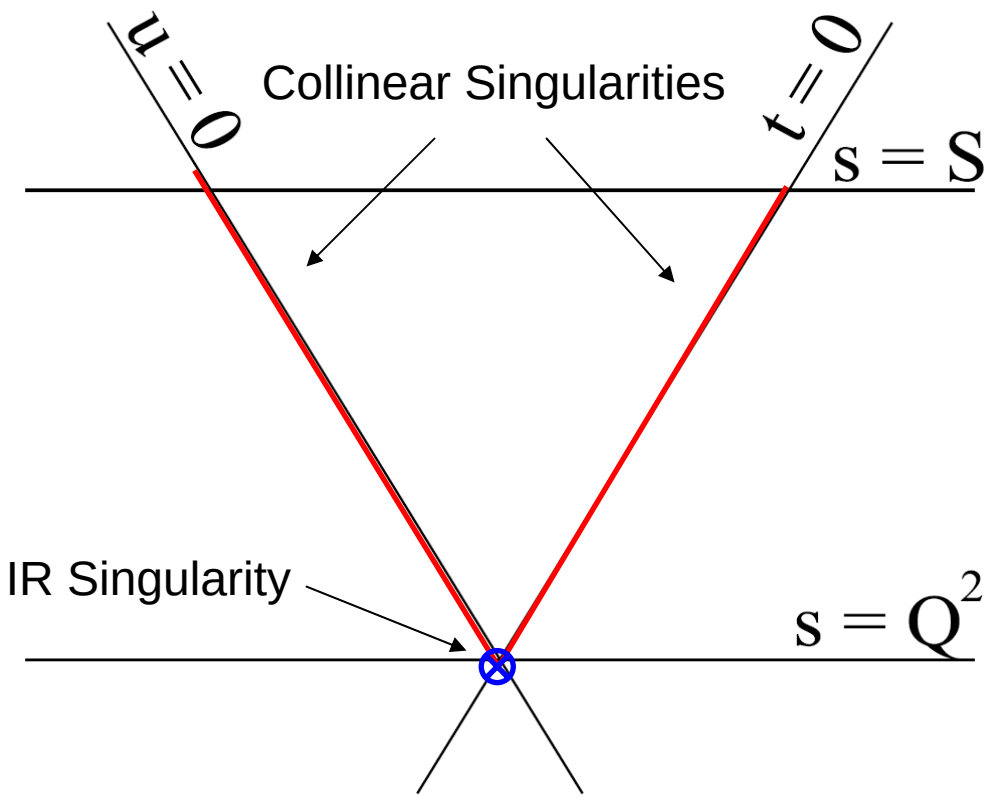
where $P_{qq}(z) = \frac{4}{3}\left[(1+z^2)/(1-z)\right]_+$ is the DGLAP splitting function.

Real-gluon emission: Cross section

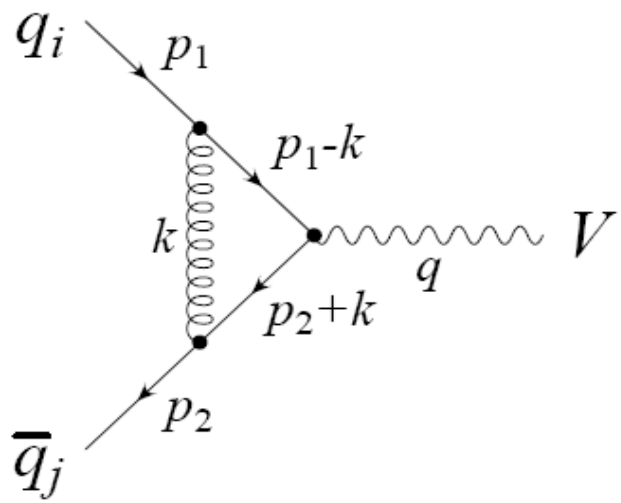
$$\sigma_{q\bar{q}}^{(1,\text{real})} = \frac{8\pi\alpha\alpha_s}{9S}\sum_{i,j} C_{ij}\int_{Q^2}^{S}\frac{ds}{s^2}\left(q_i\otimes\bar{q}_j+\bar{q}_j\otimes q_i\right)\left(\frac{s}{S}\right)$$

$$\times\int_{-\infty}^{0}dt\int_{-\infty}^{0}du\,\delta(s+t+u-Q^2)\left[\frac{s^2+Q^4}{s-Q^2}\left(\frac{1}{-t}+\frac{1}{-u}\right)-2\right]$$

- The soft singularity $1/(s-Q^2)$ obscures the collinear physics.
- We must turn to virtual corrections in order to cancel it.


u = 0
Collinear Singularities
t = 0
s = S
IR Singularity
s = Q^2

Vertex correction



- This diagram involves the vertex form factor

$$F_1(Q^2) = 1 + \delta F_1(Q^2),$$

- and contributes to the NLO cross section via its interference with the tree-level diagram.

Vertex correction: Form factor

$$\gamma^\mu \delta F_1(Q^2) = -\frac{16}{3}\pi i \alpha_s \int \frac{d^4k}{(2\pi)^4} \frac{\gamma_\nu(\not{k}+\not{p}_2)\gamma^\mu(\not{k}-\not{p}_1)\gamma^\nu}{k^2(k-p_1)^2(k+p_2)^2}$$

- Introduce Feynman parameters x, y, z and shift the loop momentum to $\ell \equiv k - xp_1 + yp_2$:

$$\frac{1}{k^2(k-p_1)^2(k+p_2)^2} = \int_0^1 dx \int_0^1 dy \int_0^1 dz\, \delta(x+y+z-1) \frac{2}{\left(\ell^2 + xyQ^2\right)^3}$$

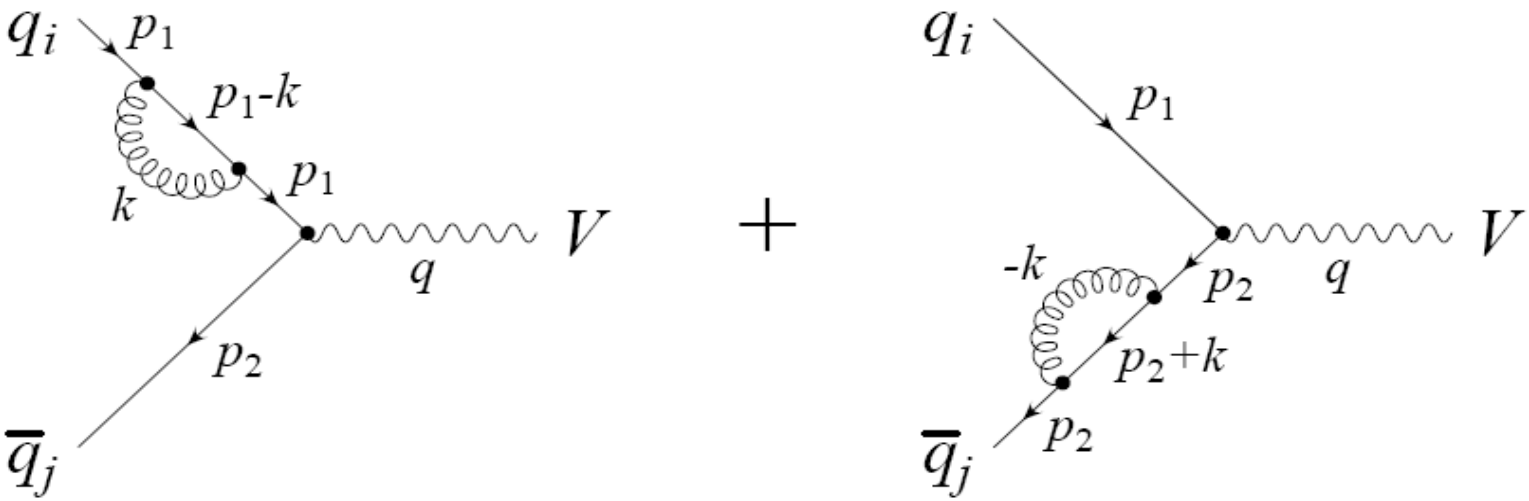
- Now we can do the loop integral. . . .

Vertex correction: Form factor

$$\delta F_1(Q^2) = -\frac{2\alpha_s}{3\pi}\int_0^1 dx \int_0^1 dy \int_0^1 dz\, \delta(x+y+z-1)$$
$$\times \left[\frac{z}{1-z}\left(\frac{1}{x}+\frac{1}{y}\right) + 1 - \ln\left(-\frac{z\Lambda^2}{xyQ^2}\right)\right]$$

- Collinear singularities at $x = 0$ and $y = 0$.
- Infrared singularity at $z = 1$.
- The $\ln \Lambda$ is an ultraviolet divergence (regulated by Pauli-Villars).

Wavefunction renormalization



- These diagrams involve the quark self-energy

$$\Sigma(p) = \Sigma'(p^2)\not{p}.$$

WFR: Self-energy

$$\Sigma(p_1) = \frac{32}{3}\pi i \alpha_s \int \frac{d^4k}{(2\pi)^4} \frac{\not{k} - \not{p}_1}{k^2 (k - p_1)^2}$$

In order to isolate the collinear singularity and combine the result with the vertex function, let's multiply by 1.

WFR: Self-energy

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In order to isolate the collinear singularity and combine the result with the vertex function, let's multiply by 1.

$$\Sigma(p_1) = \frac{32}{3}\pi i\alpha_s \int \frac{d^4k}{(2\pi)^4} \frac{(\not{k} - \not{p}_1)(k+p_2)^2}{k^2(k-p_1)^2(k+p_2)^2}$$

where $p_2^2 = 0$ and $(p_1 + p_2)^2 = Q^2$.

WFR: Self-energy

$$\Sigma'(0) = \frac{2\alpha_s}{3\pi} \int_0^1 dx \int_0^1 dy \int_0^1 dz\, \delta(x+y+z-1) \times \left[\frac{z}{y} + x + (2-3x) \ln\left(-\frac{z\Lambda^2}{xyQ^2} \right) \right]$$

- Collinear singularity at $y = 0$.
- The $\ln \Lambda$ is an ultraviolet divergence.

Virtual gluons

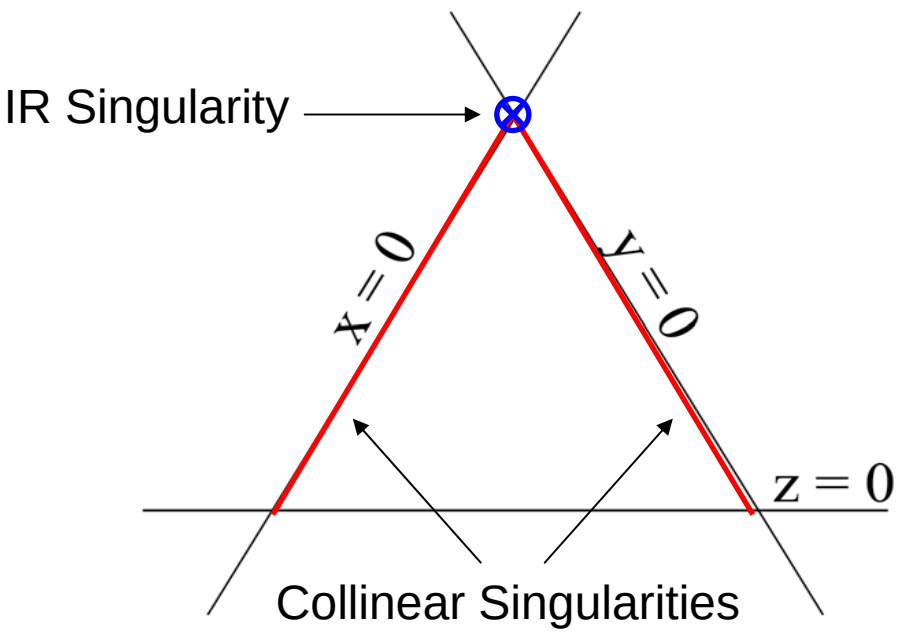
The contribution to the cross section from the vertex and WFR corrections is

$$\sigma_{q\bar{q}}^{(1,\text{virt})} = \sigma_{q\bar{q}}^{(0)} \times 2\,\mathrm{Re}\left[\delta F_1(Q^2) - \frac{1}{2}\Sigma'(0) - \frac{1}{2}\Sigma'(0)\right]$$

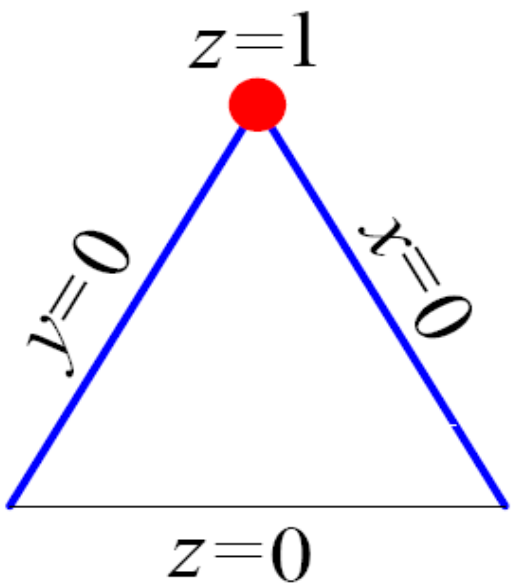
$$= -\frac{8\pi\alpha\alpha_s}{9S}\sum_{i,j} C_{ij}(q_i \otimes \bar{q}_j + \bar{q}_j \otimes q_i)(z_0)$$

$$\times \int_0^1 dx \int_0^1 dy \int_0^1 dz\, \delta(x+y+z-1)$$

$$\times \left[\frac{z(3-z)}{1-z}\left(\frac{1}{x}+\frac{1}{y}\right) + 3 - z - (1-3z)\ln\frac{z}{xy}\right]$$

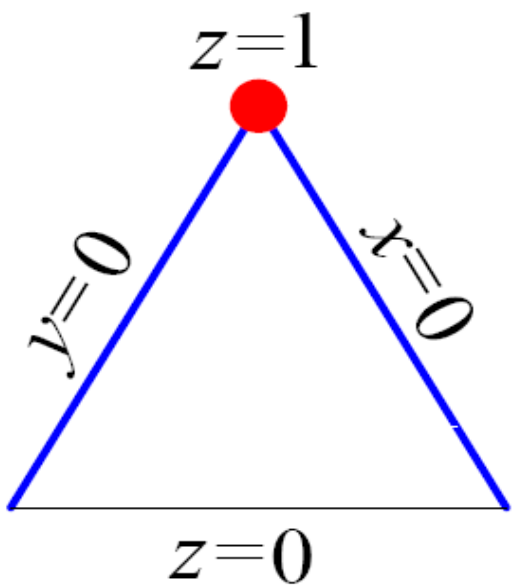

IR Singularity
x = 0
y = 0
z = 0
Collinear Singularities

Feynman



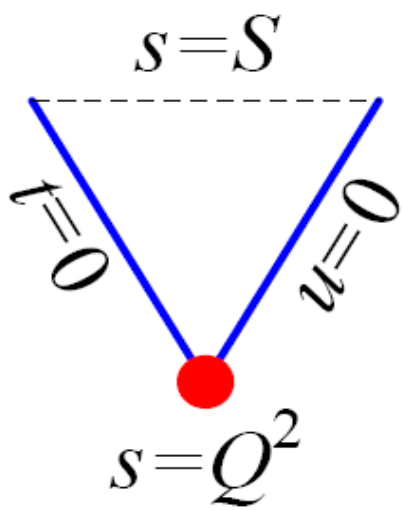
$$x + y + z = 1$$

Feynman



$$x + y + z = 1$$

Mandelstam

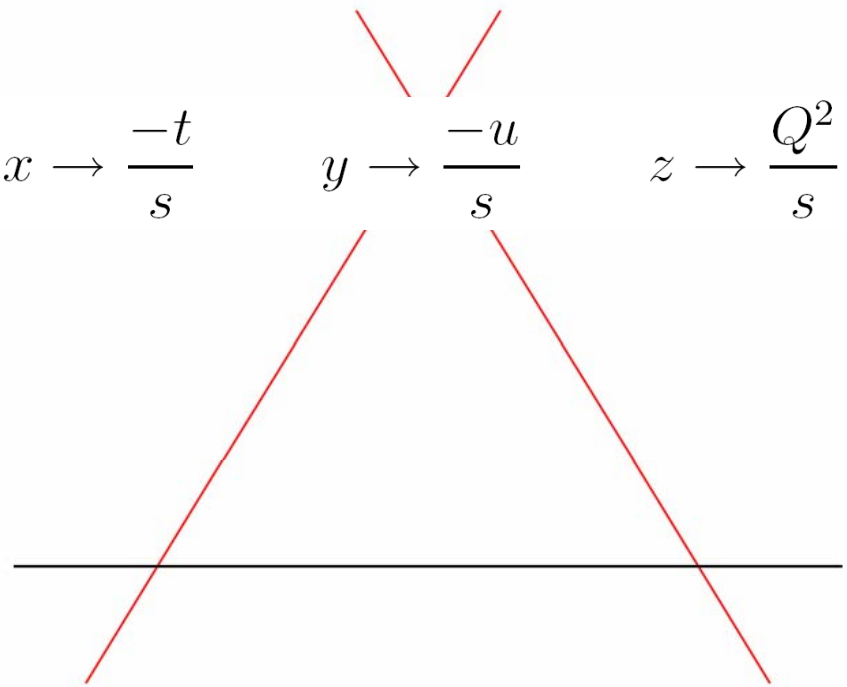


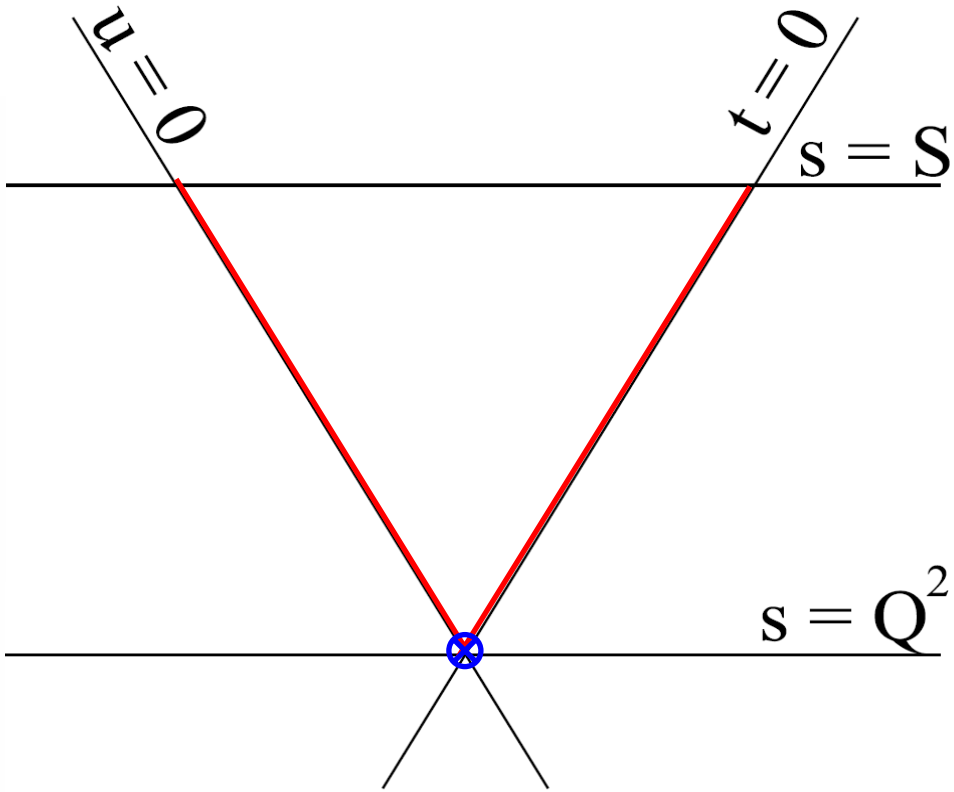
$$s + t + u = Q^2$$

Feynmandelstam

We now want to map the spaces of integration onto each other so the IR divergences cancel.

$$x \to \frac{-t}{s} \qquad y \to \frac{-u}{s} \qquad z \to \frac{Q^2}{s}$$





Real+virtual gluons: Cross section

$$\sigma_{q\bar{q}}^{(1)} = \frac{8\pi\alpha\alpha_s}{9S}\sum_{i,j} C_{ij}\int_{Q^2}^{\infty}\frac{ds}{s^2}\int_{-\infty}^{0}dt\int_{-\infty}^{0}du\,\delta(s+t+u-Q^2)$$

$$\times\left\{\theta(S-s)(q_i\otimes\bar{q}_j+\bar{q}_j\otimes q_i)\left(\frac{s}{S}\right)\left[\frac{s^2+Q^4}{s-Q^2}\left(\frac{1}{-t}+\frac{1}{-u}\right)-2\right]\right.$$

$$-(q_i\otimes\bar{q}_j+\bar{q}_j\otimes q_i)\left(\frac{Q^2}{S}\right)\frac{Q^2}{s}\left[\frac{Q^2(3s-Q^2)}{s-Q^2}\left(\frac{1}{-t}+\frac{1}{-u}\right)\right.$$

$$\left.\left.+\frac{3s-Q^2}{s}-\frac{s-3Q^2}{s}\ln\frac{sQ^2}{tu}\right]\right\}$$

Real+virtual gluons: Cross section

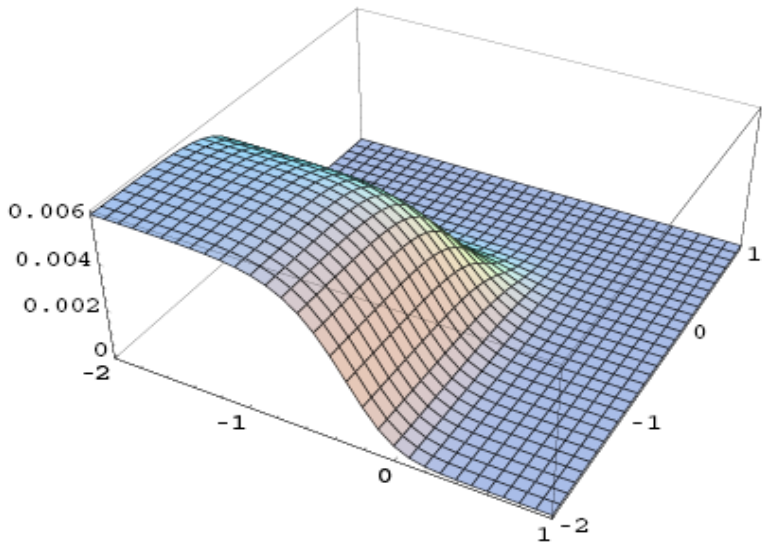
$$\sigma^{(1)}_{q\bar{q}} = \frac{8\pi\alpha\alpha_s}{9S}\sum_{i,j} C_{ij}\int_{Q^2}^{\infty}\frac{ds}{s^2}\int_{-\infty}^{0}dt\int_{-\infty}^{0}du\,\delta(s+t+u-Q^2)$$

$$\times\left\{\theta(S-s)(q_i\otimes\bar{q}_j+\bar{q}_j\otimes q_i)\left(\frac{s}{S}\right)\left[\frac{s^2+Q^4}{s-Q^2}\left(\frac{1}{-t}+\frac{1}{-u}\right)-2\right]\right.$$

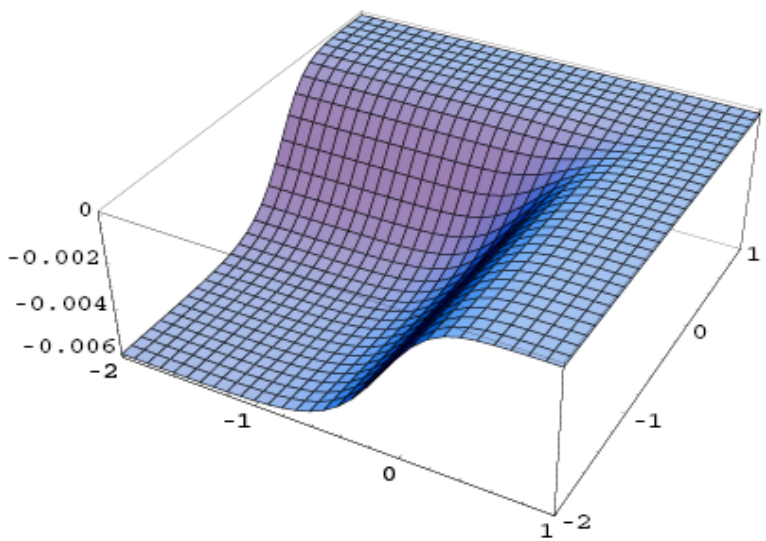
$$-(q_i\otimes\bar{q}_j+\bar{q}_j\otimes q_i)\left(\frac{Q^2}{S}\right)\frac{Q^2}{s}\left[\frac{Q^2(3s-Q^2)}{s-Q^2}\left(\frac{1}{-t}+\frac{1}{-u}\right)\right.$$

$$\left.\left.+\frac{3s-Q^2}{s}-\frac{s-3Q^2}{s}\ln\frac{sQ^2}{tu}\right]\right\}$$

Infrared cancellation

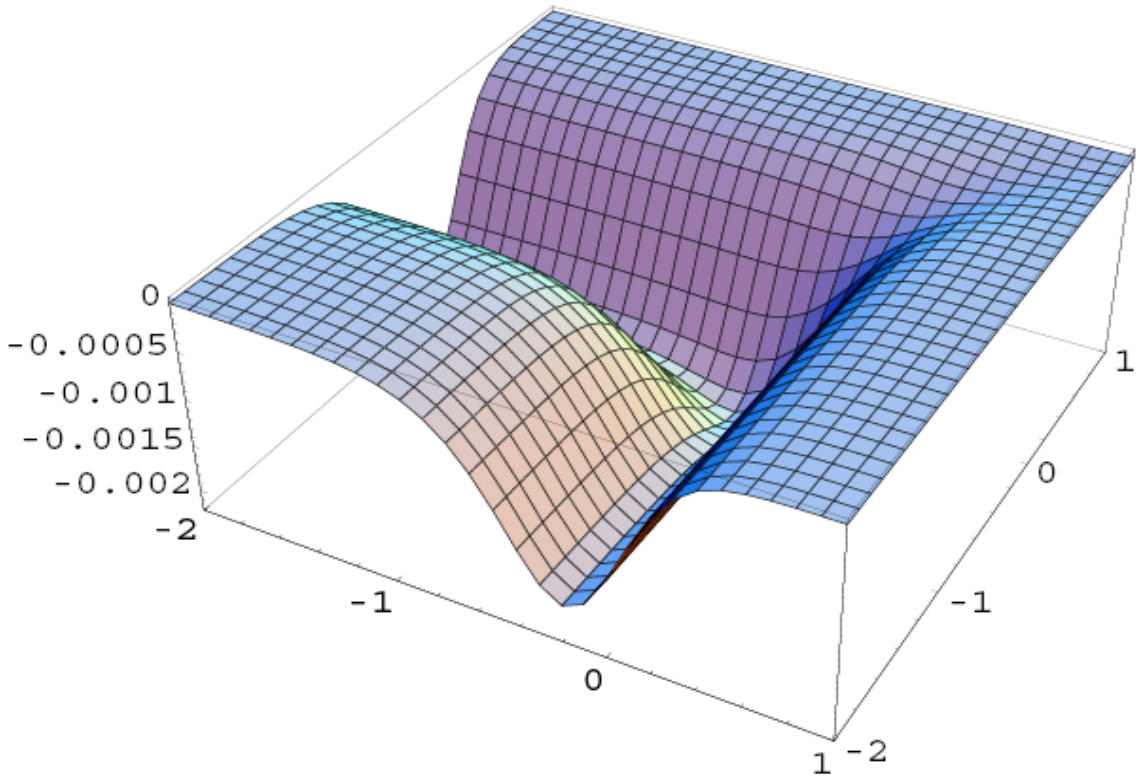


Real



Virtual

Infrared cancellation



Real+virtual gluons: Counterterm

$$\bar{\sigma}_{q\bar{q}}^{(1,t)} = \frac{8\pi\alpha\alpha_s}{9S}\sum_{i,j} C_{ij}\int_{-\mu^2}^{0}\frac{dt}{-t}$$

$$\times\int_{Q^2}^{\infty}\frac{ds}{s}\left[\theta(S-s)(q_i\otimes\bar{q}_j+\bar{q}_j\otimes q_i)\left(\frac{s}{S}\right)\frac{s^2+Q^4}{s(s-Q^2)}\right.$$

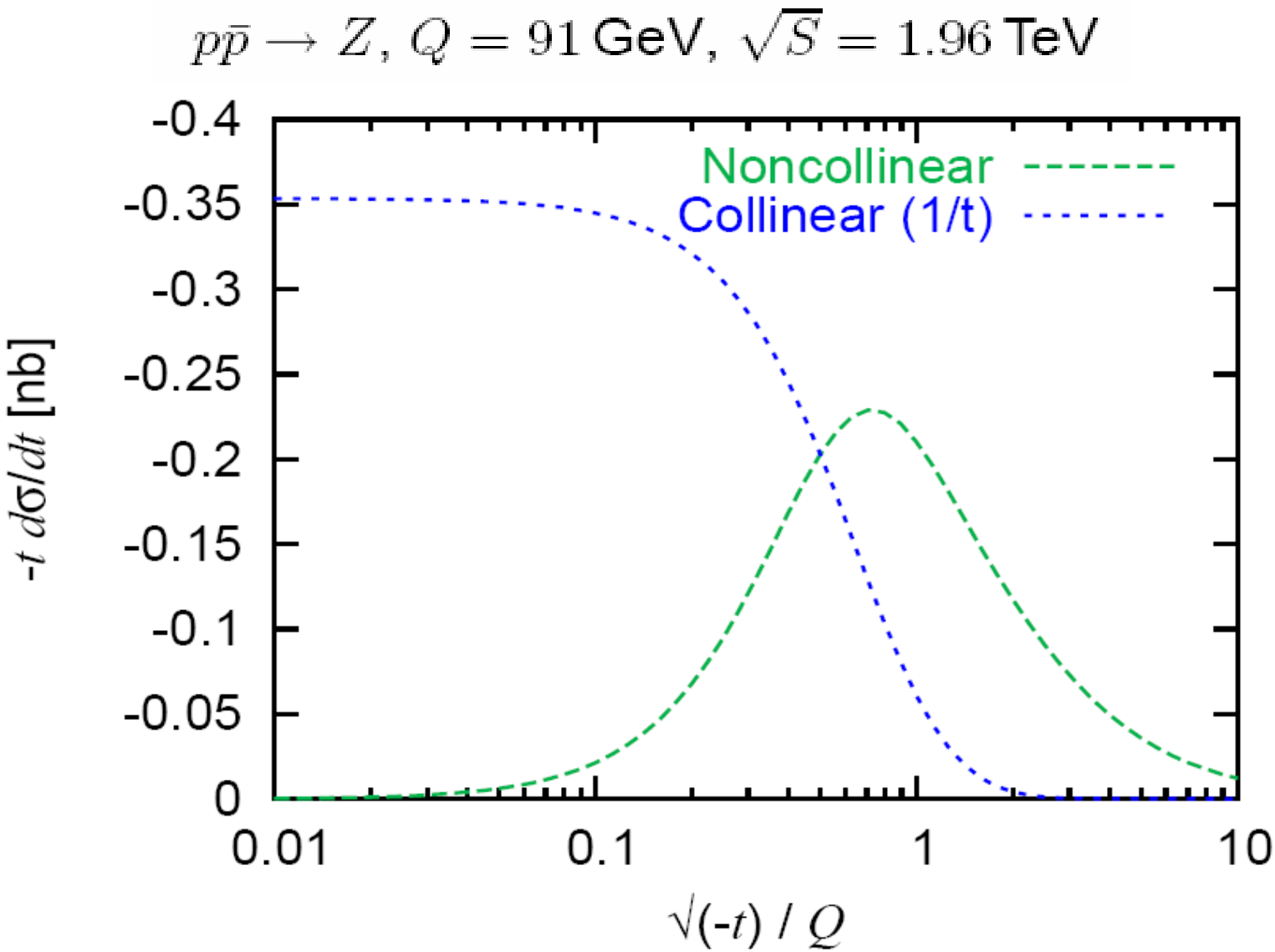
$$\left.-(q_i\otimes\bar{q}_j+\bar{q}_j\otimes q_i)\left(\frac{Q^2}{S}\right)\frac{Q^4(3s-Q^2)}{s^2(s-Q^2)}\right]$$

Real+virtual gluons: Counterterm

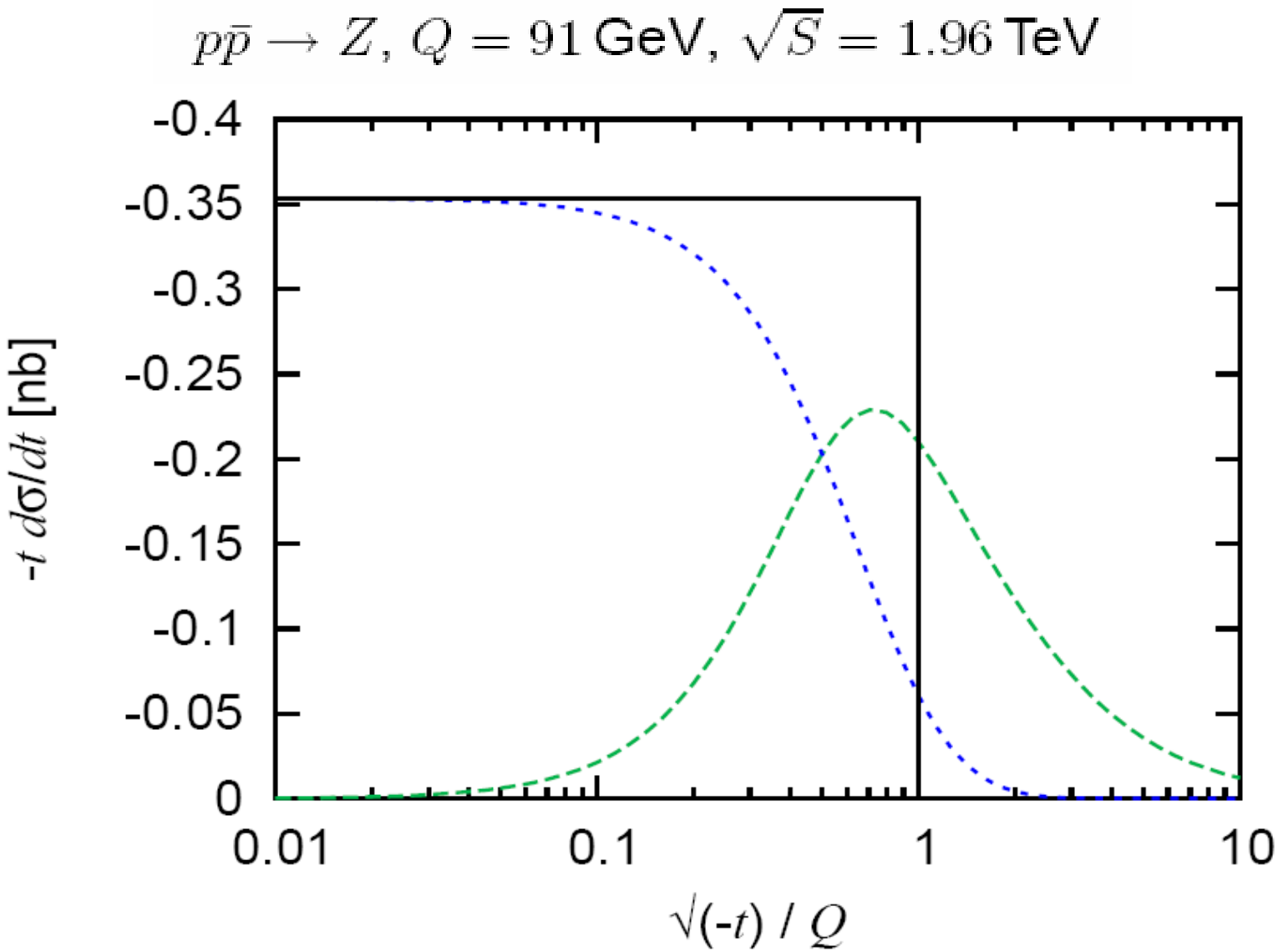
$$\bar{\sigma}_{q\bar{q}}^{(1,t)} = \frac{8\pi\alpha\alpha_s}{9S}\sum_{i,j} C_{ij}\int_{-\mu^2}^{0}\frac{dt}{-t}$$
$$\times\int_{Q^2}^{S}\frac{ds}{s}\left(q_i\otimes\bar{q}_j+\bar{q}_j+\bar{q}_j\otimes q_i\right)\left(\frac{s}{S}\right)P_{qq}\left(\frac{Q^2}{s}\right),$$

where $P_{qq}(z) = \frac{4}{3}\left[(1+z^2)/(1-z)\right]_+$ is the DGLAP splitting function.

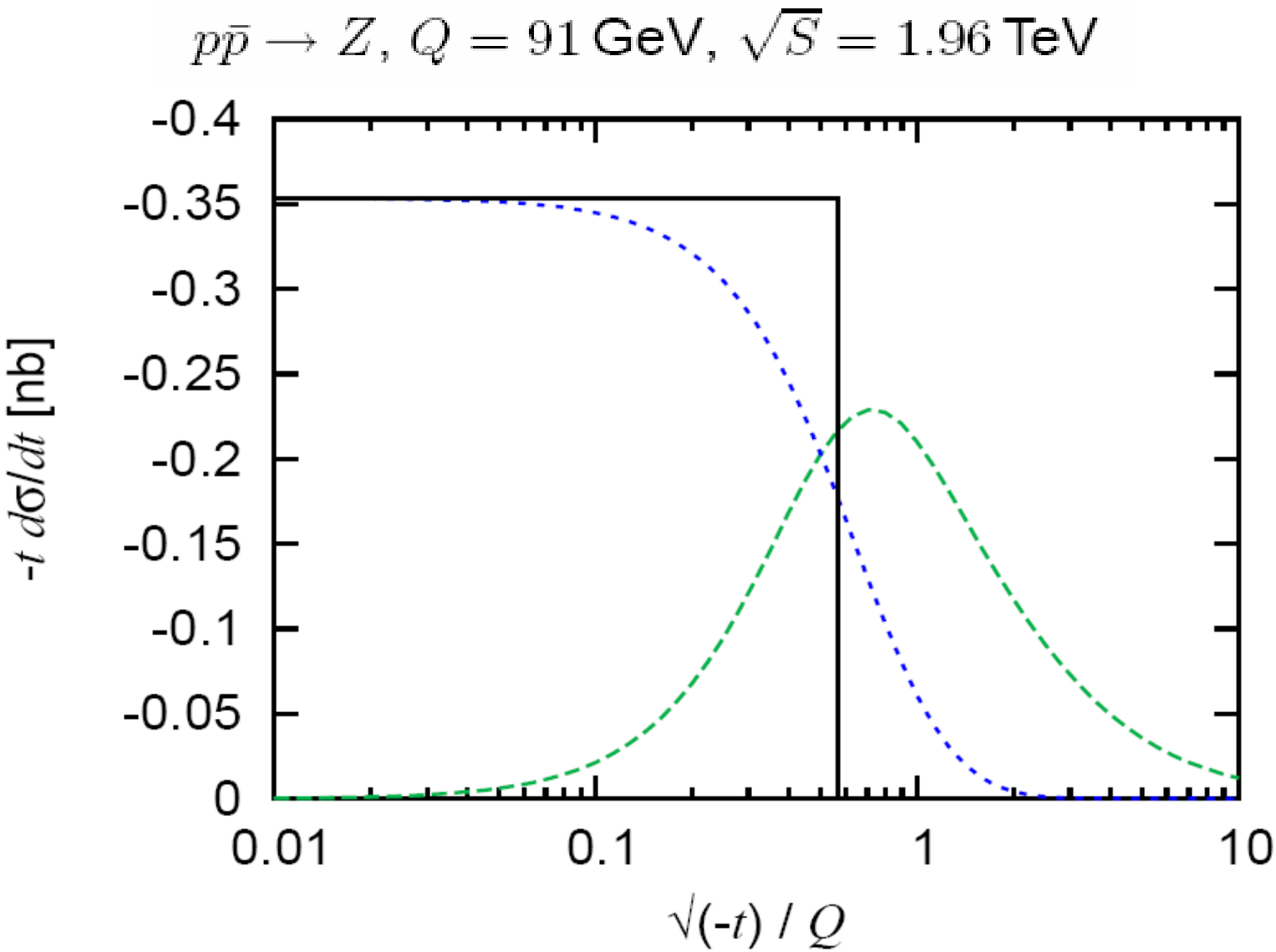
Real+virtual gluons: Collinear plateau



Real+virtual gluons: Collinear plateau



Real+virtual gluons: Collinear plateau



Real+virtual gluons: Explicit correction

$$\sigma^{(1)}_{q\bar{q}} - \bar{\sigma}^{(1,t)}_{q\bar{q}} - \bar{\sigma}^{(1,u)}_{q\bar{q}} = \frac{2\pi\alpha\alpha_s}{3S} \sum_{i,j} C_{ij} \int_{z_0}^{1} \frac{dz}{z} \left(q_i \otimes \bar{q}_j + \bar{q}_j \otimes q_i\right) \left(\frac{z_0}{z}\right)$$

$$\times \left[2P_{qq}(z) \ln \frac{Q^2}{\mu^2} + \frac{8}{3}(1+z^2) \left(\frac{\ln(1-z)}{1-z}\right)_+ \right.$$

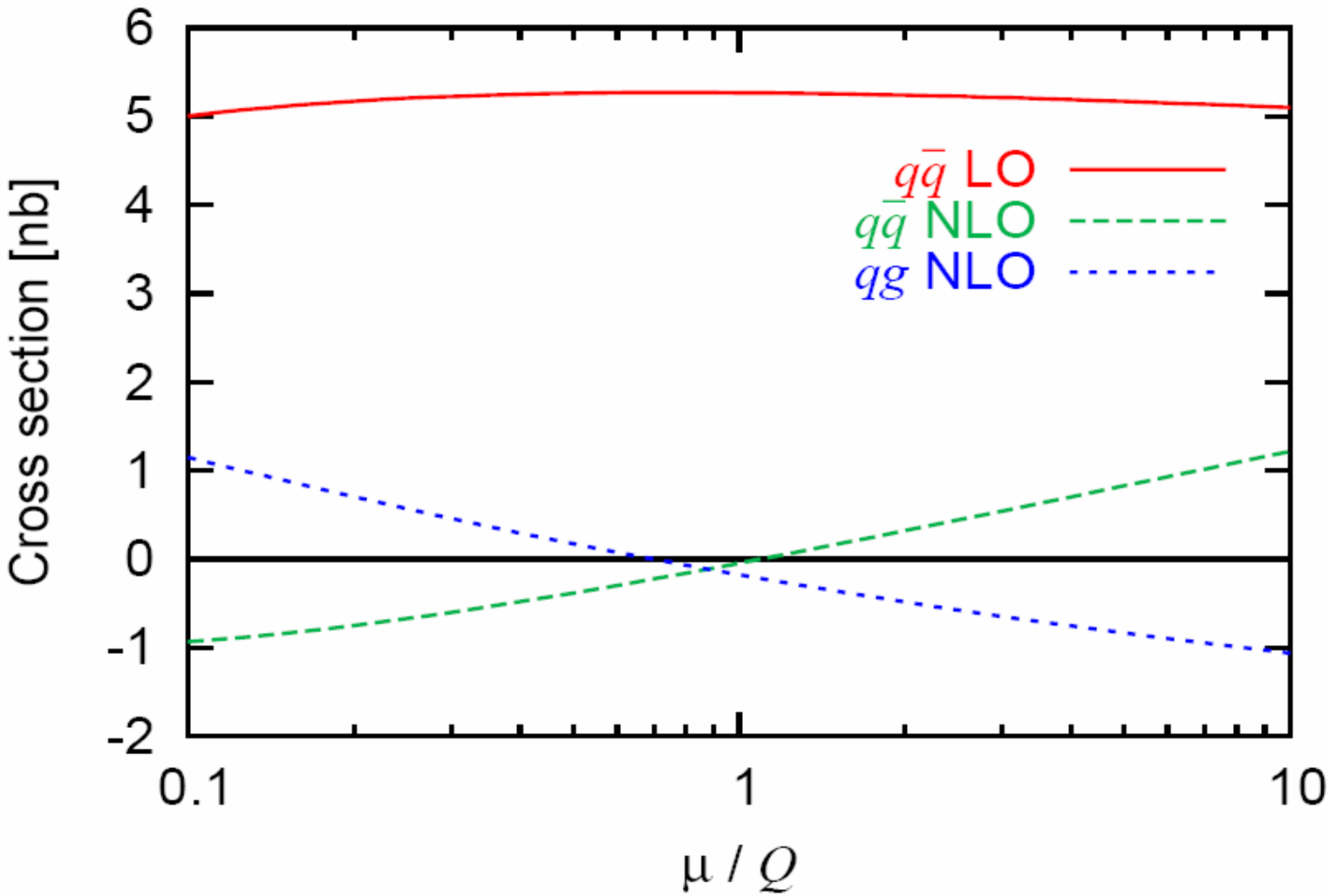
$$\left. - \left(\frac{8\pi^2}{9} + \frac{4}{3}\right) \delta(1-z) - \frac{8}{3} \frac{1+z^2}{1-z} \ln z - \frac{8}{3}(1-z) \right],$$

Real+virtual gluons: Correction to PDF

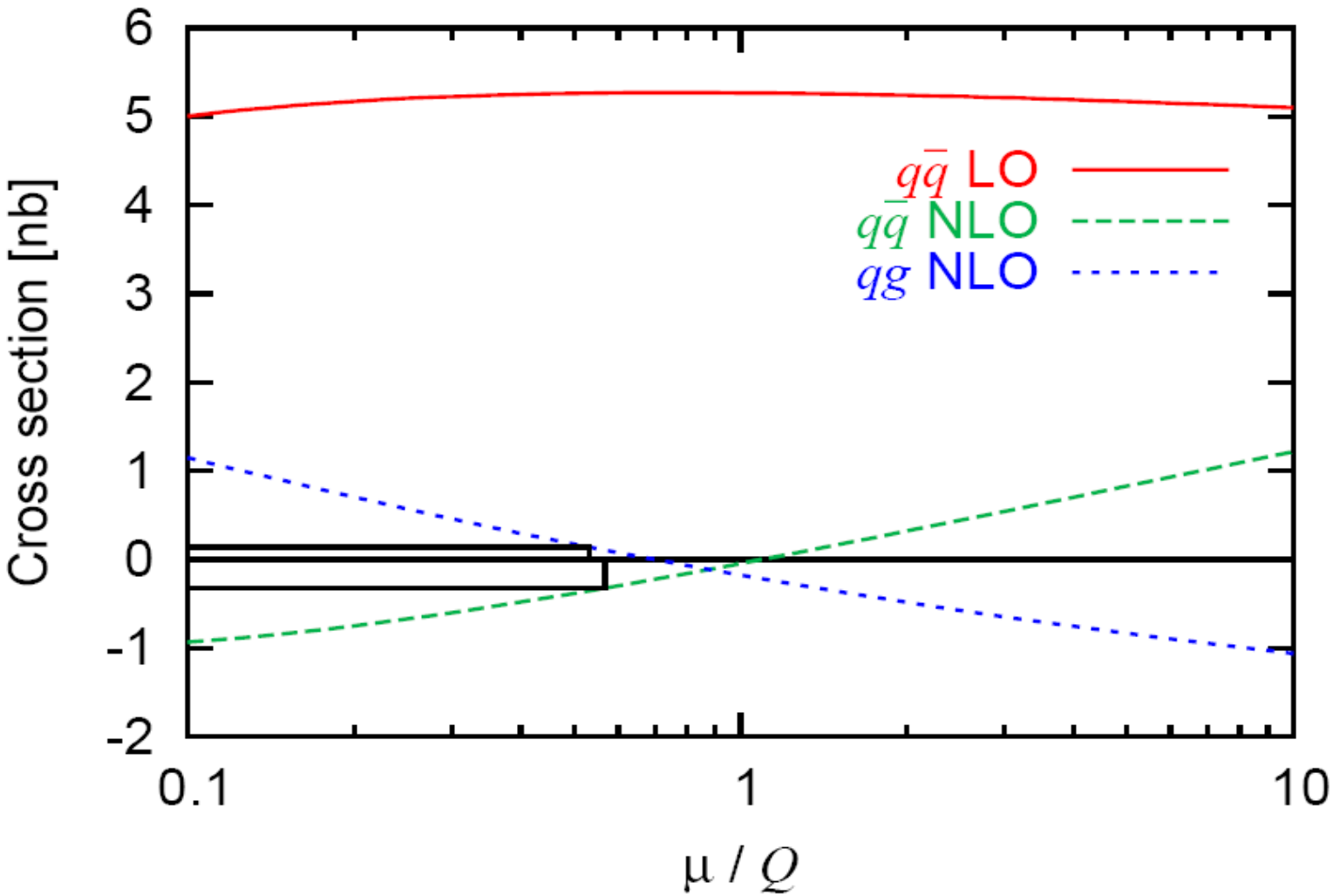
In dimensional regularization, the correction to the quark PDF is

$$\delta q_i(x) = -\frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\, q_i\left(\frac{x}{z}\right)$$
$$\times\left[\left(\frac{1}{\epsilon} - \gamma + \ln\frac{4\pi\mu_D^2}{\mu^2}\right)P_{qq}(z) - \frac{4}{3}(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+\right.$$
$$\left. - \left(\frac{8\pi^2}{9} - \frac{14}{3}\right)\delta(1-z) - \frac{4}{3}(1-z)\right].$$

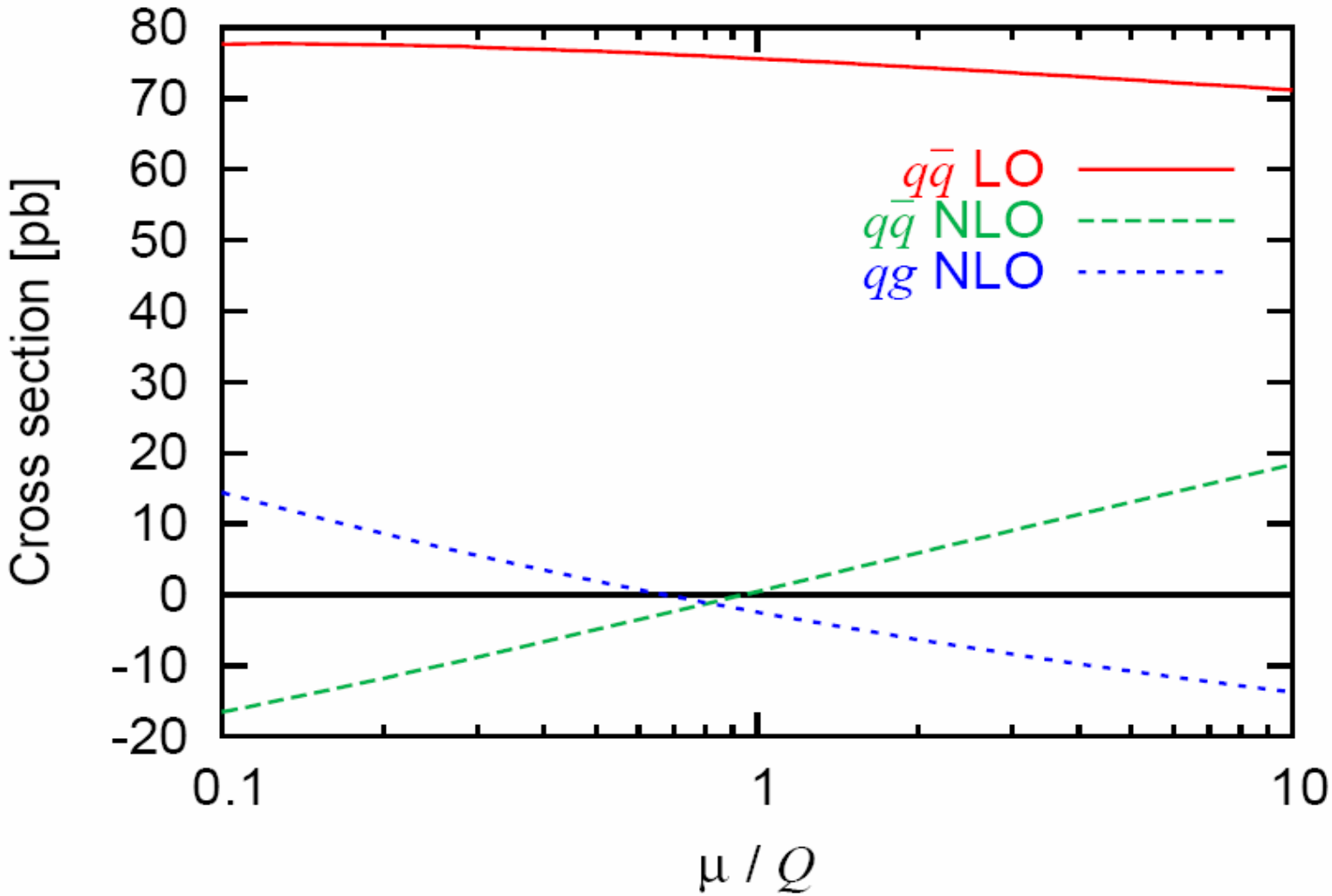
$$p\bar{p} \to Z, Q = 91 \text{ GeV}, \sqrt{S} = 1.96 \text{ TeV}$$



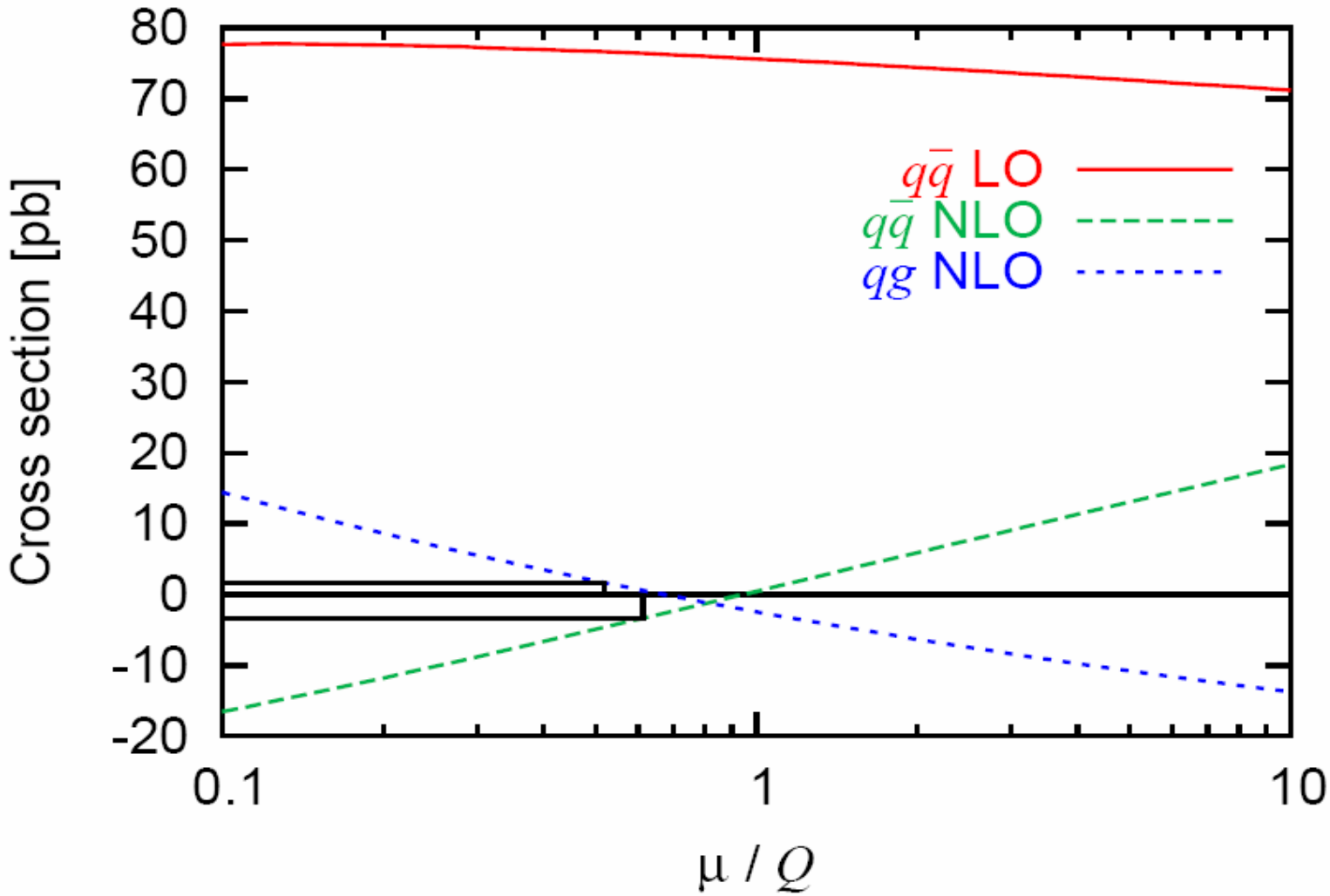
$$p\bar{p} \to Z, Q = 91 \text{ GeV}, \sqrt{S} = 1.96 \text{ TeV}$$



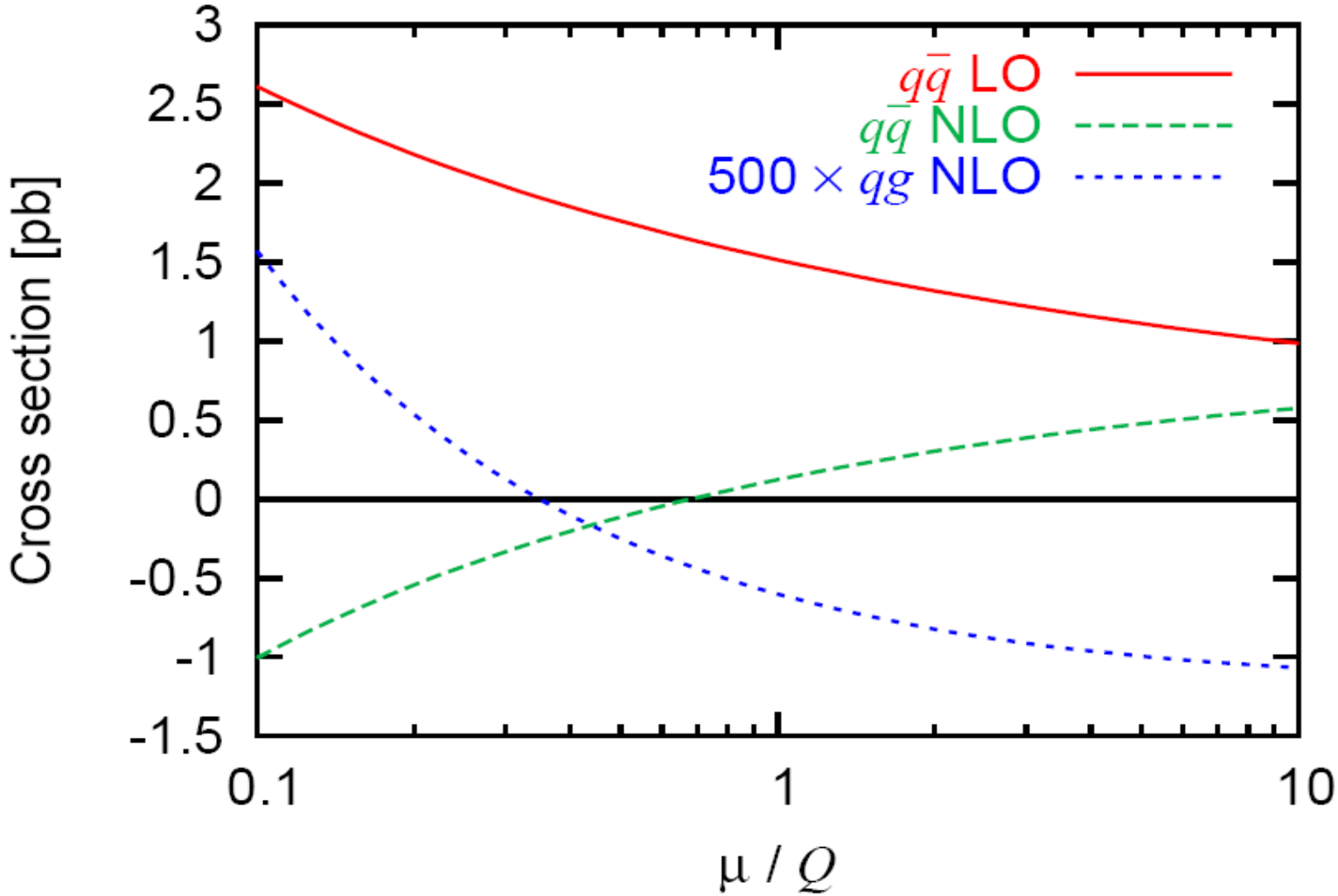
$pp \to Z^*, Q = 650\ \mathrm{GeV}, \sqrt{S} = 14\ \mathrm{TeV}$



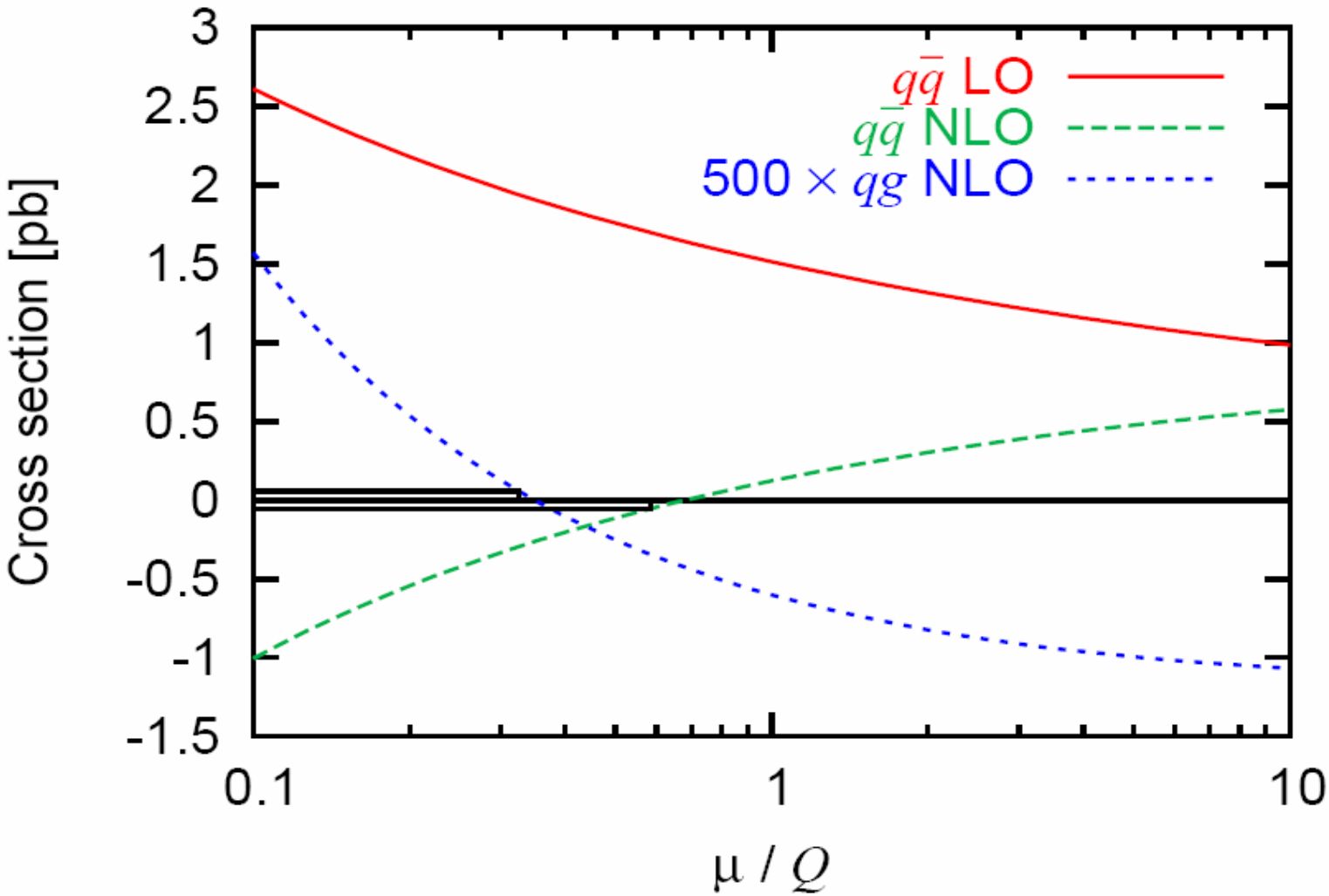
$pp \to Z^*, Q = 650\ \mathrm{GeV}, \sqrt{S} = 14\ \mathrm{TeV}$



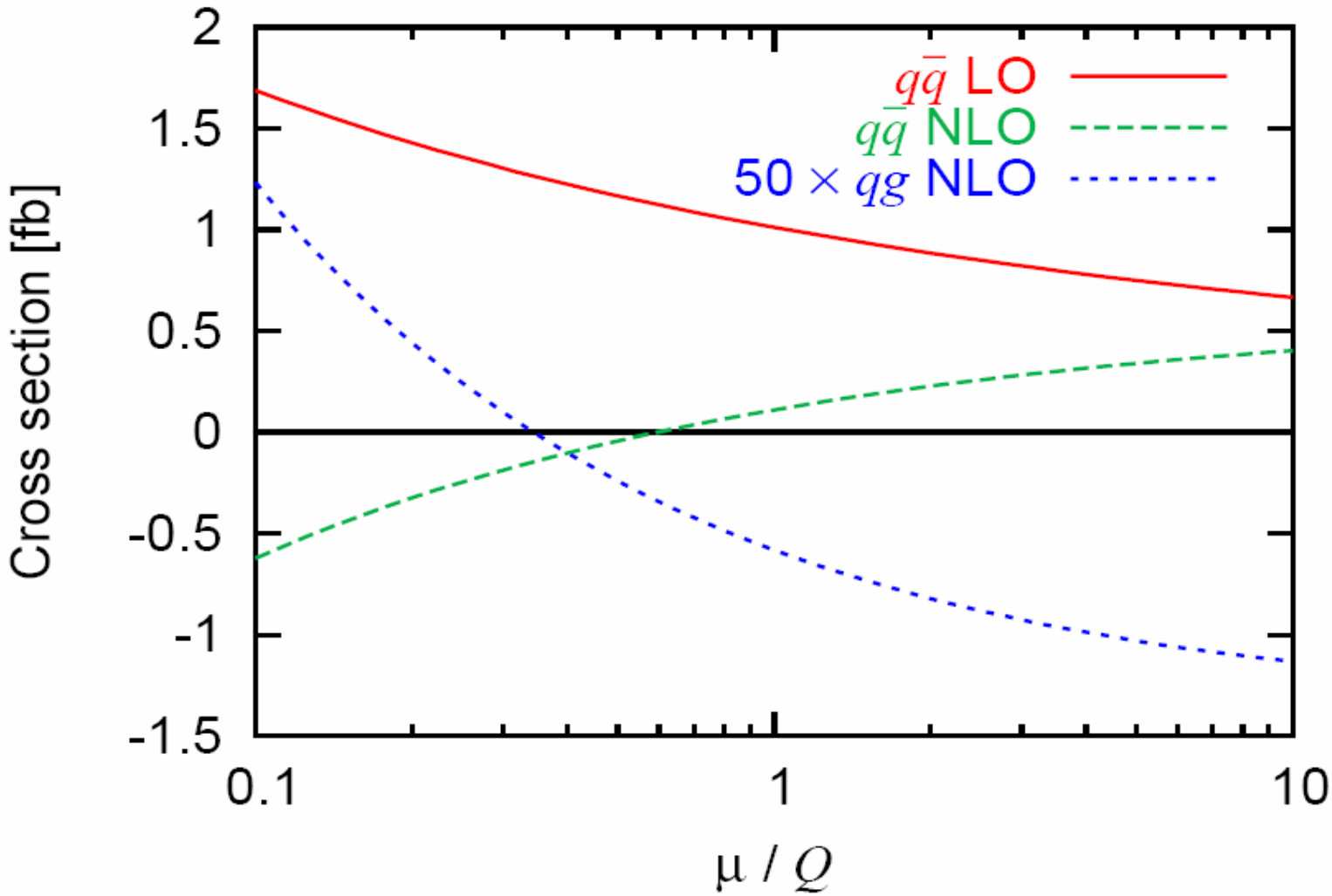
$$p\bar{p} \to Z^*, Q = 700 \text{ GeV}, \sqrt{S} = 1.96 \text{ TeV}$$



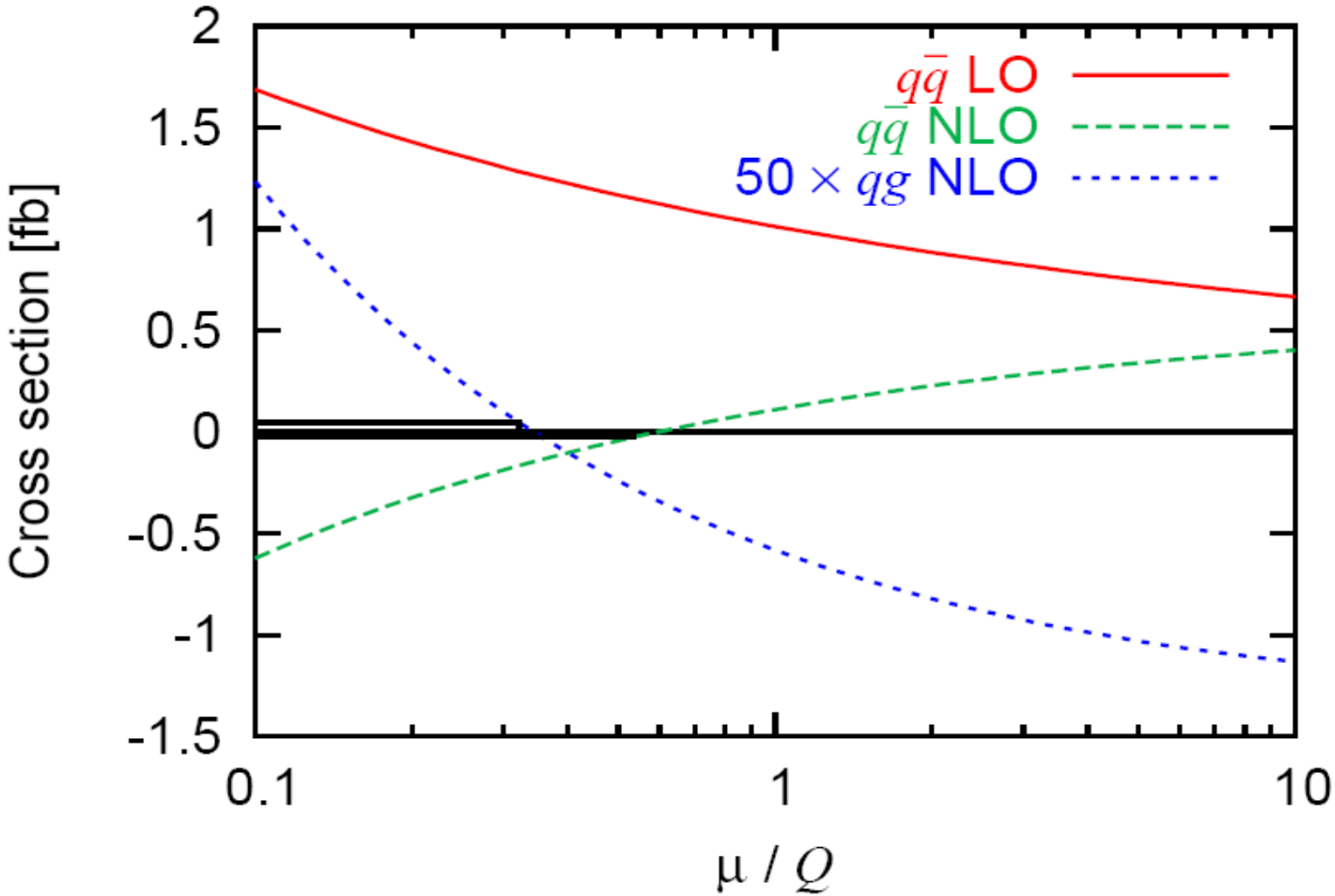
$$p\bar{p} \to Z^*, Q = 700 \text{ GeV}, \sqrt{S} = 1.96 \text{ TeV}$$



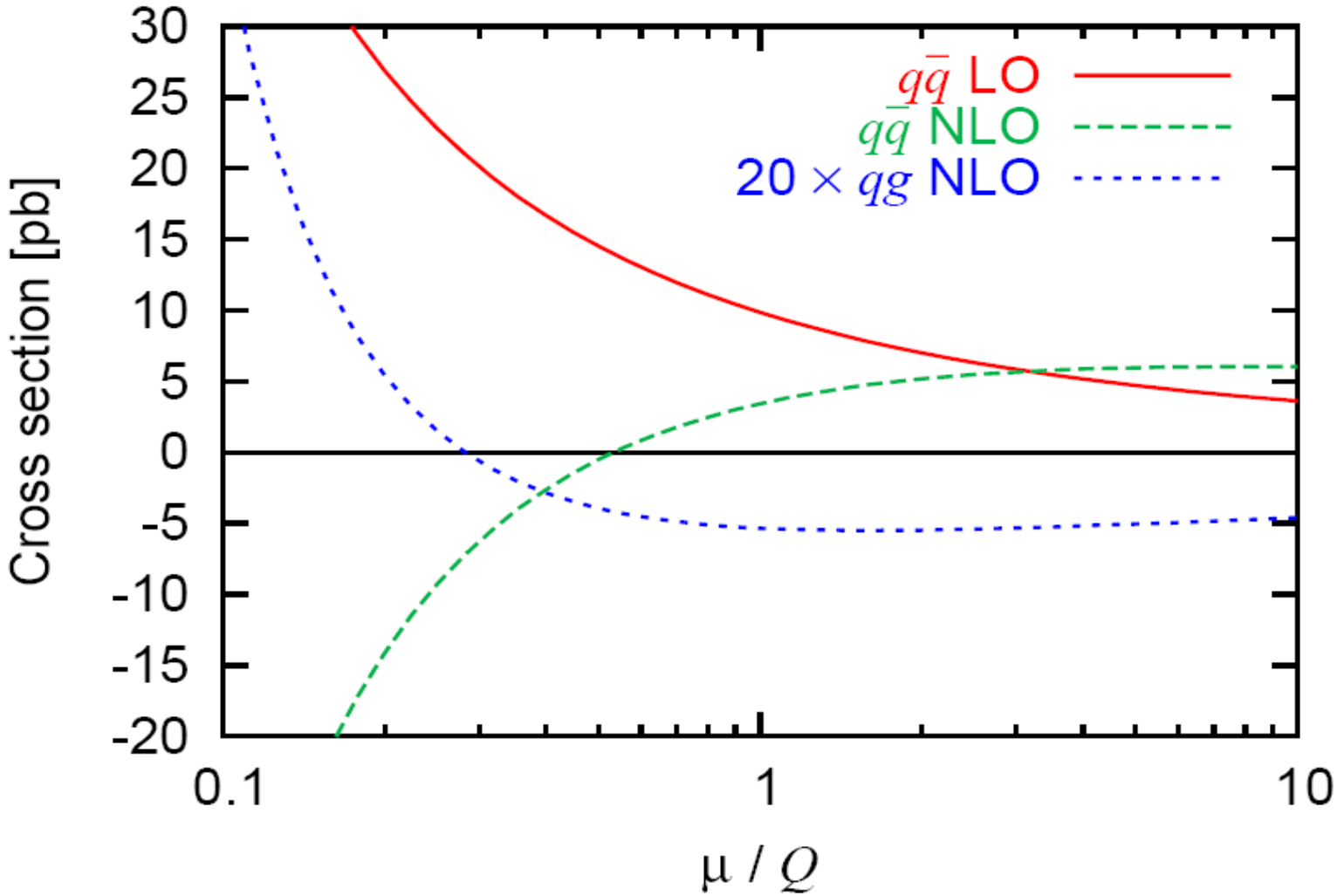
$$pp \to Z^*, Q = 5\ \mathrm{TeV}, \sqrt{S} = 14\ \mathrm{TeV}$$



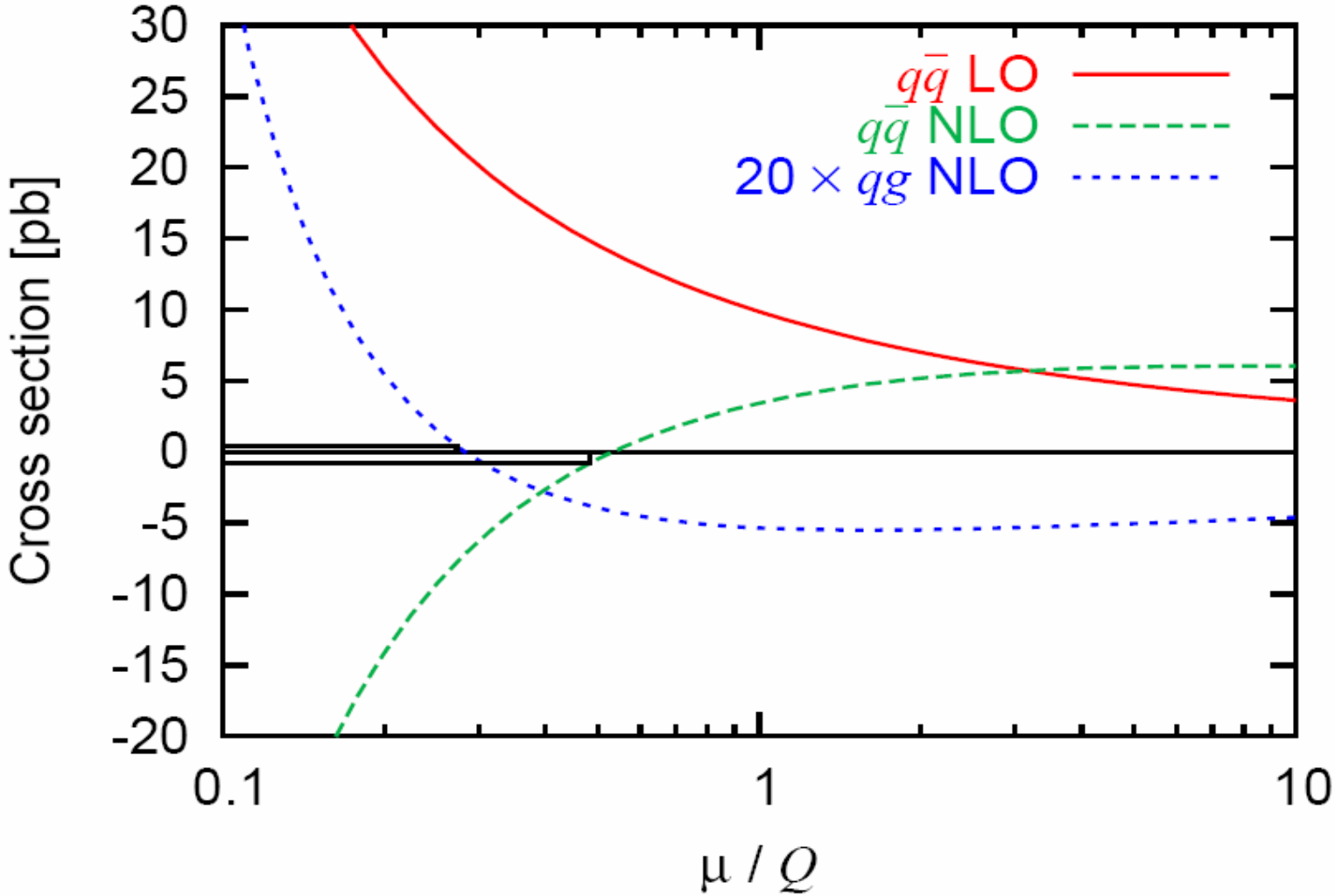
$$pp \to Z^*, Q = 5\ \mathrm{TeV}, \sqrt{S} = 14\ \mathrm{TeV}$$



$pp \to \gamma^*, Q = 20\ \mathrm{GeV}, \sqrt{S} = 38.8\ \mathrm{GeV}$



$pp \to \gamma^*, Q = 20\ \mathrm{GeV}, \sqrt{S} = 38.8\ \mathrm{GeV}$

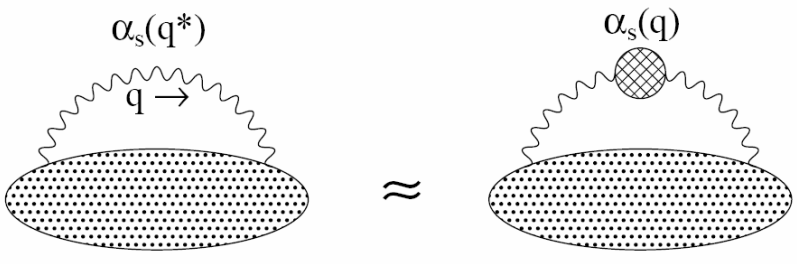


Summary and Outlook

- The collinear factorization scheme absorbs only collinear physics into the PDFs.
- A physically motivated choice of factorization scale is possible in this scheme: $\mu \approx Q/2$ for Drell-Yan.
- Explicit NLO corrections are small for $\mu \approx Q/2$.
- A full collinear-scheme calculation requires the LO cross section to be evaluated with collinear-scheme PDFs.
- The goal: a simple technique to choose the factorization scale for any process.
- It may be possible to translate these results into $\overline{\text{MS}}$.

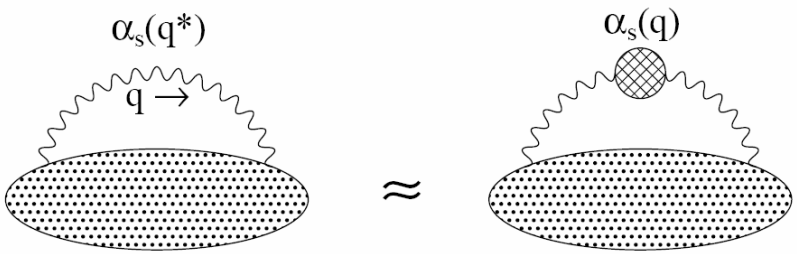
Backup slides

Comparison to BLM



$$\ln(q^{*\,2}) \equiv \frac{\int d^4q\, f(q)\, \ln(q^2)}{\int d^4q\, f(q)}$$

Comparison to BLM



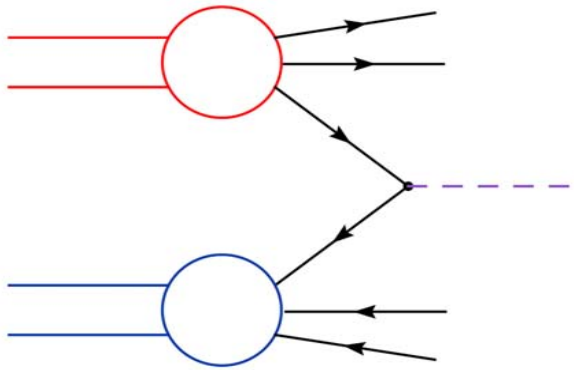
$$\ln(q^{*\,2}) \equiv \frac{\int d^4q\, f(q)\, \ln(q^2)}{\int d^4q\, f(q)}$$

$$\ln \frac{\mu^2}{Q^2} = \frac{\int_{z_0}^1 \frac{dz}{z}\, (g \otimes \bar{q})\left(\frac{z_0}{z}\right) \Delta_{qg}(z)}{\int_{z_0}^1 \frac{dz}{z}\, (g \otimes \bar{q})\left(\frac{z_0}{z}\right) P_{qg}(z)}$$

$$\Delta_{qg}(z) = P_{qg}(z) \ln \frac{1-z}{z}$$

$$\Delta_{q\bar{q}}(z) = \left[\hat{P}_{qq}(z) \ln \frac{1-z}{z}\right]_+ + \frac{4}{3}\delta(1-z)$$

The Parton Model



QCD Processes Factorize at High Energies:

Parton Distribution Function (PDF) of type i

Hard Scattering Cross Section

Hadronic Cross Section

$$\sigma_X(S) = \int_{Q^2}^{S} \frac{ds}{S} (f_i \otimes f_j) \left(\frac{s}{S}\right) \hat{\sigma}_{ij \to X}(s)$$

Final State X

Invariant Mass Squared of Final State X

Partonic CM Energy Squared

Machine CM Energy Squared

The convolution is defined as:

$$(f \otimes g)(x) \equiv \int_0^1 dx_1 \int_0^1 dx_2\, \delta(x - x_1 x_2) f(x_1) g(x_2)$$

The Parton Model Beyond LO

As we calculate in perturbation theory we will encounter three types of divergences:

UV – We know how to deal with these using standard ideas of renormalization.

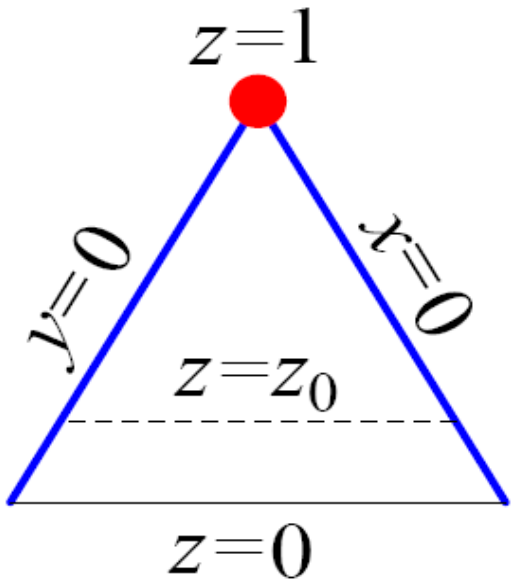
IR – These will cancel in the final cross section.

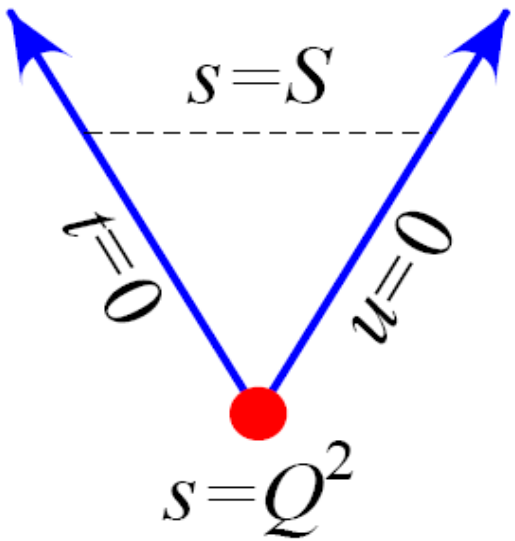
Collinear – These are singularities associated with partons splitting collinearly. To deal with these we redefine our PDFs so that they contain a piece that will cancel off the divergent part of the cross section. This is called *factorization.*

Factorizing (just like UV renormalizing) introduces a momentum scale called the *factorization scale* which the cross section will depend on at any finite order in perturbation theory.

$$\sigma_X(S,\mu) = \int_{Q^2}^{S} \frac{ds}{S} (f_i \otimes f_j)\left(\frac{s}{S},\mu\right) \hat{\sigma}_{ij\to X}(s,\mu)$$

Feynmandelstam





$$x \leftrightarrow \frac{-u}{s}, \qquad y \leftrightarrow \frac{-t}{s}, \qquad z \leftrightarrow \frac{Q^2}{s}$$