What can we learn from semi-inclusive radiative decays of heavy quarkonium?

Xavier Garcia i Tormo

Argonne National Laboratory

(work done with Joan Soto and Nora Brambilla and Antonio Vairo)
Outline of the talk

- Introduction – Heavy quarkonium
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- The decay $\Upsilon(nS) \rightarrow X\gamma$
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- What can we learn from the photon spectrum?
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What can we learn from the photon spectrum?

The nature of heavy quarkonia
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  - The nature of heavy quarkonia
  - $\alpha_s$ extraction
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  - The nature of heavy quarkonia
  - $\alpha_s$ extraction
- Conclusions
Introduction

\[
\begin{align*}
\{ & u, d, s \} \quad \text{light}(m < \Lambda_{\text{QCD}})\text{quarks} \\
& c, b, t \quad \text{heavy}(m > \Lambda_{\text{QCD}})\text{quarks}
\end{align*}
\]
Introduction

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\begin{align*}
\{ & u, d, s \} \quad \text{light} (m < \Lambda_{\text{QCD}}) \text{quarks} \\
& c, b, t \quad \text{heavy} (m > \Lambda_{\text{QCD}}) \text{quarks}
\end{align*}
\]

Heavy quarkonium \rightarrow \text{Meson containing a heavy quark and antiquark} (Q\bar{Q})
Relevant scales in heavy quarkonium

\[ m \quad \text{mass of the heavy quark} \]
\[ mv \quad \text{typical 3-momentum} \]
\[ mv^2 \quad \text{binding energy} \]

Well separated scales
\[ m \gg mv \gg mv^2 \]
Relevant scales in heavy quarkonium

\[
\begin{align*}
    m & \quad \text{hard scale} \\
    mv & \quad \text{soft scale} \\
    mv^2 & \quad \text{ultrasoft scale}
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\]

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Λ_{QCD}
EFT for heavy quarkonium

NRQCD $m \gg mv, mv^2, \Lambda_{QCD}$

Bodwin, Braaten, Lepage '94
\[ \mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \bar{q}_i (\not{D} - m_i) q_i - \frac{1}{4} G^{\mu \nu a} G_{\mu \nu}^a \]

\[ \mathcal{L}_{NRQCD} = \psi^\dagger \left( iD_0 + \frac{1}{2m} \not{D}^2 \right) \psi + \chi^\dagger \left( iD_0 - \frac{1}{2m} \not{D}^2 \right) \chi + \]

\[ + \sum_{i=1}^{n_f} \bar{q}_i \not{D} q_i - \frac{1}{4} G^{\mu \nu a} G_{\mu \nu}^a + \delta \mathcal{L}_{4f} + \cdots \]
EFT for heavy quarkonium

NRQCD $m \gg \mu v, \mu v^2, \Lambda_{QCD}$  
Bodwin, Braaten, Lepage '94

pNRQCD $m \gg \mu v \gg \mu v^2$  
Brambilla, Pineda, Soto, Vairo '99

- Weak coupling regime: $\Lambda_{QCD} \lesssim \mu v^2$
- Strong coupling regime: $\mu v^2 \ll \Lambda_{QCD} \lesssim \mu v$
\[ \mathcal{L}_{\text{NRQCD}} = \psi^\dagger \left( iD_0 + \frac{1}{2m} D^2 \right) \psi + \chi^\dagger \left( iD_0 - \frac{1}{2m} D^2 \right) \chi + \]
\[ + \sum_{i=1}^{n_f} \bar{q}_i i \slashed{D} q_i - \frac{1}{4} G^{\mu\nu} a G^a_{\mu\nu} + \delta \mathcal{L}_{4f} + \cdots \]

\[ \mathcal{L}_{\text{pNRQCD}} = \int d^3 r \text{ Tr } \left\{ S^\dagger \left( i\partial_0 + \frac{C_f \alpha_s}{r} \right) S + \right\}
\[ + O^\dagger \left( iD_0 - \frac{1}{2N_c} \frac{\alpha_s}{r} \right) O \right\} - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu} a + \sum_{i=1}^{n_f} \bar{q}_i i \slashed{D} q_i \]
EFT for heavy quarkonium

- **NRQCD** \( m \gg mv, mv^2, \Lambda_{QCD} \)
  
  Bodwin, Braaten, Lepage '94

- **pNRQCD** \( m \gg mv \gg mv^2 \)
  
  Brambilla, Pineda, Soto, Vairo '99

  - Weak coupling regime: \( \Lambda_{QCD} \lesssim mv^2 \)
  - Strong coupling regime: \( mv^2 \ll \Lambda_{QCD} \lesssim mv \)

**SCET**: describes the interactions of very energetic (collinear) modes with soft degrees of freedom

Bauer et.al. '00; Beneke et.al. '02
\[ \Upsilon(nS) \rightarrow X\gamma \]

That decay has been subject of investigation since the early days of QCD.
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    - Field '83
- We will analyze the decay from an EFT point of view
Data

Recent CLEO measurements of the photon spectra.
Data (1S)

Data (1S)

Data from CLEO III, 2005

Data (2S)

Data from CLEO III, 2005

Data (3S)

Data from CLEO III, 2005

Two types of contributions:
- Fragmentation contributions
- Direct Contributions
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- Fragmentation contributions
- Direct Contributions

\[
\frac{d\Gamma}{dz} = \frac{d\Gamma^{\text{frag}}}{dz} + \frac{d\Gamma^{\text{dir}}}{dz}
\]
Fragmentation Contributions

Electromagnetic couplings to light quarks.

\[
\frac{d\Gamma^{frag}}{dz} = \sum_a C_a \otimes D_{a\rightarrow\gamma}
\]
This type of contributions become important at low $z$.
Direct contributions

Electromagnetic couplings to heavy quarks.
The NRQCD formalism organizes the decay as:

\[
\frac{d\Gamma}{dz} = \sum_i C_i(M, z) \langle \Upsilon | \mathcal{O}_i | \Upsilon \rangle
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Expansion in powers of \( v \):
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\frac{d\Gamma}{dz} = \sum_i C_i(M, z) \langle \Upsilon | O_i | \Upsilon \rangle
\]

Expansion in powers of \( v \):

- **Leading Order (LO) operator** (Color Singlet Model)

\[
O_1 (^3S_1) \rightarrow O (\alpha_s^2 \alpha_{em})
\]
The NRQCD formalism organizes the decay as:

\[ \frac{d\Gamma}{dz} = \sum_i C_i(M, z) \langle \Upsilon | O_i | \Upsilon \rangle \]

Expansion in powers of \( \nu \):

- **Leading Order (LO)** operator (Color Singlet Model)
  
  \[ O_1 \left( ^3S_1 \right) \rightarrow O \left( \alpha_s^2 \alpha_{em} \right) \]

- **Next-to-Leading Order (NLO)** (\( \nu^4 \) suppressed)
  
  \[ O_8 \left( ^1S_0 \right), O_8 \left( ^3P_J \right) \alpha \delta(1-z) \]
But this is only true in the central $z$ region.

For $z \rightarrow 0$ the photon can only cause transitions within the bound state. But we can ignore that effect, since the fragmentation contributions dominate in this region.
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For $z \to 1$

Higher order corrections to the coefficients have large logs $\log(1 - z)$

Krämer '99; Maltoni and Petrelli '98
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- For $z \rightarrow 0$ the photon can only cause transitions within the bound state. But we can ignore that effect, since the fragmentation contributions dominate in this region.

- For $z \rightarrow 1$
  - Higher order corrections to the coefficients have large logs $\log(1 - z)$
  - NRQCD Operator Product Expansion breaks down
  - Shape functions resumming a certain class of operators must be introduced

Rothstein and Wise '97

Krämer '99; Maltoni and Petrelli '98
The upper end-point region

Collinear degrees of freedom are also relevant in the upper end-point region. Need combined NRQCD+SCET analysis
The decay rate has been expressed in the factorized form:

\[
\frac{d\Gamma}{dz} = \sum_\omega H(M, \omega, \mu) \int dk^+ S(k^+, \mu) \text{Im} J_\omega \left( k^+ + M (1 - z), \mu \right)
\]
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There are contributions from color singlet and color octet (P- and S-wave) operators.

Sudakov logarithms have been resummed for both the color singlet and the color octet operators.

Bauer et al. '01; Fleming and Leibovich '02 '04
If one considers the $\Upsilon(1S)$ as a Coulombic state ($m\alpha_s \gg \Lambda_{\text{QCD}}$) the octet shape functions can be calculated.
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We will calculate the octet shape functions in the weak (small) coupling regime

The calculation is reliable for $z \gtrsim 0.7 \ (1 - z \ll 1)$ and $z \lesssim 0.92 \ (M(1 - z) \gtrsim 1 GeV)$

X.G.T. and Soto '04
The calculation is a combination of potential NRQCD (pNRQCD) and SCET
Singlet field

Octet field
SCET octet operators

ultrasoft gluon $p \sim (m_\alpha^2, m_\alpha^2, m_\alpha^2)$

$$\frac{igc_F}{\sqrt{N_cT_F}} \frac{(\sigma_1 - \sigma_2)}{2m} \text{Tr} \left[ T^b B \right]$$
Collinear gluon $p_+, p_\perp \ll M\sqrt{1-z}$

$$\frac{1}{M^2(1-z) - Mk_+ - k^2_\perp + i\varepsilon}$$
\[ T_{(8, 1^S_0)}^{\alpha \alpha'}(z) = -i \eta_{\alpha \alpha'}(4\pi) \frac{32}{3} T_F^2 \left( \frac{e_F}{2m} \right)^2 \alpha_s(\mu_u) C_f \times \]

\[ \times \int d^3x \int d^3x' \psi_{n0}^*(x') \psi_{n0}(x) \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{k^2 + i\epsilon} \times \]

\[ \times \left( \frac{1}{-k_0 + E_n - h_o + i\epsilon} \right)_{x',0} \left( \frac{1}{M(1 - z) - k_+} \right) \left( \frac{1}{M - k_0^2 + i\epsilon} \right)_{0,x} \]
\[
\text{Im} \left( T_{(8, 1 S_0)}^{\alpha \alpha'} (z) \right) = -\eta_{\perp}^{\alpha \alpha'} \frac{16}{3} T_F^2 \left( \frac{c_F}{2m} \right)^2 \alpha_s(\mu_u) C_f \frac{1}{M} \int_0^\infty dk_+ \delta(M(1 - z) - k_+) \times \\
\times \int_0^\infty dx \left( 2\psi_{10}(0) I_S \left( \frac{k_+}{2} + x \right) - I_S^2 \left( \frac{k_+}{2} + x \right) \right) 
\]

\[
\text{Im} \left( T_{(8, 1 S_0)}^{\alpha \alpha'} (z) \right) = -\eta_{\perp}^{\alpha \alpha'} \int dl_+ S_S (l_+) \text{Im} J_M (l_+ - M(1 - z)) 
\]

\[
\text{Im} J_M (l_+ - M(1 - z)) = T_F^2 \left( N_c^2 - 1 \right) \frac{2\pi}{M} \delta(M(1 - z) - l_+) 
\]

\[
S_S (l_+) = \frac{4\alpha_s(\mu_u)}{3\pi N_c} \left( \frac{c_F}{2m} \right)^2 \int_0^\infty dx \left( 2\psi_{10}(0) I_S \left( \frac{l_+}{2} + x \right) - I_S^2 \left( \frac{l_+}{2} + x \right) \right) 
\]
We obtain the shape functions

\[ S_S(l_+) := \frac{4\alpha_s(\mu_u)}{3\pi N_c} \left( \frac{c_F}{2m} \right)^2 \int_0^\infty dx \left( 2\psi_{10}(0)I_S\left(\frac{l_+}{2} + x\right) - I_S^2\left(\frac{l_+}{2} + x\right) \right) \]

\[ S_{P1}(l_+) := \frac{\alpha_s(\mu_u)}{6\pi N_c} \int_0^\infty dx \left( 2\psi_{10}(0)I_P\left(\frac{l_+}{2} + x\right) - I_P^2\left(\frac{l_+}{2} + x\right) \right) \]

\[ S_{P2}(l_+) := \frac{\alpha_s(\mu_u)}{6\pi N_c} \int_0^\infty dx \frac{8l_+x}{(l_+ + 2x)^2} \left( \psi_{10}^2(0) - 2\psi_{10}(0)I_P\left(\frac{l_+}{2} + x\right) + I_P^2\left(\frac{l_+}{2} + x\right) \right) \]

where

\[ I_S\left(\frac{k_+}{2} + x\right) := m\sqrt{\frac{\gamma}{\pi}} \frac{\alpha_s N_c}{2} \frac{1}{1 - z'} \left( 1 - \frac{2z'}{1 + z'} \right) _2F_1 \left( -\frac{1}{z'}, 1, 1 - \frac{\lambda}{z'}, \frac{1 - z'}{1 + z'} \right) \]

\[ I_P\left(\frac{k_+}{2} + x\right) := \sqrt{\frac{\gamma^3}{\pi}} \frac{8}{3} \frac{(2 - \lambda)}{4(1 + z')^3} \left( 2(1 + z')(2 + z') + (5 + 3z')(-1 + \lambda) + 2(-1 + \lambda)^2 + \frac{1}{(1 - z')^2} \left( 4z'(1 + z')(z'^2 - \lambda^2) \left( -1 + \frac{\lambda(1 - z')}{(1 + z')(z' - \lambda)} \right) + 2F_1 \left( -\frac{\lambda}{z'}, 1, 1 - \frac{\lambda}{z'}, \frac{1 - z'}{1 + z'} \right) \right) \right) \]

\[ \gamma = \frac{mC_f\alpha_s}{2} \quad z' = \frac{\kappa}{\gamma} \quad \frac{\kappa^2}{m} = E_1 - \frac{k_+}{2} - x \quad \lambda = -\frac{1}{2N_cC_f} \]
We obtain the shape functions

\[ S_S(l_+) := \frac{4\alpha_s(\mu_u)}{3\pi N_c} \left( \frac{c_F}{2m} \right)^2 \int_0^\infty dx \left( 2\psi_{10}(0)I_S\left(\frac{l_+}{2} + x\right) - I_S^2\left(\frac{l_+}{2} + x\right) \right) \]

\[ S_{P1}(l_+) := \frac{\alpha_s(\mu_u)}{6\pi N_c} \int_0^\infty dx \left( 2\psi_{10}(0)I_P\left(\frac{l_+}{2} + x\right) - I_P^2\left(\frac{l_+}{2} + x\right) \right) \]

\[ S_{P2}(l_+) := \frac{\alpha_s(\mu_u)}{6\pi N_c} \int_0^\infty dx \frac{8l_+x}{(l_+ + 2x)^2} \left( \psi_{10}^2(0) - 2\psi_{10}(0)I_P\left(\frac{l_+}{2} + x\right) + I_P^2\left(\frac{l_+}{2} + x\right) \right) \]

The shape functions are UV divergent and need to be regularized and renormalized

\[ \frac{1}{\varepsilon} + \ln \left( \frac{\mu}{\frac{l_+}{2} + \frac{\gamma^2}{m}} \right) \]
\[ \Lambda = 300 \text{ MeV} \]

\[ z = \frac{2E_\gamma}{M_\gamma} \]

The complete photon spectrum

- The end-point region of the spectrum can be well described
- For $z$ in the central region the NRQCD formulas should work
The complete photon spectrum

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- Different approximations are needed in the different regions
The complete photon spectrum

- The end-point region of the spectrum can be well described.
- For $z$ in the central region the NRQCD formulas should work.
- Different approximations are needed in the different regions.
  - Results for the central region in the end-point → miss Sudakov and Coulomb resummations.
  - Results for the end-point in the central region → miss non-trivial functions of $z$. 
Choose cuts $z_1, z_2$ and interpolate

Introduce theoretical ambiguities
Central region

\[ \nu \text{ counting} \rightarrow \alpha_s (\mu_h) \sim \nu^2 \quad \alpha_s (\mu_s) \sim \nu \]
Central region

$v$ counting $\rightarrow \alpha_s (\mu_h) \sim v^2$ $\alpha_s (\mu_s) \sim v$

LO $\rightarrow \cdot C_1 \left[ ^3S_1 \right] \frac{\langle \Upsilon(1S)|O_1(^3S_1)|\Upsilon(1S)\rangle}{m^2}$

NLO ($v^2$ suppressed) $\rightarrow \cdot C'_1 \left[ ^3S_1 \right] \frac{\langle \Upsilon(1S)|P_1(^3S_1)|\Upsilon(1S)\rangle}{m^4}$

$\cdot \alpha_s (\mu_h)$ corrections to $C_1 \left[ ^3S_1 \right]$  

$\cdot \alpha_s^2 (\mu_s)$ corrections to wave function
End-point region

\[ 1 - z \ SCET\ counting \]
End-point region

$1 - z$ SCET counting

- Both color singlet and color octet contributions appear at leading order
- Final result incorporates Sudakov and Coulomb resummations and involves $\alpha_s$ at several different scales: $M \sqrt{(1 - z)}$, $mv$, ...
Mismatch between the two expressions
Expand the end-point expressions for \( z \) in the central region
Expand the end-point expressions for $z$ in the central region

Check: the $(1 - z) \log(1 - z)$, the logs of the relativistic corrections and the $\alpha_s \log(1 - z)$ are correctly reproduced (in the limit $z \rightarrow 1$)
Expand the end-point expressions for \( z \) in the central region

Check: the \((1 - z) \log(1 - z)\), the logs of the relativistic corrections and the \( \alpha_s \log(1 - z) \) are correctly reproduced (in the limit \( z \to 1 \))

Use the formula

\[
\frac{1}{\Gamma_0} \frac{d\Gamma_{\text{dir}}}{dz} = \frac{1}{\Gamma_0} \frac{d\Gamma_c}{dz} + \left( \frac{1}{\Gamma_0} \frac{d\Gamma^e}{dz} - \frac{1}{\Gamma_0} \frac{d\Gamma^e}{dz} \bigg|_c \right)
\]

X.G.T. and Soto ’05
\[ \frac{dF_{\text{dir}}}{dz} \]

\( \nu^2 \) NRQCD (central region) expression

end-point expression
Sum the direct and fragmentation contributions to obtain the total spectrum
The photon spectrum can be well described when all the contributions are consistently included.
The photon spectrum can be well described when all the contributions are consistently included

So, now the question is...
What can we learn from it?
What can we learn from it?

- The nature of heavy quarkonium
- $\alpha_s$ extraction
The nature of heavy quarkonia

Remember

weak coupling regime  strong coupling regime

\[ \Lambda_{QCD} \lesssim m v^2 \]
\[ m v^2 \ll \Lambda_{QCD} \lesssim m v \]
The nature of heavy quarkonia

Remember

weak coupling regime \[ \Lambda_{QCD} \lesssim m v^2 \]
strong coupling regime \[ m v^2 \ll \Lambda_{QCD} \lesssim m v \]

But those scales are not directly accessible experimentally
The nature of heavy quarkonia

Remember

weak coupling regime   strong coupling regime

\[ \Lambda_{QCD} \lesssim \mu v^2 \]
\[ \mu v^2 \ll \Lambda_{QCD} \lesssim \mu v \]

But those scales are not directly accessible experimentally

*Given a heavy quarkonium state, to which regime does it belong to?*
The nature of heavy quarkonia

Remember

**weak coupling regime**  **strong coupling regime**

\[ \Lambda_{QCD} \lesssim mv^2 \]
\[ mv^2 \ll \Lambda_{QCD} \lesssim mv \]

But those scales are not directly accessible experimentally

*Given a heavy quarkonium state, to which regime does it belong to?*

The photon spectra can tell you.
In the central region

\[ \frac{d\Gamma_n}{dz} = \left( C_1 \left[ ^3S_1 \right](z) + f\mathcal{O}_1(^3S_1)(z) \right) \frac{\langle\mathcal{O}_1(^3S_1)\rangle_n}{m^2} + \]

\[ + C_1' \left[ ^3S_1 \right](z) \frac{\langle\mathcal{P}_1(^3S_1)\rangle_n}{m^4} + f\mathcal{O}_8(^3S_1)(z) \frac{\langle\mathcal{O}_8(^3S_1)\rangle_n}{m^2} + \]

\[ + f\mathcal{O}_8(^1S_0)(z) \frac{\langle\mathcal{O}_8(^1S_0)\rangle_n}{m^2} + f\mathcal{O}_8(^3P_J)(z) \frac{\langle\mathcal{O}_8(^3P_0)\rangle_n}{m^4} + \]

\[ + f\mathcal{O}_8(^3P_J)(z) \frac{\langle\mathcal{O}_8(^3P_J)\rangle_n}{m^4} + \]
**Strong coupling:**

- Octet matrix elements reduce to wave function at the origin times bound-state independent parameters

\[
\langle \Upsilon(nS)|O_8(^1S_0)|\Upsilon(nS)\rangle = C_A \frac{|R_n(0)|^2}{2\pi} \left(\frac{(C_f - C_A/2)c_F^2B_1}{3m^2}\right)
\]

**Weak coupling:**

- Octet matrix elements depends non-trivially on \( n \)
Two states in the strong coupling regime

\[
\frac{d\Gamma_n}{dz} = \frac{\langle \mathcal{O}_1 (^3S_1) \rangle_n}{\langle \mathcal{O}_1 (^3S_1) \rangle_r} \left( 1 + \frac{C'_1 [^3S_1] (z)}{C_1 [^3S_1] (z)} \frac{1}{m} (E_n - E_r) \right)
\]

Model-independent formula. Holds at NLO
Two states in the strong coupling regime

\[
\frac{d\Gamma_n}{d\Gamma_r} \frac{dz}{dz} = \frac{\langle O_1(3S_1) \rangle_n}{\langle O_1(3S_1) \rangle_r} \left( 1 + \frac{C_1' [3S_1] (z)}{C_1 [3S_1] (z)} \frac{1}{m} (E_n - E_r) \right)
\]

if not

\[
\frac{d\Gamma_n}{d\Gamma_r} \frac{dz}{dz} = \frac{\langle O_1(3S_1) \rangle_n}{\langle O_1(3S_1) \rangle_r} \left( 1 + \frac{C_1' [3S_1] (z)}{C_1 [3S_1] (z)} \frac{R^{nr}_{P1(3S_1)}}{m^2} \right)
\]

\[
\frac{f_{O_8(3S_1)} (z)}{C_1 [3S_1] (z)} R^{nr}_{O_8(3S_1)} + \frac{f_{O_8(1S_0)} (z)}{C_1 [3S_1] (z)} R^{nr}_{O_8(1S_0)} + \frac{f_{O_8(3P_J)} (z)}{C_1 [3S_1] (z)} R^{nr}_{O_8(3P_0)}
\]

X.G.T. and Soto '05
\[ \chi^2 = 1.2 \rightarrow 18\% CL \]
\[ \chi^2 = 0.9 \rightarrow 68\% \text{CL} \]
\[ \chi^2 = 0.75 \rightarrow 89\%\text{CL} \]
Errors are large

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Same analysis can be performed for charmonium ($J/\Psi$ and $\Psi'$)
The $\alpha_s$ extraction
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Extract $\alpha_s$ from the ratio

$$R_\gamma = \frac{\Gamma_{gg\gamma}^{\text{direct}}}{\Gamma_{ggg}} = \frac{N_{gg\gamma}^{\text{direct}}}{N_{ggg}}$$
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From the measured photon spectrum plus a theoretical model (to extrapolate down to $z = 0$)

$N_{gg\gamma}$ can be extracted
Extract $\alpha_s$ from the ratio

$$R_\gamma = \frac{\Gamma_{\text{direct}}^{gg\gamma}}{\Gamma_{ggg}} = \frac{N_{\text{direct}}^{gg\gamma}}{N_{ggg}}$$

From the measured photon spectrum plus a theoretical model (to extrapolate down to $z = 0$)

- $N_{gg\gamma}$ can be extracted
- $N_{ggg}$ can also be determined experimentally
Extract $\alpha_s$ from the ratio

$$R_\gamma = \frac{\Gamma_{gg\gamma}^{\text{direct}}}{\Gamma_{ggg}} = \frac{N_{gg\gamma}^{\text{direct}}}{N_{ggg}}$$

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- $N_{gg\gamma}$ can be extracted
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Compare then with a theoretical expression for $R_\gamma$ to extract $\alpha_s$
Relativistic corrections cancel at first order (they are the same for $\Upsilon \rightarrow ggg$ and $\Upsilon \rightarrow gg\gamma$)
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The CLEO paper used
  - Field’s model and our calculation to extrapolate down to $z = 0$.
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The idea is to use the same counting employed for the calculation of the spectrum to extract $\alpha_s$
NRQCD is OK for the total width. The counting used

\[ \alpha_s(\mu_h) \sim v^2 \quad \alpha_s(\mu_s) \sim v \]
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We include all the terms up to \( O(v^2) \): radiative corrections, relativistic corrections and octet operators.
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We include all the terms up to \( \mathcal{O}(v^2) \): radiative corrections, relativistic corrections and octet operators

Inclusion of all those terms well known to be potentially important

Data is now very precise and we have all the necessary theoretical ingredients to include all the pieces
Schematically

\[
\frac{\Gamma_{gg\gamma}}{\Gamma_{ggg}} = \frac{C_{\gamma O_1(3S_1)O_1(3S_1)} + C_{\gamma P_1(3S_1)P_1(3S_1)} + C_{\gamma O_8(1S_0,3P_0)O_8(1S_0,3P_0)}}{C_{O_1(3S_1)O_1(3S_1)} + C_{P_1(3S_1)P_1(3S_1)} + C_{O_8(1S_0,3P_0,3S_1)O_8(1S_0,3P_0,3S_1)}}
\]
Schematically

\[
\frac{\Gamma_{gg\gamma}}{\Gamma_{ggg}} = \frac{C_{\gamma O_1(3S_1) O_1(3S_1)} + C_{\mathcal{P}_1(3S_1) \mathcal{P}_1(3S_1)} + C_{\gamma O_8(1S_0, 3P_0) O_8(1S_0, 3P_0)}}{C_{O_1(3S_1) O_1(3S_1)} + C_{\mathcal{P}_1(3S_1) \mathcal{P}_1(3S_1)} + C_{O_8(1S_0, 3P_0, 3S_1) O_8(1S_0, 3P_0, 3S_1)}}
\]

- \(\mathcal{O}_8(1S_0)\) and \(\mathcal{O}_8(3P_0)\) have been estimated in the continuum (weak coupling)
  
  X.G.T. and Soto '04

- \(\mathcal{O}_8(3S_1)\) and \(\mathcal{O}_8(1S_0)\) have been calculated on the lattice
  
  Bodwin, Lee and Sinclair '05
Two different extractions

- C (for continuum)

- L (for lattice)
Two different extractions

- **C** (for continuum)
  - Uses all the weak-coupling expressions available and lattice calculation for $\mathcal{O}_8(3S_1)$

- **L** (for lattice)
  - Uses all lattice calculations available and NRQCD $\nu$-scaling for $\mathcal{O}_8(3P_0)$
Two different extractions

- C (for continuum)
  - Uses all the weak-coupling expressions available and lattice calculation for $\mathcal{O}_8(^3S_1)$

- L (for lattice)
  - Uses all lattice calculations available and NRQCD $v$-scaling for $\mathcal{O}_8(^3P_0)$

The two procedures give very similar results. We take the average as the final value
Error estimation

- C (for continuum)

- L (for lattice)
Error estimation

C (for continuum)

\[ 0.18 \leq \alpha_s(m_b v) \leq 0.38 \]
\[ 0.32 \leq \alpha_s(m_b v^2) \leq 1.3 \]
\[ 0 \leq \mathcal{R}_{O_8(3S_1)} \leq 1.6 \times 10^{-4} \]

L (for lattice)

\[ 0 \leq \mathcal{R}_{O_8(1S_0)} \leq 4.8 \times 10^{-3} \]
\[ 0 \leq \mathcal{R}_{O_8(3S_1)} \leq 1.6 \times 10^{-4} \]
\[ -2.4 \times 10^{-4} \leq \mathcal{R}_{O_8(3P_0)} \leq 2.4 \times 10^{-4} \]
\[ -0.052 \leq \mathcal{R}_{P_1(3S_1)} \leq -0.035 \]

Plus errors associated to higher order terms (\(v^3\)) and experimental errors
The results are rather insensitive to the values of $O_8(1S_0)$ and $O_8(3P_0)$.
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Much more sensitive to $O_8(\,^3S_1\,)$ (but this matrix element is small according to the lattice)
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The error in the final result is taken as the full range of the two determinations
Our results (always from the $\Upsilon(1S)$ data. We have used $R_\gamma$ extracted with our calculation, not Field)
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\alpha_s(M_{\Upsilon(1S)}) = 0.184^{+0.014}_{-0.013} \rightarrow \alpha_s(M_Z) = 0.119^{+0.006}_{-0.005}
\]

N.Brambilla, X.G.T., J.Soto, A.Vairo ’07
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Recall, PDG average \rightarrow \alpha_s(M_{Z}) = 0.1176 \pm 0.0020
As/\( \Lambda \)M './RParen1\( \)CΑpUpsilon\( \)decay Lattice Τ decays DIS DIS Polarized DIS ep event shapes Z width e'\( \)e' rates Hadronic jets Lattice Y decay \( \alpha_s(M_Z) \)

Fermilab - March 1, 2007  •  What can we learn from semi-inclusive radiative decays of heavy quarkonium? – p.46/47
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Conclusions

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**What can we learn from semi-inclusive radiative decays of heavy quarkonium?**

- Precise measurements of the photon spectrum will help in identifying the nature of heavy quarkonium.
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What can we learn from semi-inclusive radiative decays of heavy quarkonium?

- Precise measurements of the photon spectrum will help in identifying the nature of heavy quarkonium.
- We can obtain a consistent and precise determination of $\alpha_s$ from the radiative decay width.
- Much to be learned from the photon spectrum!