What can we learn from semi-inclusive radiative decays of heavy quarkonium?

Xavier Garcia i Tormo

Argonne National Laboratory

(work done with Joan Soto
and Nora Brambilla and Antonio Vairo)

Outline of the talk

- Introduction – Heavy quarkonium

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- The decay $\Upsilon(nS) \to X\gamma$

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  - The nature of heavy quarkonia
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- Conclusions

Introduction

$$\left.\begin{array}{c} u \\ d \\ s \end{array}\right\} \text{light}(m < \Lambda_{\mathrm{QCD}})\text{quarks} \quad \left.\begin{array}{c} c \\ b \\ t \end{array}\right\} \text{heavy}(m > \Lambda_{\mathrm{QCD}})\text{quarks}$$

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Heavy quarkonium $\longrightarrow$ Meson containing a heavy quark and antiquark$(Q\bar{Q})$

Relevant scales in heavy quarkonium

m	mass of the heavy quark	Well separated scales
mv	typical 3-momentum	$m \gg mv \gg mv^2$
mv^2	binding energy	

Relevant scales in heavy quarkonium

$$\left.\begin{array}{ll} m & \textit{hard}\text{ scale} \\ mv & \textit{soft}\text{ scale} \\ mv^2 & \textit{ultrasoft}\text{ scale} \end{array}\right\}$$

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$$\Lambda_{\text{QCD}}$$

EFT for heavy quarkonium

- NRQCD $m \gg mv, mv^2, \Lambda_{QCD}$ Bodwin, Braaten,Lepage '94

$$\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \bar{q}_i (i\not{D} - m_i) q_i - \frac{1}{4} G^{\mu\nu\,a} G^a_{\mu\nu}$$

$$\downarrow$$

$$\mathcal{L}_{NRQCD} = \psi^\dagger \left(iD_0 + \frac{1}{2m}\mathbf{D}^2 \right) \psi + \chi^\dagger \left(iD_0 - \frac{1}{2m}\mathbf{D}^2 \right) \chi +$$

$$+ \sum_{i=1}^{n_f} \bar{q}_i \, i\not{D} \, q_i - \frac{1}{4} G^{\mu\nu\,a} G^a_{\mu\nu} + \delta\mathcal{L}_{4f} + \cdots$$

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- pNRQCD $m \gg mv \gg mv^2$ Brambilla, Pineda, Soto, Vairo '99
  - Weak coupling regime: $\Lambda_{QCD} \lesssim mv^2$
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$$\Downarrow$$

$$\mathcal{L}_{pNRQCD} = \int d^3\mathbf{r} \; \mathrm{Tr} \left\{ \mathrm{S}^\dagger \left(i\partial_0 + \frac{C_f \alpha_{\mathrm{s}}}{r} \right) \mathrm{S} + \right.$$

$$\left. + \mathrm{O}^\dagger \left(iD_0 - \frac{1}{2N_c} \frac{\alpha_{\mathrm{s}}}{r} \right) \mathrm{O} \right\} - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu\, a} + \sum_{i=1}^{n_f} \bar{q}_i \, i\not{D} q_i$$

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SCET: describes the interactions of very energetic (collinear) modes with soft degrees of freedom

Bauer et.al. '00; Beneke et.al. '02

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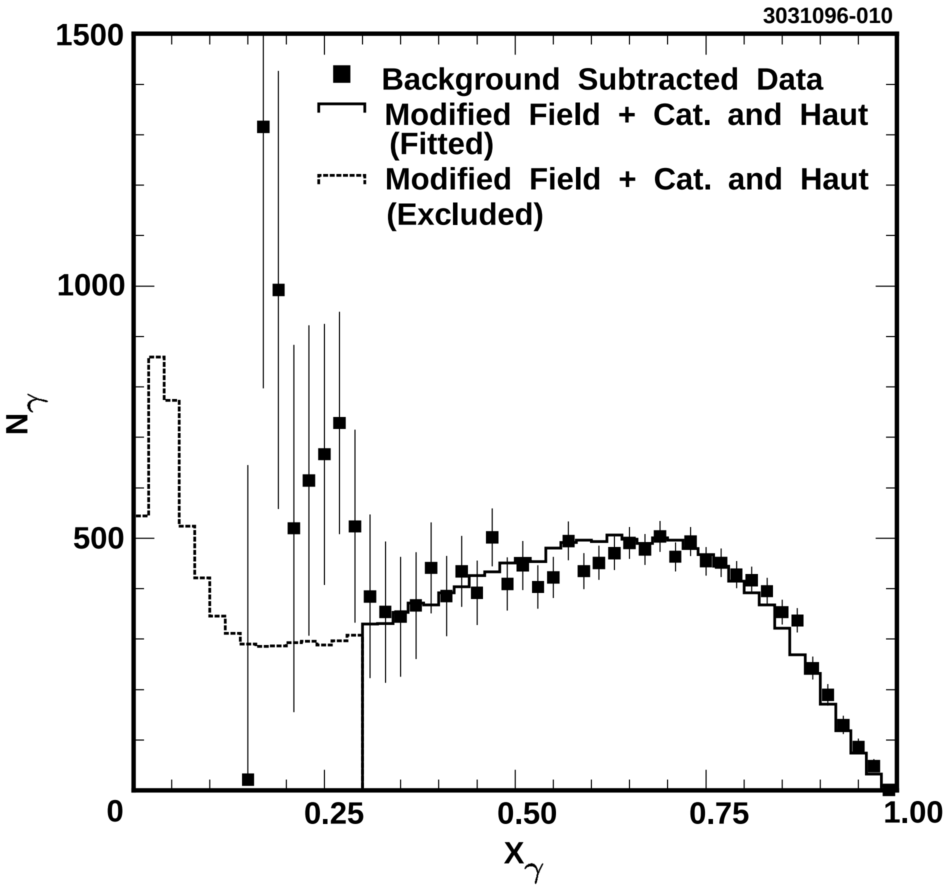
- We will analyze the decay from an EFT point of view

Data

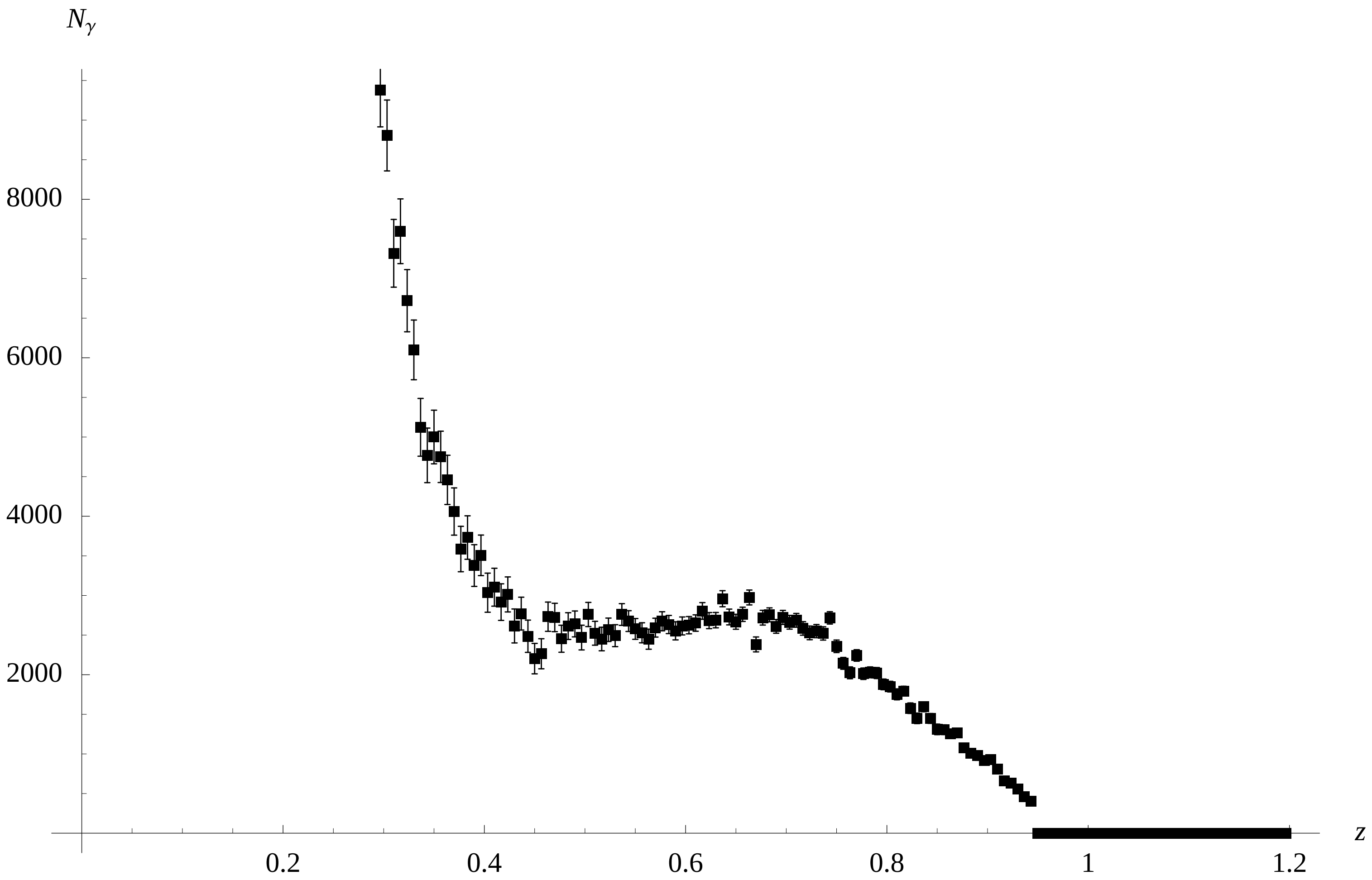
Recent CLEO measurements of the photon spectra.

Data (1S)



From B. Nemati *et al.* [CLEO Collaboration], Phys. Rev. D **55** (1997) 5273 (hep-ex/9611020)

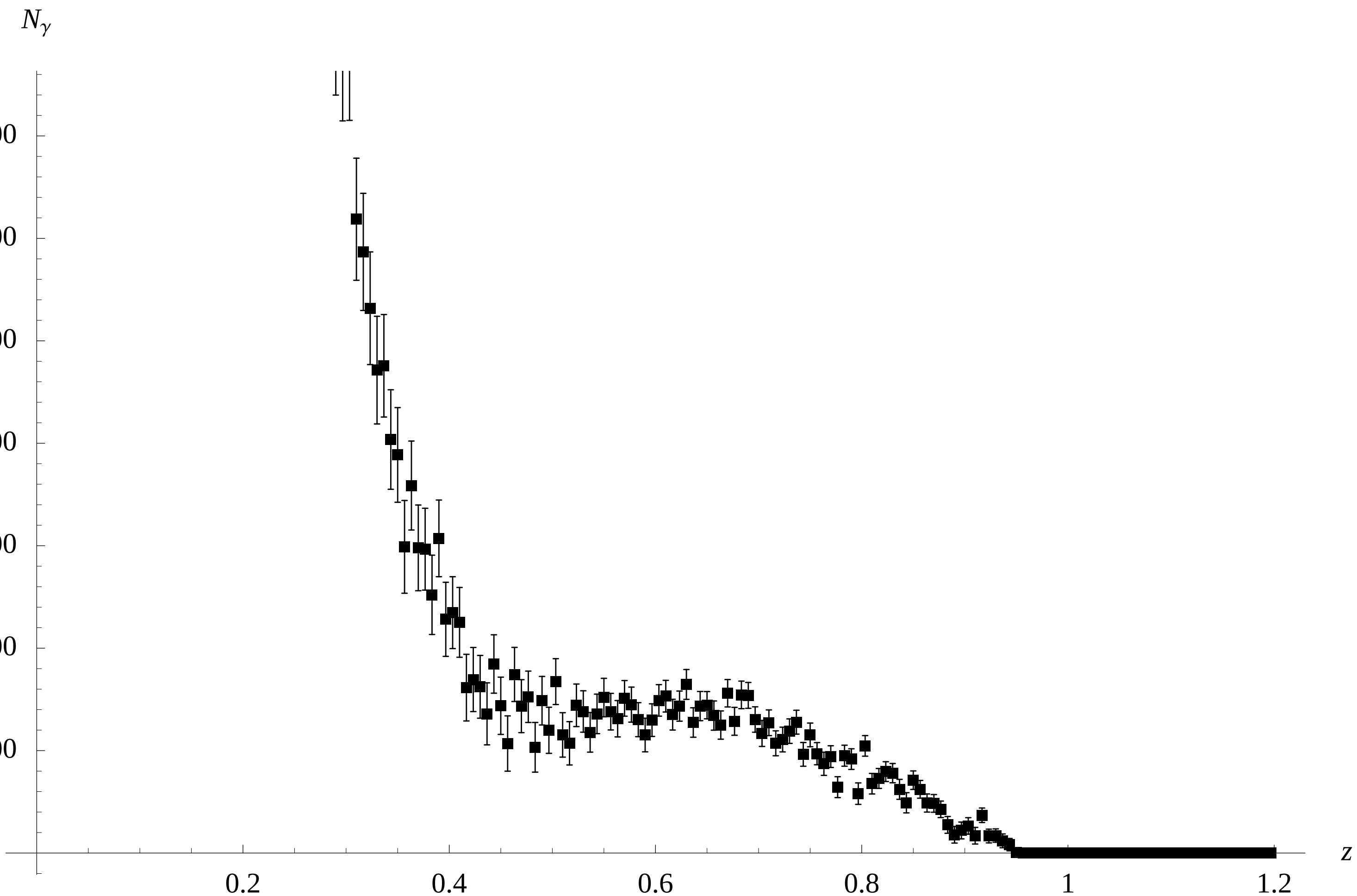
Data (1S)



Data from CLEO III, 2005

[D. Besson *et al.* [CLEO Collaboration], Phys. Rev. D **74** (2006) 012003 (hep-ex/0512061)]

Data (2S)



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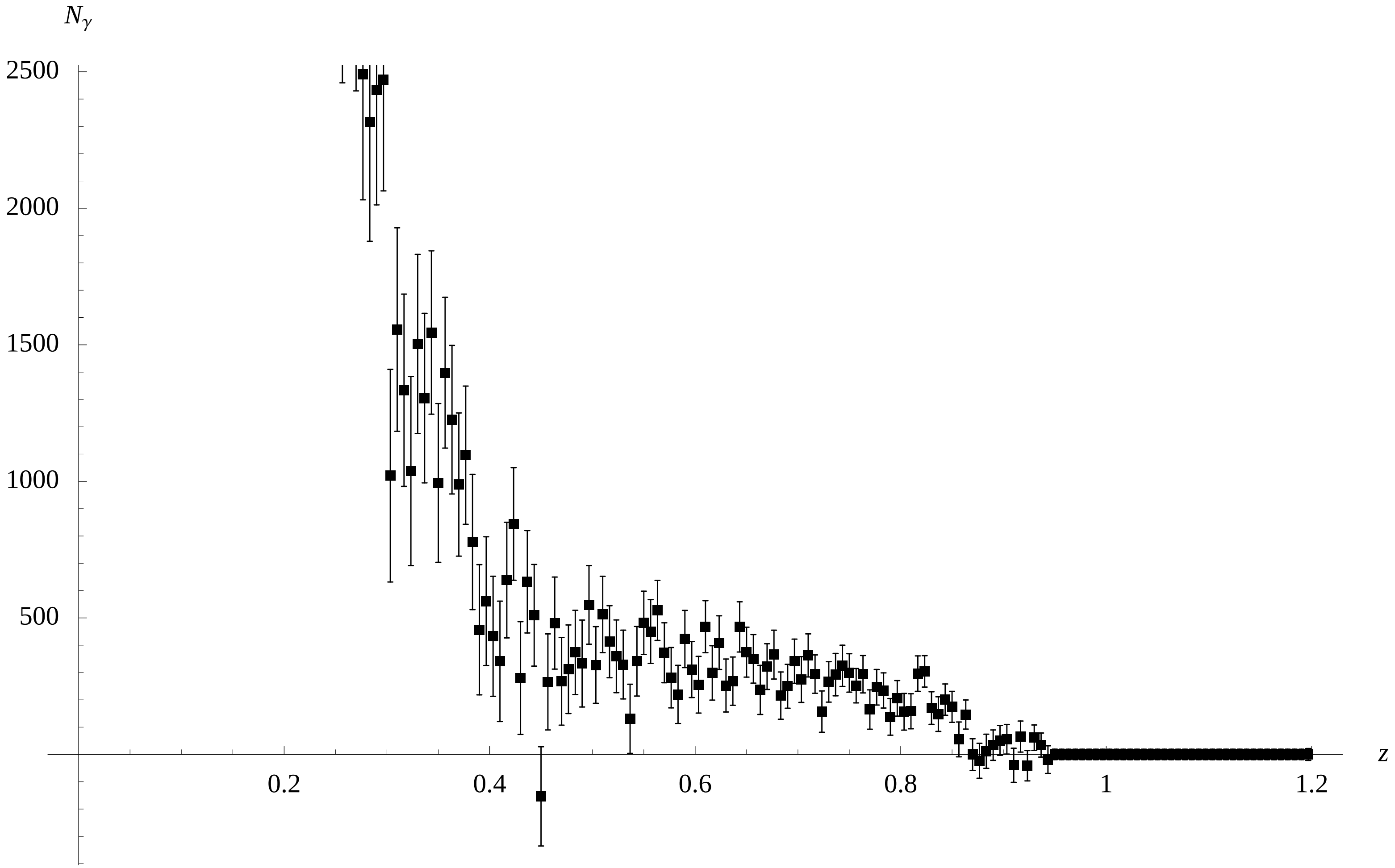
Data (3S)



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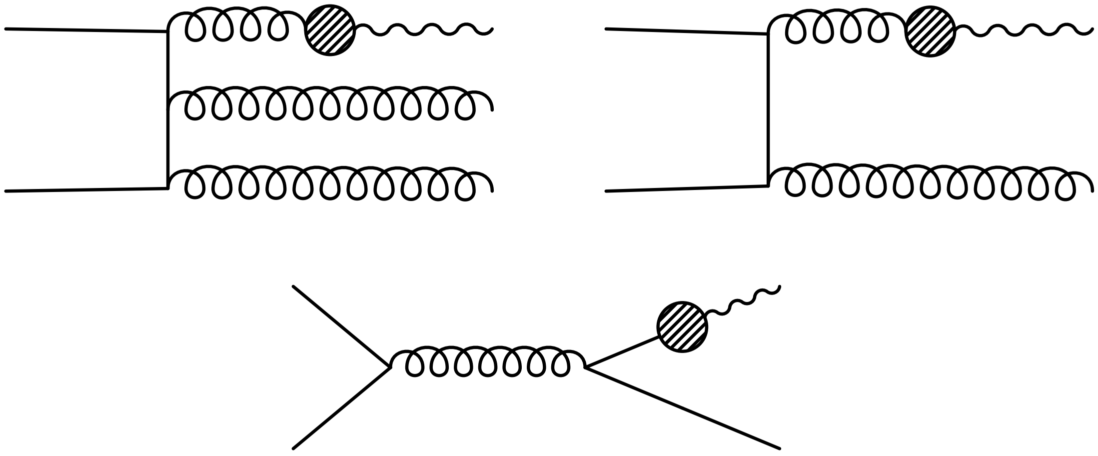
$$\frac{d\Gamma}{dz} = \frac{d\Gamma^{frag}}{dz} + \frac{d\Gamma^{dir}}{dz}$$

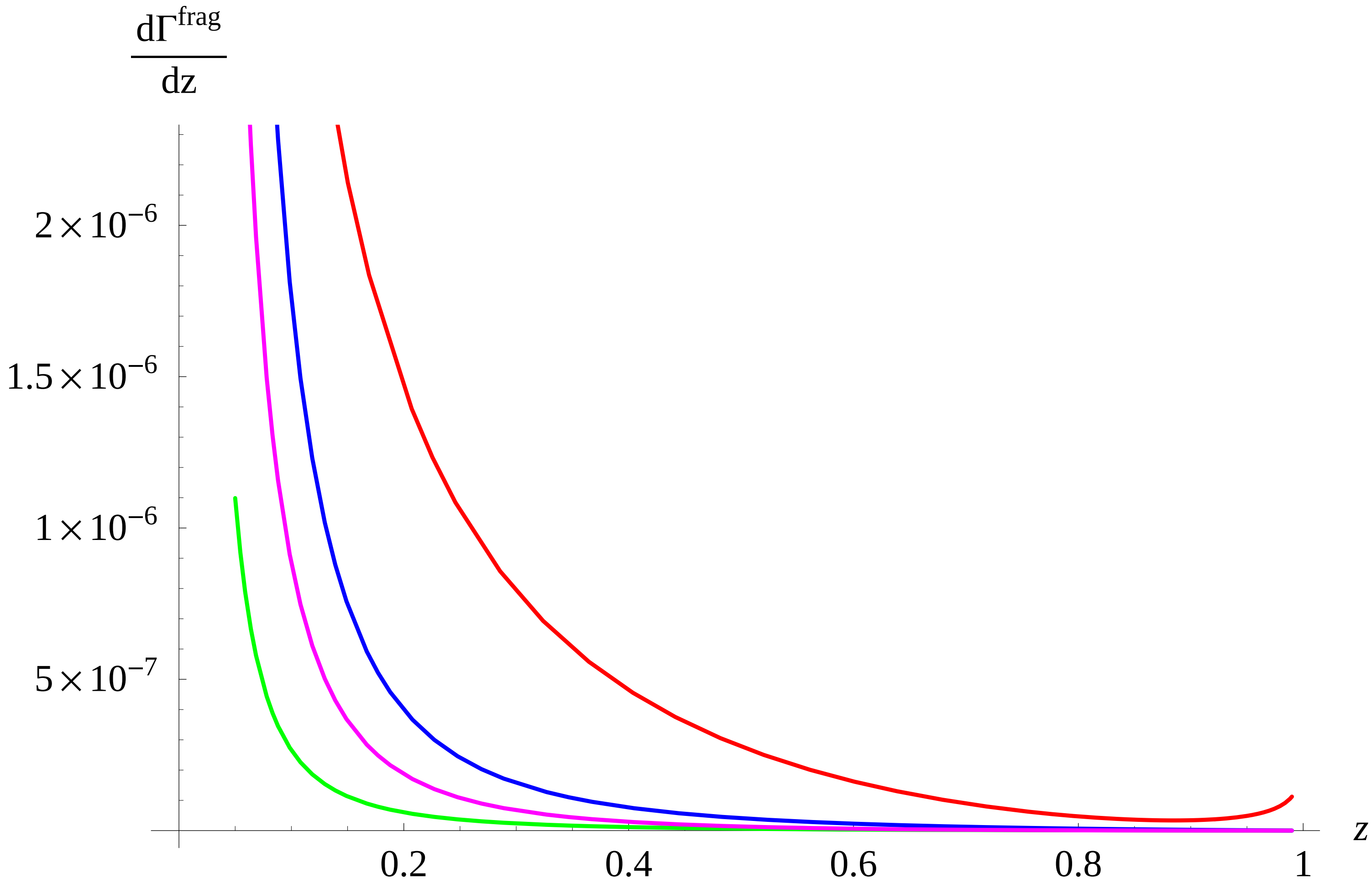
Fragmentation Contributions

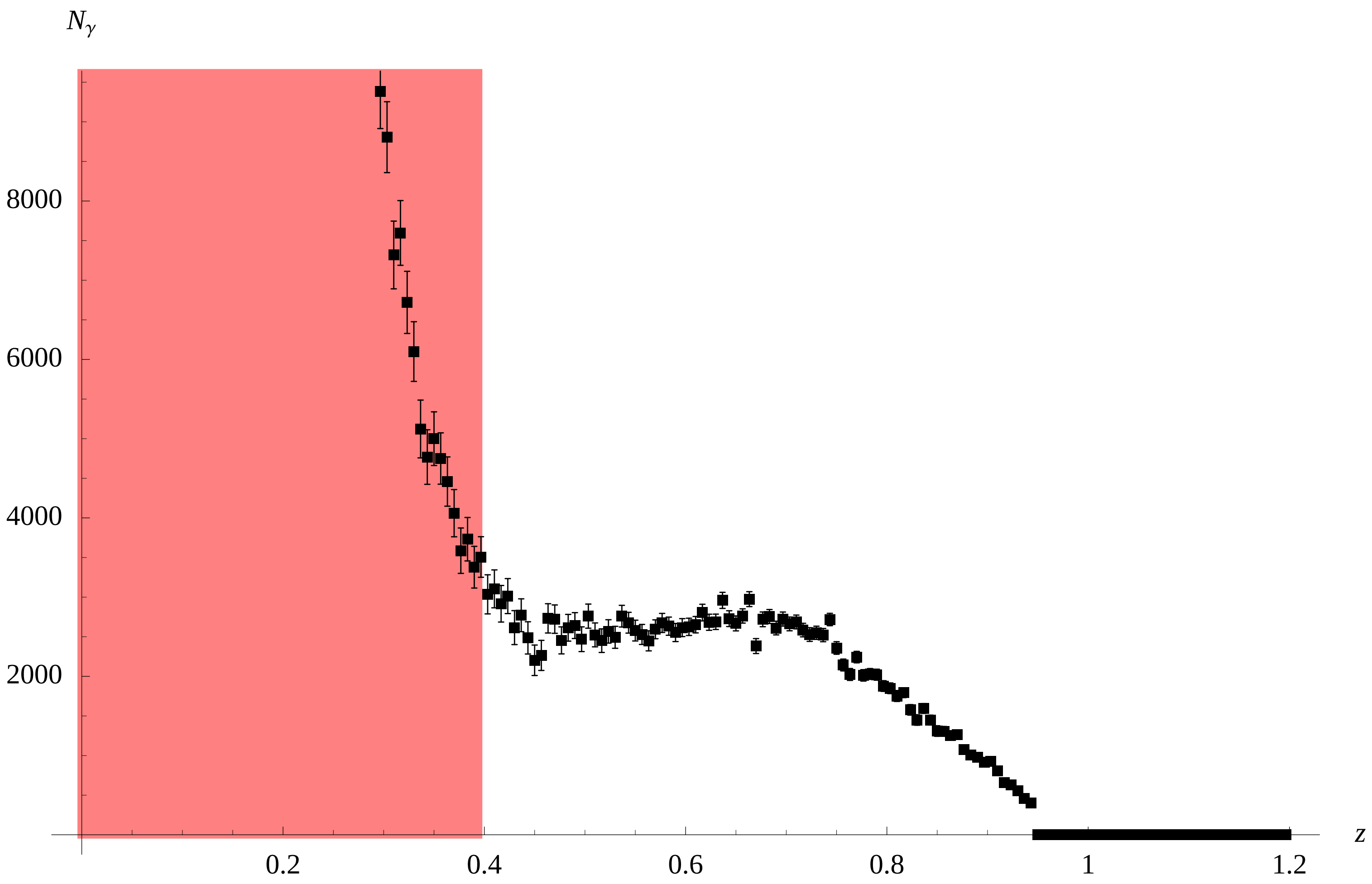
Electromagnetic couplings to light quarks.

$$\frac{d\Gamma^{frag}}{dz} = \sum_a C_a \otimes D_{a\to\gamma}$$



This type of contributions become important at low z

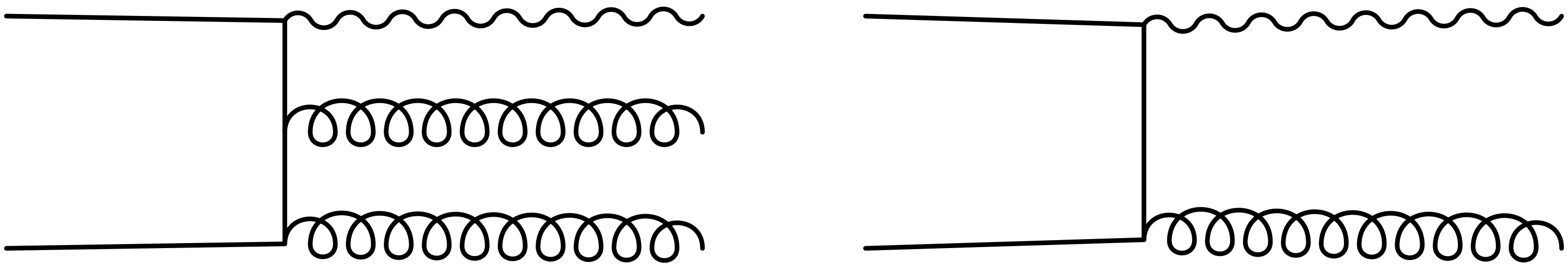




Fragmentation region

Direct contributions

Electromagnetic couplings to heavy quarks.



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$$\mathcal{O}_1 \left({}^3S_1 \right) \quad \rightarrow \quad O\left(\alpha_s^2 \alpha_{em} \right)$$

- Next-to-Leading Order (NLO) (v^4 suppressed)

$$\mathcal{O}_8 \left({}^1S_0 \right), \mathcal{O}_8 \left({}^3P_J \right) \; \alpha \; \delta(1-z)$$

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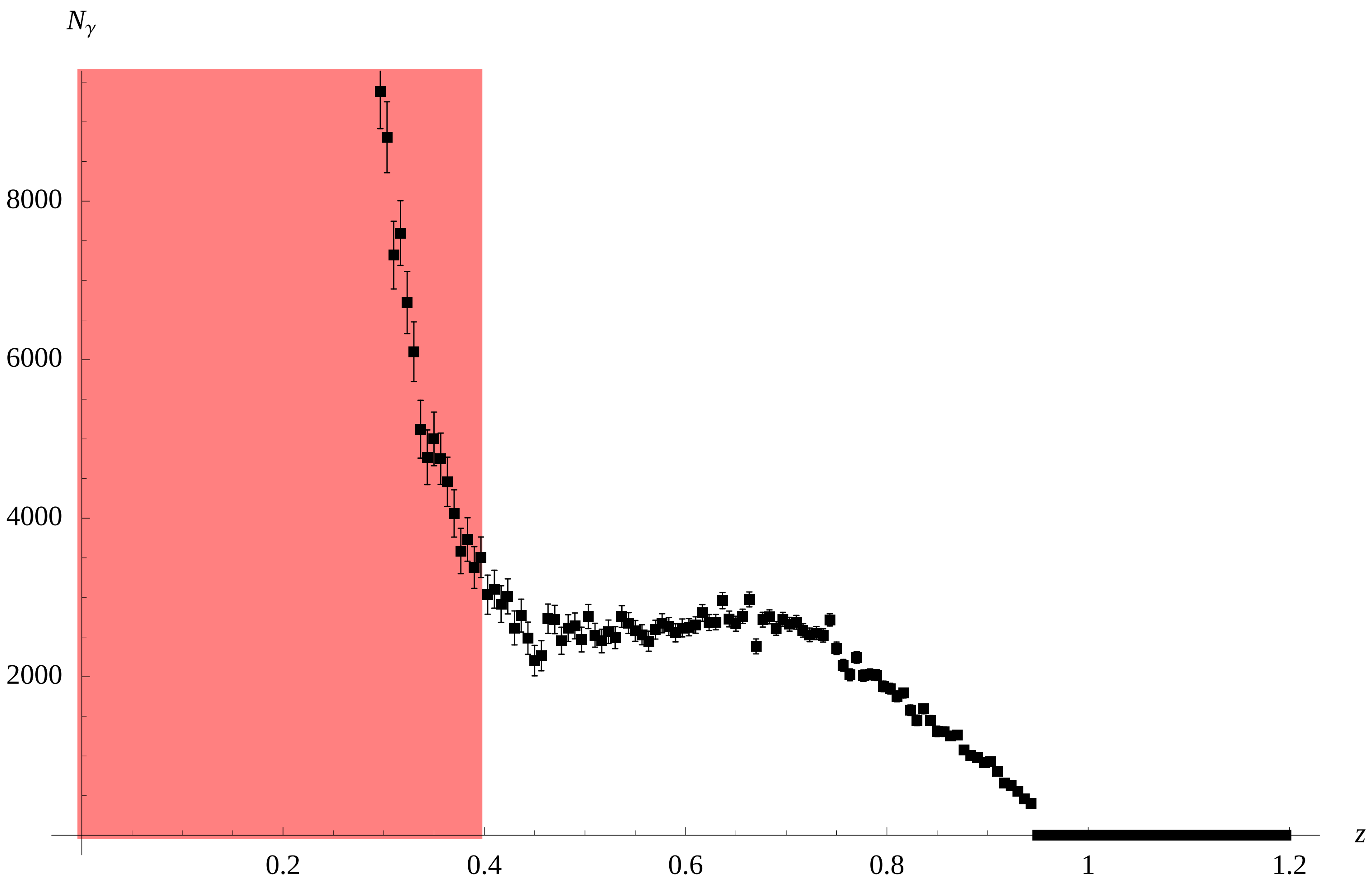
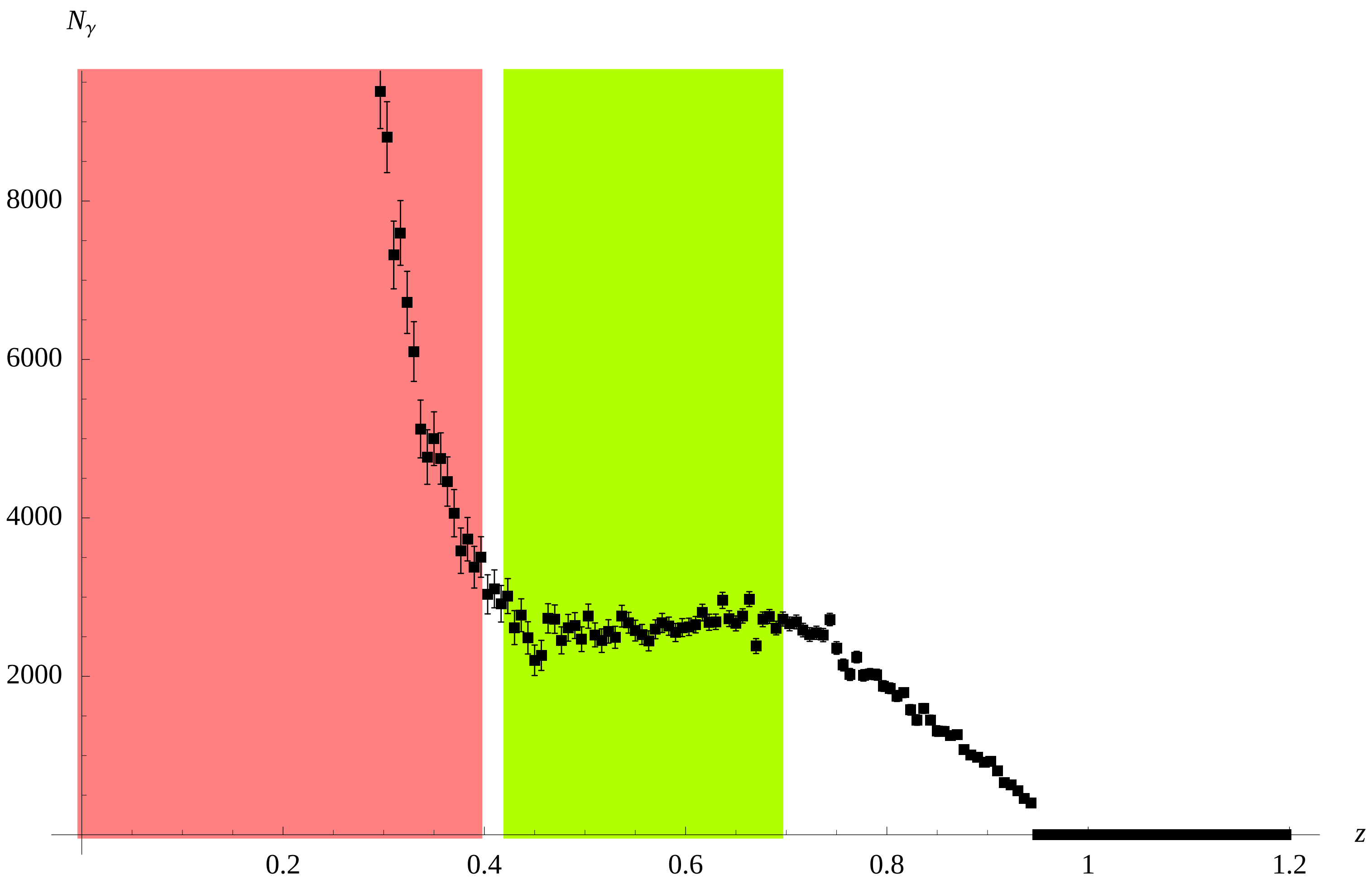
  - Higher order corrections to the coefficients have large logs $\log(1-z)$ Krämer '99; Maltoni and Petrelli '98
  - NRQCD Operator Product Expansion breaks down
    - Shape functions resumming a certain class of operators must be introduced

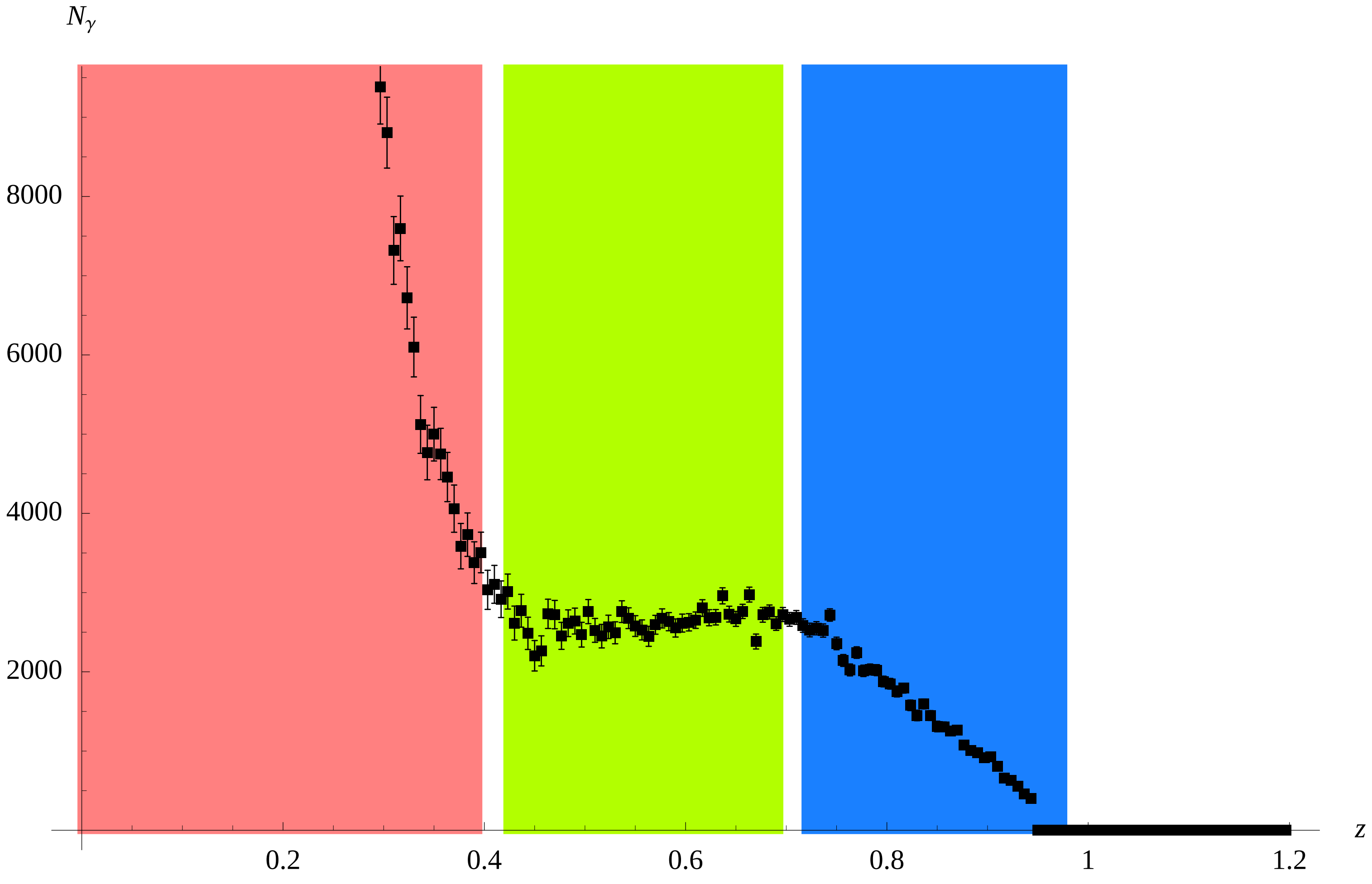
      Rothstein and Wise '97



Fragmentation region



NRQCD (central) region



upper end-point region

The upper end-point region

- Collinear degrees of freedom are also relevant in the upper end-point region. Need combined NRQCD+SCET analysis

- The decay rate has been expressed in the factorized form:

$$\frac{d\Gamma}{dz} = \sum_{\omega} H(M,\omega,\mu) \int dk^+ S(k^+,\mu) \mathrm{Im} J_{\omega}(k^+ + M(1-z),\mu)$$

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  - There are contributions from color singlet and color octet (P- and S-wave) operators
- Sudakov logarithms have been resummed for both the color singlet and the color octet operators

Bauer et al. ’01; Fleming and Leibovich ’02 ’04

- If one considers the $\Upsilon(1S)$ as a Coulombic state ($m\alpha_s \gg \Lambda_{\text{QCD}}$) the octet shape functions can be calculated.

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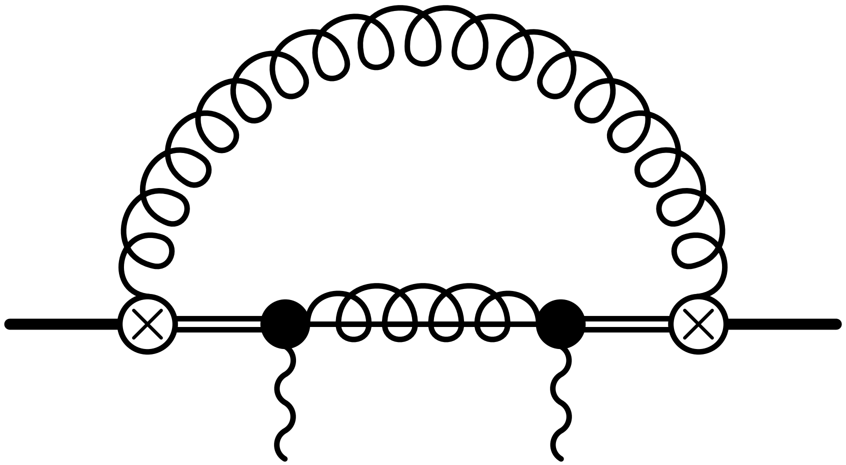
We will calculate the octet shape functions in the weak (small) coupling regime

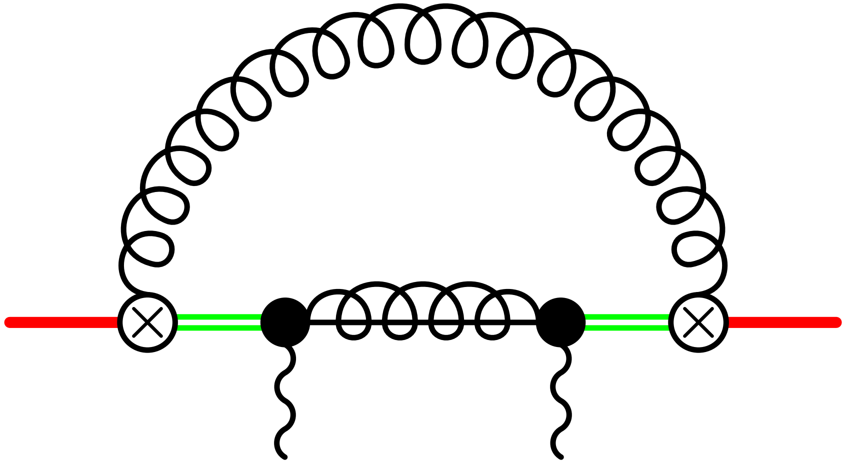
- The calculation is reliable for $z \gtrsim 0.7$ ($1 - z \ll 1$) and $z \lesssim 0.92$ ($M(1-z) \gtrsim 1GeV$)

X.G.T. and Soto '04

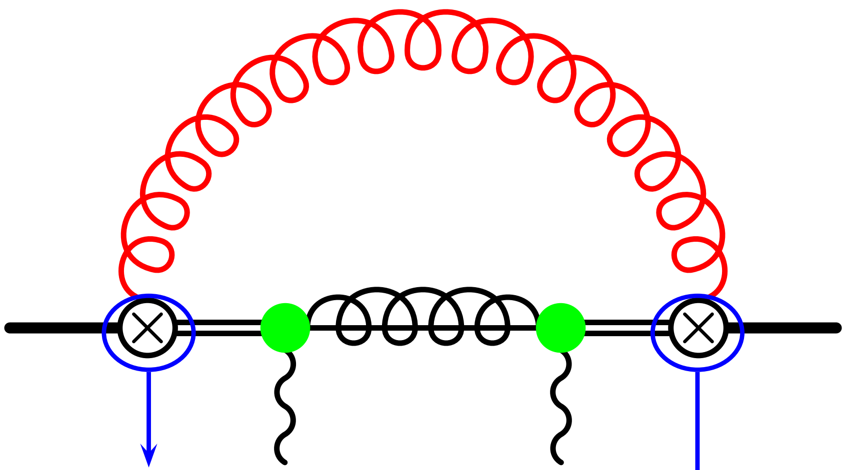
The calculation is a combination of potential NRQCD (pNRQCD) and SCET



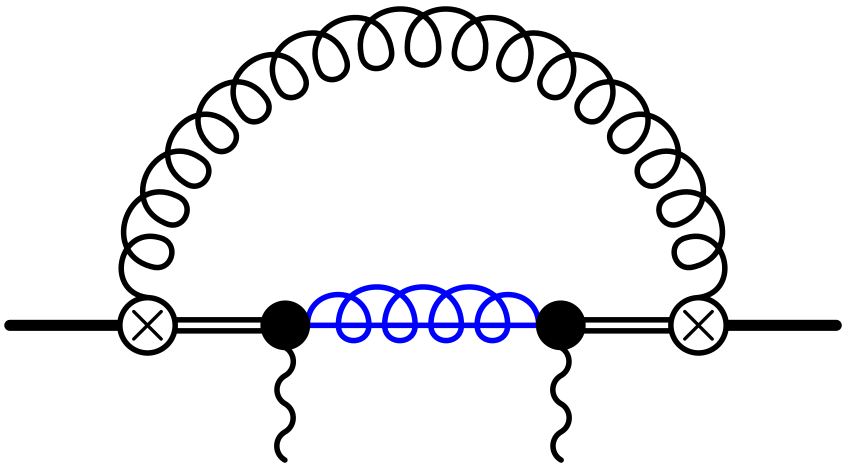
- Singlet field
- Octet field



- SCET octet operators
- ultrasoft gluon $p \sim (m\alpha_s^2, m\alpha_s^2, m\alpha_s^2)$



$$\frac{igc_F}{\sqrt{N_c T_F}} \frac{(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)}{2m} \, \mathrm{Tr}\left[T^b \mathbf{B}\right]$$



Collinear gluon $p_+, p_\perp \ll M\sqrt{1-z}$

$$\frac{1}{M^2(1-z) - Mk_+ - \mathbf{k}_\perp^2 + i\varepsilon}$$

$$T^{\alpha\alpha'}_{(8,{}^1S_0)}(z) = -i\eta^{\alpha\alpha'}_{\perp}(4\pi)\frac{32}{3}T_F^2\left(\frac{c_F}{2m}\right)^2\alpha_{\rm s}(\mu_u)C_f\times$$

$$\times\int d^3\mathbf{x}\int d^3\mathbf{x}'\psi^*_{n0}(\mathbf{x}')\psi_{n0}(\mathbf{x})\int\frac{d^4k}{(2\pi)^4}\frac{\mathbf{k}^2}{k^2+i\epsilon}\times$$

$$\times\left(\frac{1}{-k_0+E_n-h_o+i\epsilon}\right)_{\mathbf{x}',\mathbf{0}}\frac{1}{(M(1-z)-k_+)M-\mathbf{k}_\perp^2+i\epsilon}\left(\frac{1}{-k_0+E_n-h_o+i\epsilon}\right)_{\mathbf{0},\mathbf{x}}$$

$$\mathrm{Im}\left(T^{\alpha\alpha'}_{(8,{}^1S_0)}(z)\right) = -\eta^{\alpha\alpha'}_{\perp}\frac{16}{3}T_F^2\left(\frac{c_F}{2m}\right)^2\alpha_{\mathrm{s}}(\mu_u)C_f\frac{1}{M}\int_0^\infty dk_+\delta(M(1-z)-k_+)\times$$
$$\times\int_0^\infty dx\left(2\psi_{10}(\mathbf{0})I_S(\frac{k_+}{2}+x)-I_S^2(\frac{k_+}{2}+x)\right)$$

$$\mathrm{Im}\left(T^{\alpha\alpha'}_{(8,{}^1S_0)}(z)\right) = -\eta^{\alpha\alpha'}_{\perp}\int dl_+S_S(l_+)\mathrm{Im}J_M(l_+-M(1-z))$$

$$\mathrm{Im}J_M(l_+-M(1-z)) = T_F^2\left(N_c^2-1\right)\frac{2\pi}{M}\delta(M(1-z)-l_+)$$

$$S_S(l_+) = \frac{4\alpha_{\mathrm{s}}(\mu_u)}{3\pi N_c}\left(\frac{c_F}{2m}\right)^2\int_0^\infty dx\left(2\psi_{10}(\mathbf{0})I_S(\frac{l_+}{2}+x)-I_S^2(\frac{l_+}{2}+x)\right)$$

We obtain the shape functions

$$S_S(l_+) := \frac{4\alpha_s(\mu_u)}{3\pi N_c}\left(\frac{c_F}{2m}\right)^2\int_0^\infty dx\left(2\psi_{10}(\mathbf{0})I_S(\frac{l_+}{2}+x) - I_S^2(\frac{l_+}{2}+x)\right)$$

$$S_{P1}(l_+) := \frac{\alpha_s(\mu_u)}{6\pi N_c}\int_0^\infty dx\left(2\psi_{10}(\mathbf{0})I_P(\frac{l_+}{2}+x) - I_P^2(\frac{l_+}{2}+x)\right)$$

$$S_{P2}(l_+) := \frac{\alpha_s(\mu_u)}{6\pi N_c}\int_0^\infty dx\frac{8l_+x}{(l_+ + 2x)^2}\left(\psi_{10}^2(\mathbf{0}) - 2\psi_{10}(\mathbf{0})I_P(\frac{l_+}{2}+x) + I_P^2(\frac{l_+}{2}+x)\right)$$

where

$$I_S(\frac{k_+}{2}+x) := m\sqrt{\frac{\gamma}{\pi}}\frac{\alpha_s N_c}{2}\frac{1}{1-z'}\left(1 - \frac{2z'}{1+z'}\,{}_2F_1\left(-\frac{\lambda}{z'}, 1, 1-\frac{\lambda}{z'}, \frac{1-z'}{1+z'}\right)\right)$$

$$I_P(\frac{k_+}{2}+x) := \sqrt{\frac{\gamma^3}{\pi}}\frac{8}{3}(2-\lambda)\frac{1}{4(1+z')^3}\left(2(1+z')(2+z') + (5+3z')(-1+\lambda) + 2(-1+\lambda)^2 + \right.$$

$$\left. + \frac{1}{(1-z')^2}\left(4z'(1+z')(z'^2-\lambda^2)\left(-1 + \frac{\lambda(1-z')}{(1+z')(z'-\lambda)} + {}_2F_1\left(-\frac{\lambda}{z'}, 1, 1-\frac{\lambda}{z'}, \frac{1-z'}{1+z'}\right)\right)\right)\right)$$

$$\gamma = \frac{mC_f\alpha_s}{2} \quad z' = \frac{\kappa}{\gamma} \quad -\frac{\kappa^2}{m} = E_1 - \frac{k_+}{2} - x \quad \lambda = -\frac{1}{2N_cC_f}$$

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$$S_S(l_+) := \frac{4\alpha_s(\mu_u)}{3\pi N_c}\left(\frac{c_F}{2m}\right)^2 \int_0^\infty dx \left(2\psi_{10}(\mathbf{0}) I_S(\frac{l_+}{2}+x) - I_S^2(\frac{l_+}{2}+x)\right)$$

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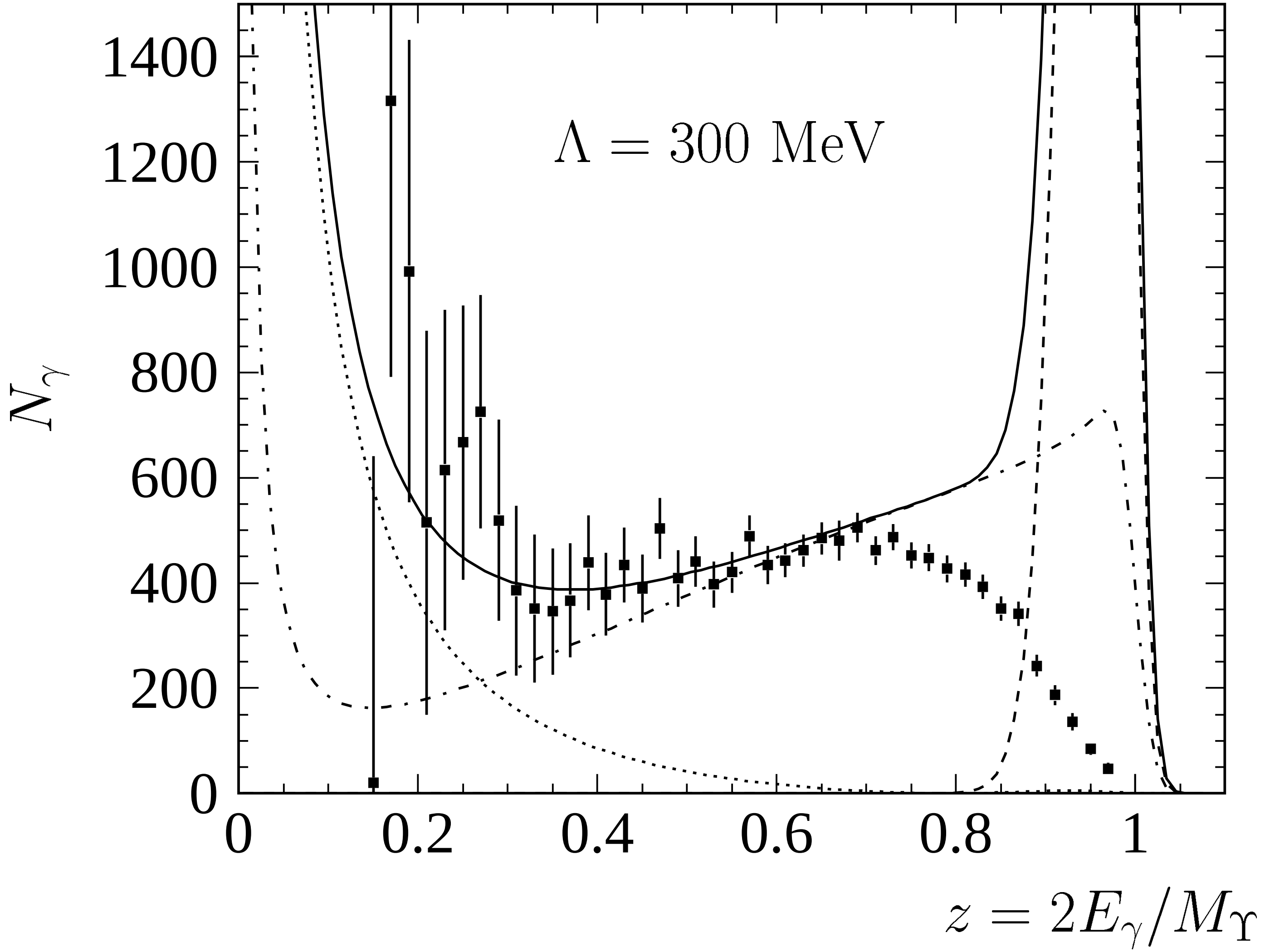
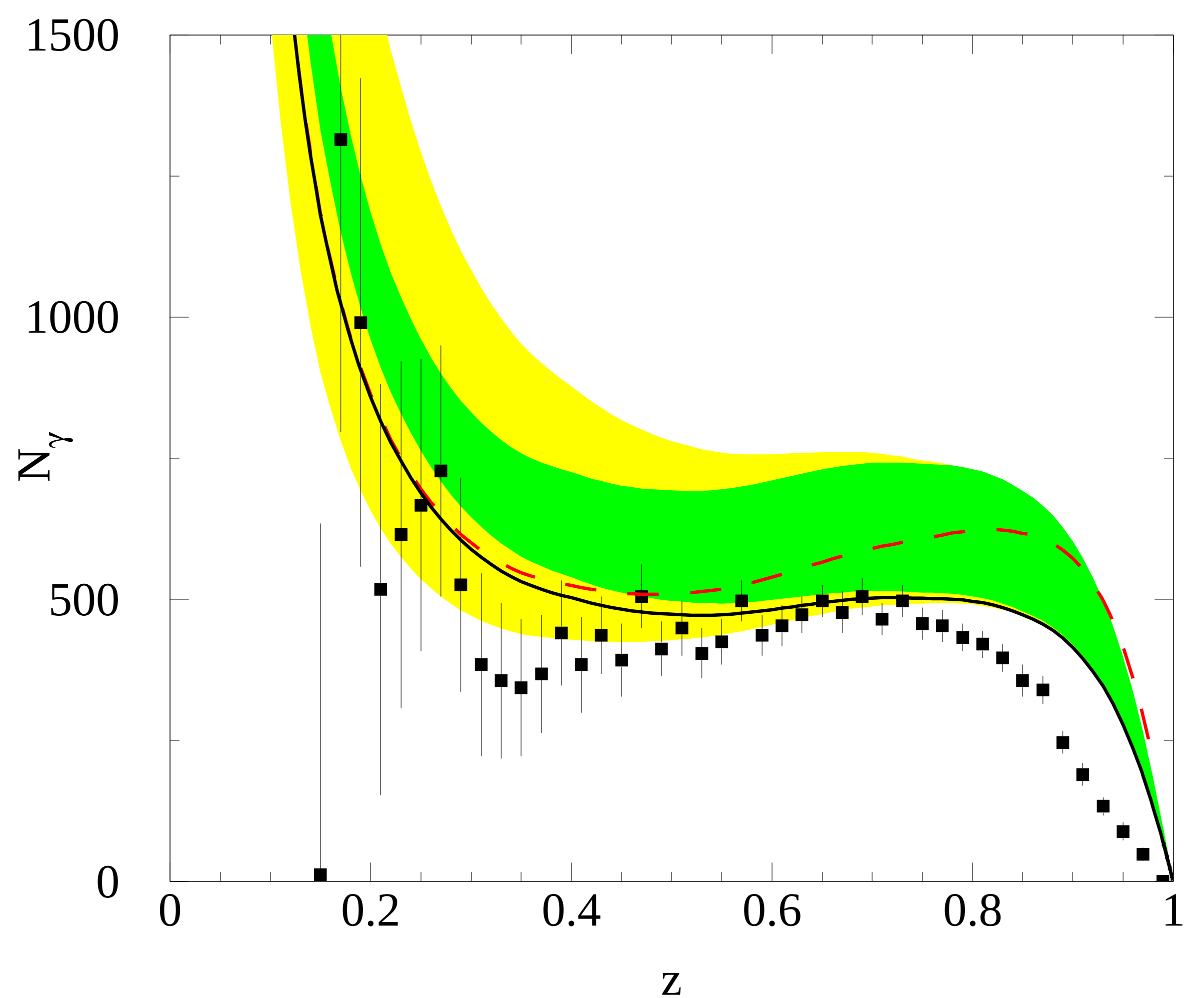
The shape functions are UV divergent and need to be regularized and renormalized

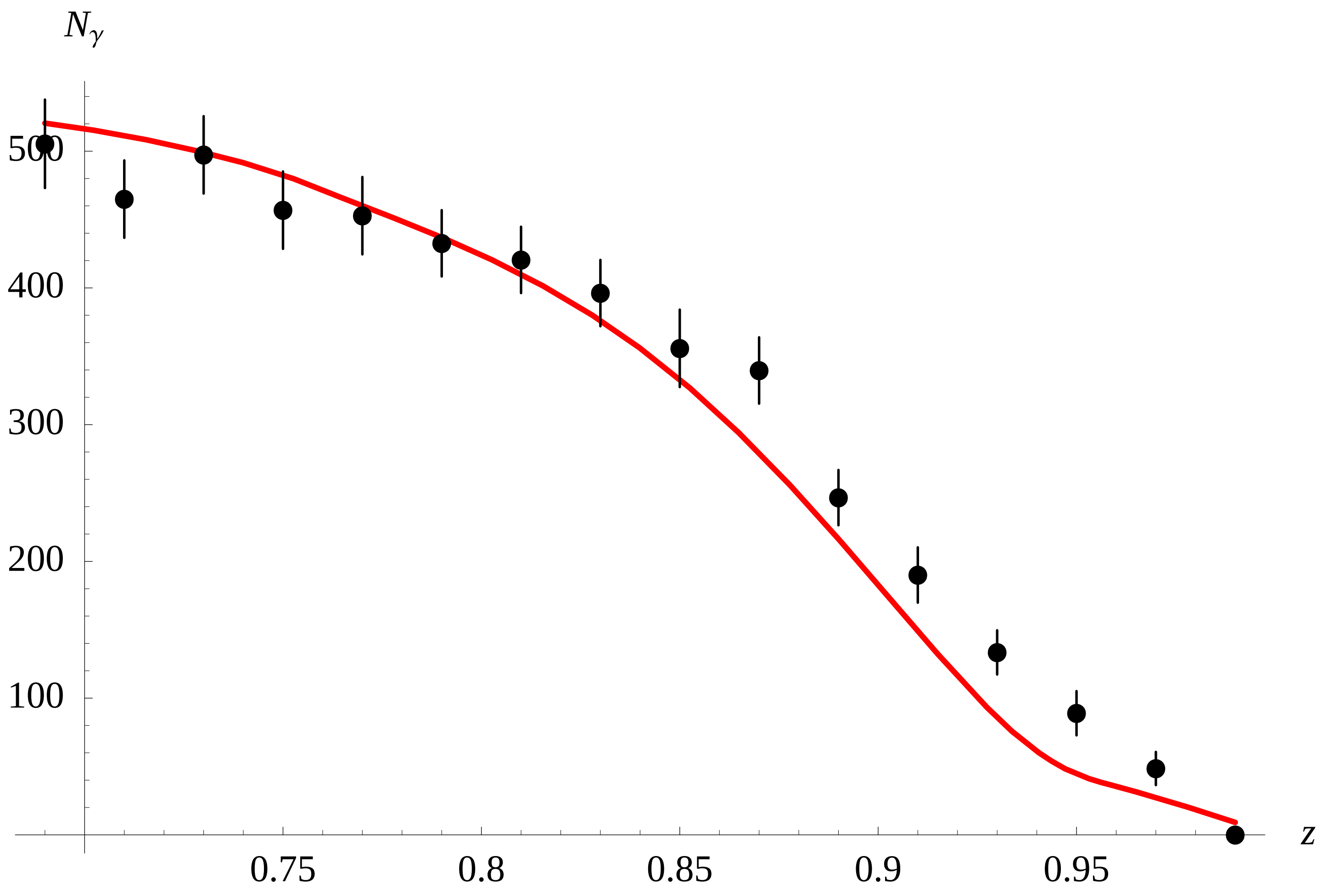
$$\frac{1}{\varepsilon} + \ln\left(\frac{\mu}{\frac{l_+}{2}+\frac{\gamma^2}{m}}\right)$$



From S. Wolf, Phys. Rev. D **63** (2001) 074020 (arXiv:hep-ph/0010217)



From S. Fleming and A. K. Leibovich, Phys. Rev. D **67** (2003) 074035 (arXiv:hep-ph/0212094)



N_γ
500
400
300
200
100
0.75
0.8
0.85
0.9
0.95
z

The complete photon spectrum

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- For z in the central region the NRQCD formulas should work
- Different approximations are needed in the different regions
  - Results for the central region in the end-point $\longrightarrow$ **miss Sudakov and Coulomb resummations**
  - Results for the end-point in the central region $\longrightarrow$ **miss non-trivial functions of z**

Choose cuts z_1, z_2 and interpolate

$$\Downarrow$$

Introduce theoretical ambiguities

Central region

$\boxed{v \text{ counting } \rightarrow \ \alpha_s(\mu_h) \sim v^2 \quad \alpha_s(\mu_s) \sim v}$

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$$\boxed{v \text{ counting } \rightarrow \alpha_s(\mu_h) \sim v^2 \quad \alpha_s(\mu_s) \sim v}$$

LO $\rightarrow$ $\cdot\ C_1\left[{}^3S_1\right]\dfrac{\left\langle\Upsilon(1S)|\mathcal{O}_1\left({}^3S_1\right)|\Upsilon(1S)\right\rangle}{m^2}$

NLO (v^2 suppressed) $\rightarrow$ $\cdot\ C_1'\left[{}^3S_1\right]\dfrac{\left\langle\Upsilon(1S)|\mathcal{P}_1\left({}^3S_1\right)|\Upsilon(1S)\right\rangle}{m^4}$

$\cdot\ \alpha_s(\mu_h)$ corrections to $C_1\left[{}^3S_1\right]$

$\cdot\ \alpha_s^2(\mu_s)$ corrections to wave function

End-point region

$\boxed{1-z \text{ SCET counting}}$

End-point region

$\boxed{1 - z \text{ SCET counting}}$

- Both color singlet and color octet contributions appear at leading order
- Final result incorporates Sudakov and Coulomb resummations and involves α_s at several different scales: $M\sqrt{(1-z)}$, mv, ...

Mismatch between the two expressions

- Expand the end-point expressions for z in the central region

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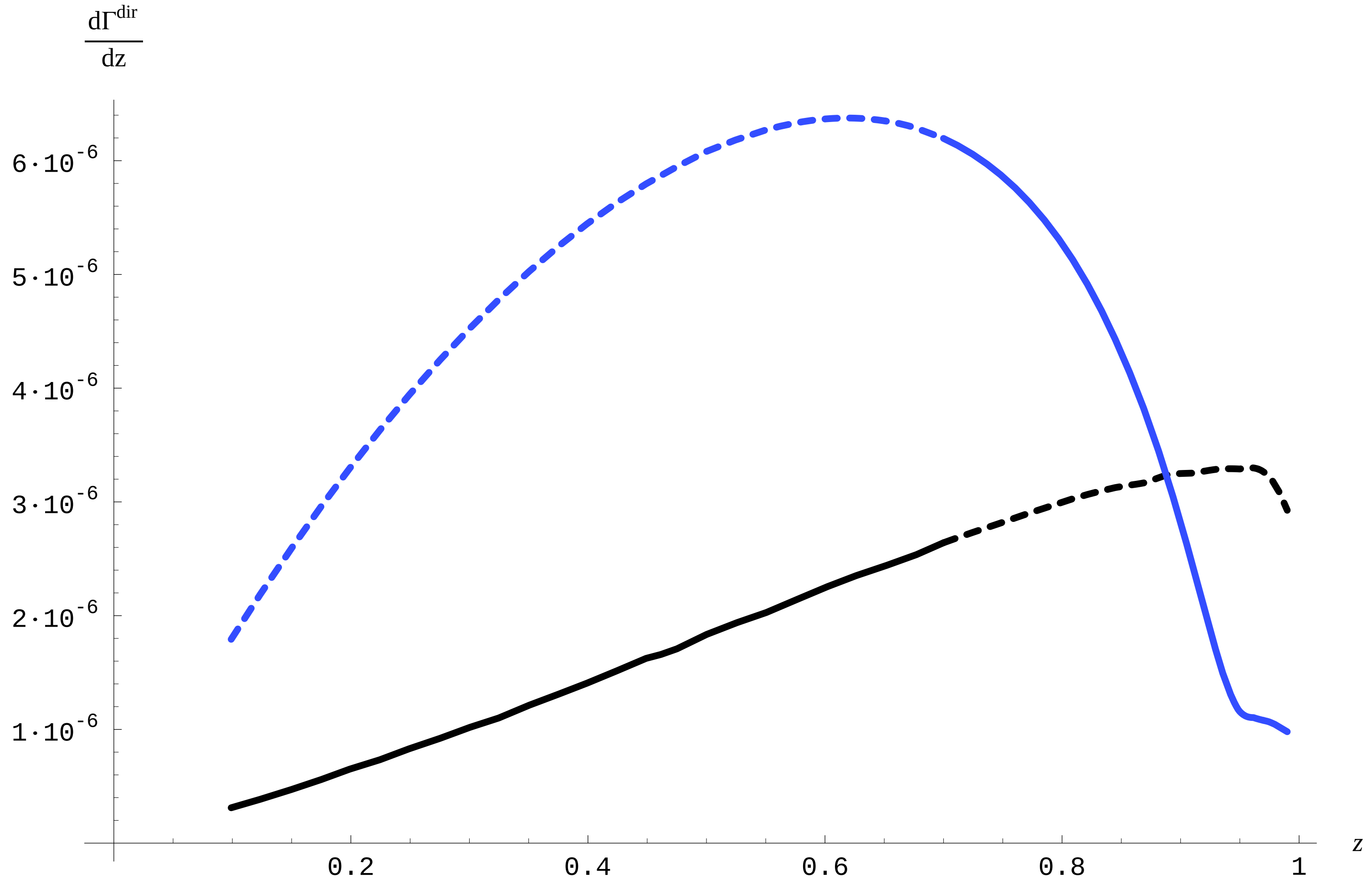
- Check: the $(1-z)\log(1-z)$, the logs of the relativistic corrections and the $\alpha_s \log(1-z)$ are correctly reproduced (in the limit $z \to 1$)

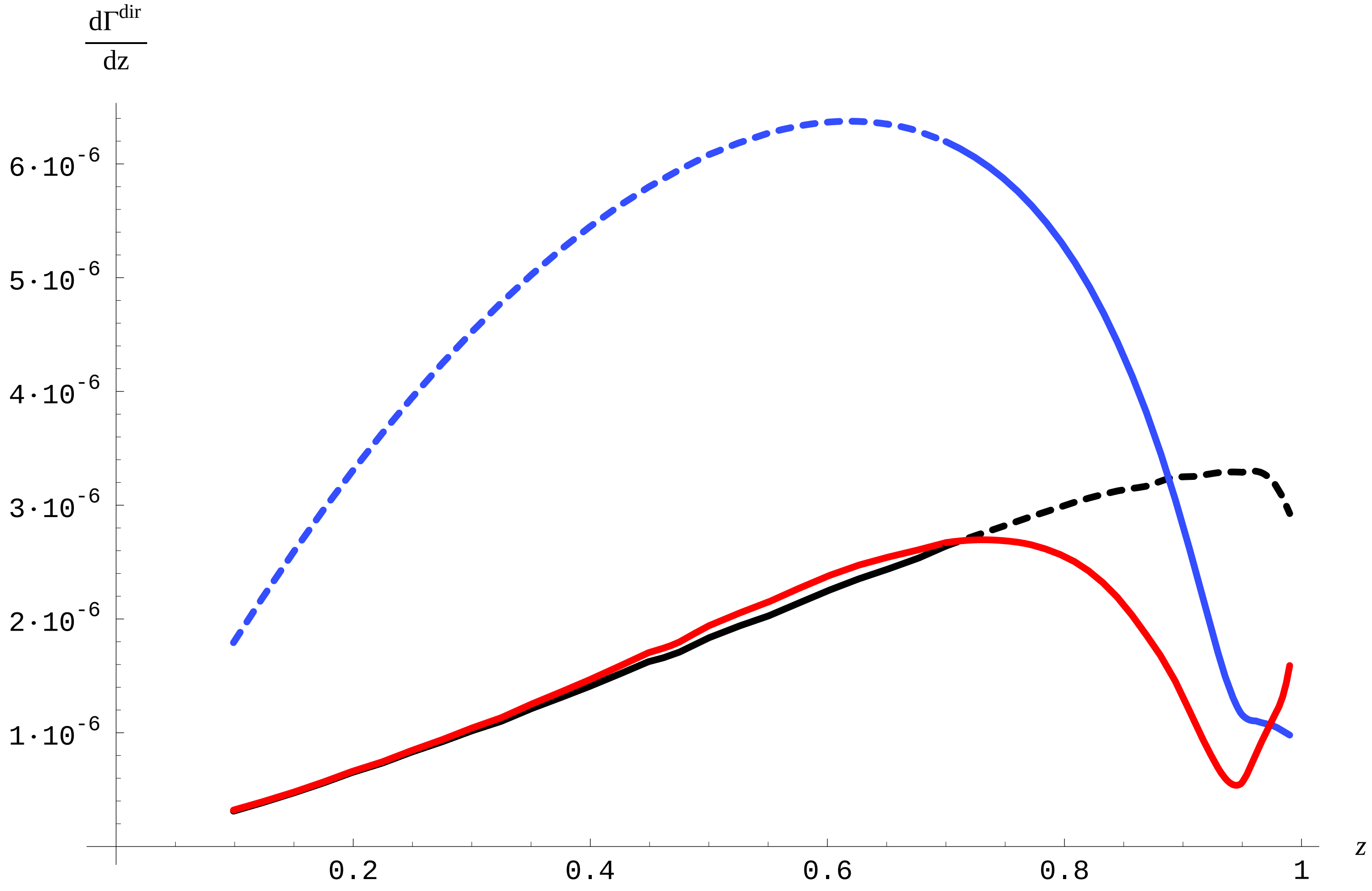
- Expand the end-point expressions for z in the central region
- Check: the $(1-z)\log(1-z)$, the logs of the relativistic corrections and the $\alpha_s \log(1-z)$ are correctly reproduced (in the limit $z \to 1$)
- Use the formula

$$\frac{1}{\Gamma_0}\frac{d\Gamma^{dir}}{dz} = \frac{1}{\Gamma_0}\frac{d\Gamma^{c}}{dz} + \left(\frac{1}{\Gamma_0}\frac{d\Gamma^{e}}{dz} - \left.\frac{1}{\Gamma_0}\frac{d\Gamma^{e}}{dz}\right|_c \right)$$

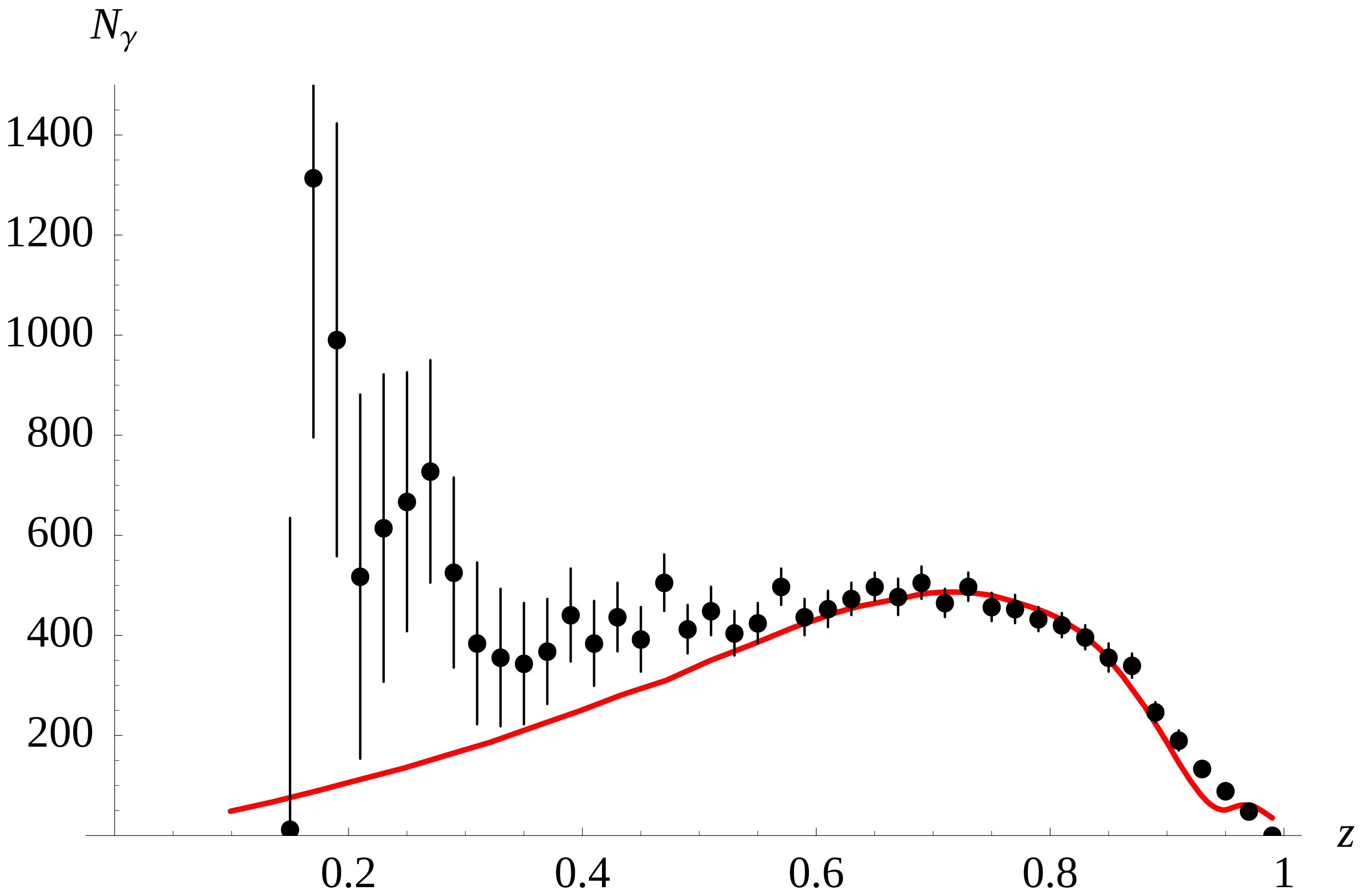
X.G.T. and Soto '05



v^2 NRQCD (central region) expression   end-point expression

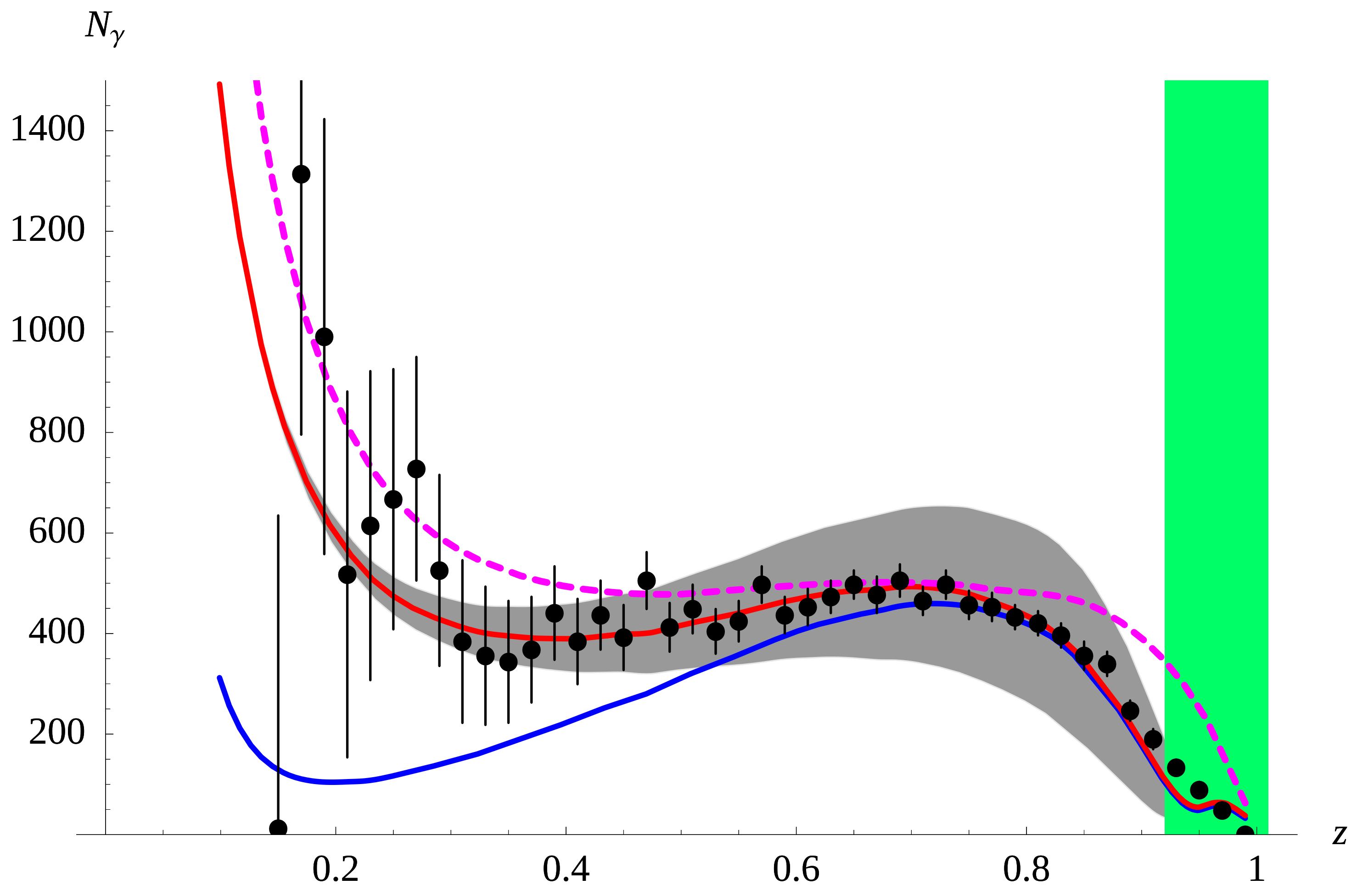


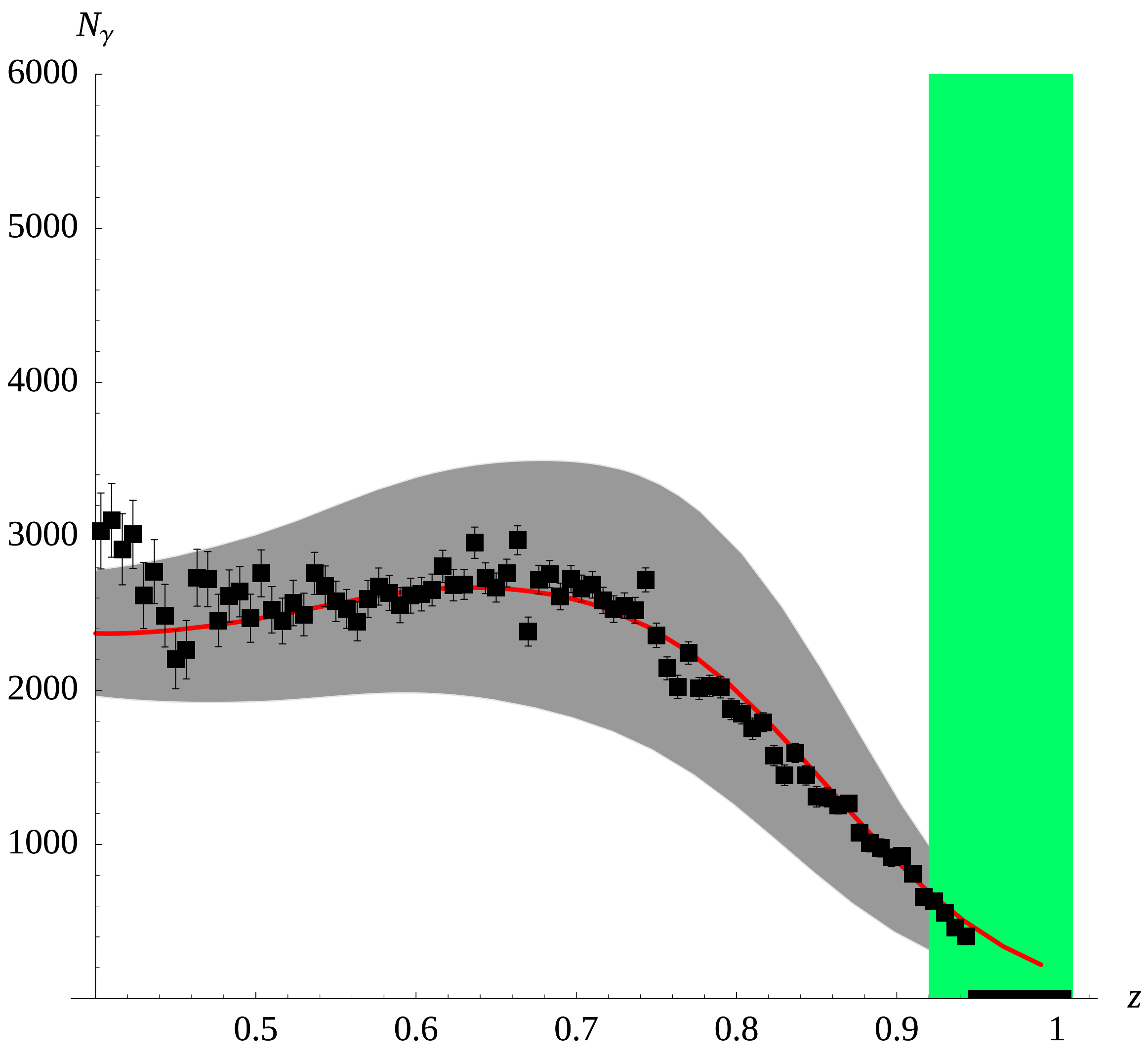
v^2 NRQCD (central region) expression   end-point expression   merging formula



N_γ
1400
1200
1000
800
600
400
200
0.2
0.4
0.6
0.8
1
z

- Sum the direct and fragmentation contributions to obtain the total spectrum


N_γ
1400
1200
1000
800
600
400
200
0.2
0.4
0.6
0.8
1
z



N_γ
6000
5000
4000
3000
2000
1000
0.5
0.6
0.7
0.8
0.9
1
z

The photon spectrum can be well described when all the contributions are consistently included

The photon spectrum can be well described when all the contributions are consistently included

So, now the question is...

What can we learn from it?

What can we learn from it?

- The nature of heavy quarkonium

- α_{s} extraction

The nature of heavy quarkonia

Remember

weak coupling regime	**strong coupling regime**
$\Lambda_{QCD} \lesssim mv^2$	$mv^2 \ll \Lambda_{QCD} \lesssim mv$

The nature of heavy quarkonia

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Given a heavy quarkonium state, to which regime does it belong to?

The nature of heavy quarkonia

Remember

weak coupling regime **strong coupling regime**

$$\Lambda_{QCD} \lesssim mv^2 \qquad\qquad mv^2 \ll \Lambda_{QCD} \lesssim mv$$

But those scales are not directly accessible experimentally

Given a heavy quarkonium state, to which regime does it belong to?

The photon spectra can tell you.

In the central region

$$\frac{d\Gamma_n}{dz} = \left(C_1\left[{}^3S_1\right](z) + f_{\mathcal{O}_1({}^3S_1)}(z)\right)\frac{\langle \mathcal{O}_1({}^3S_1)\rangle_n}{m^2} +$$

$$+C_1'\left[{}^3S_1\right](z)\frac{\langle \mathcal{P}_1({}^3S_1)\rangle_n}{m^4} + f_{\mathcal{O}_8({}^3S_1)}(z)\frac{\langle \mathcal{O}_8({}^3S_1)\rangle_n}{m^2} +$$

$$+f_{\mathcal{O}_8({}^1S_0)}(z)\frac{\langle \mathcal{O}_8({}^1S_0)\rangle_n}{m^2} + f_{\mathcal{O}_8({}^3P_J)}(z)\frac{\langle \mathcal{O}_8({}^3P_0)\rangle_n}{m^4}$$

Strong coupling:

- Octet matrix elements reduce to wave function at the origin times bound-state independent parameters

$$\langle \Upsilon(nS)|\mathcal{O}_8(^1S_0)|\Upsilon(nS)\rangle = C_A \frac{|R_n(0)|^2}{2\pi}\left(\frac{(C_f - C_A/2)c_F^2\mathcal{B}_1}{3m^2}\right)$$

Weak coupling:

- Octet matrix elements depends non-trivially on n

Two states in the strong coupling regime

$$\frac{\frac{d\Gamma_n}{dz}}{\frac{d\Gamma_r}{dz}} = \frac{\langle \mathcal{O}_1(^3S_1)\rangle_n}{\langle \mathcal{O}_1(^3S_1)\rangle_r}\left(1 + \frac{C_1'\left[^3S_1\right](z)}{C_1\left[^3S_1\right](z)}\frac{1}{m}(E_n - E_r)\right)$$

Model-independent formula. Holds at NLO

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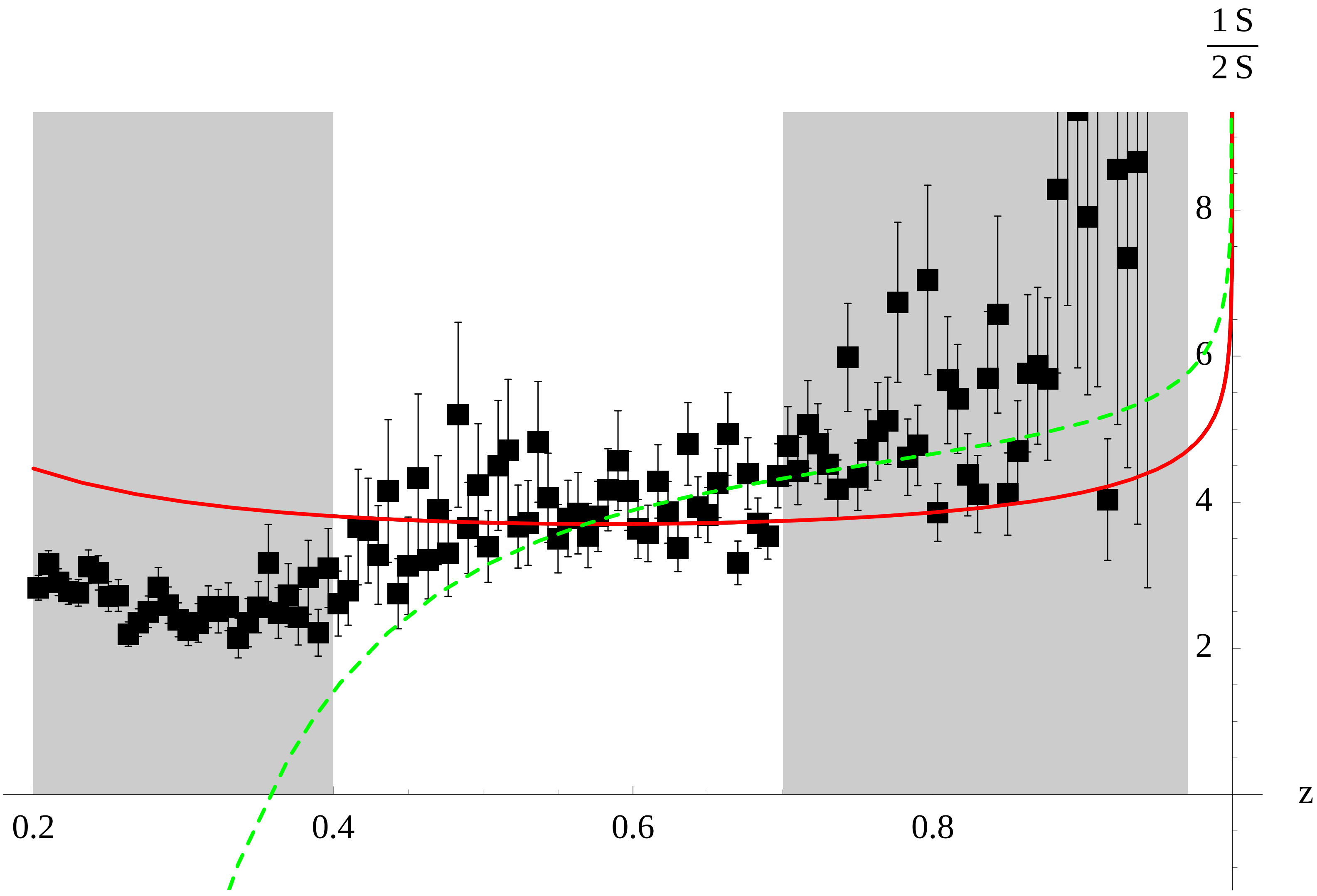
if not

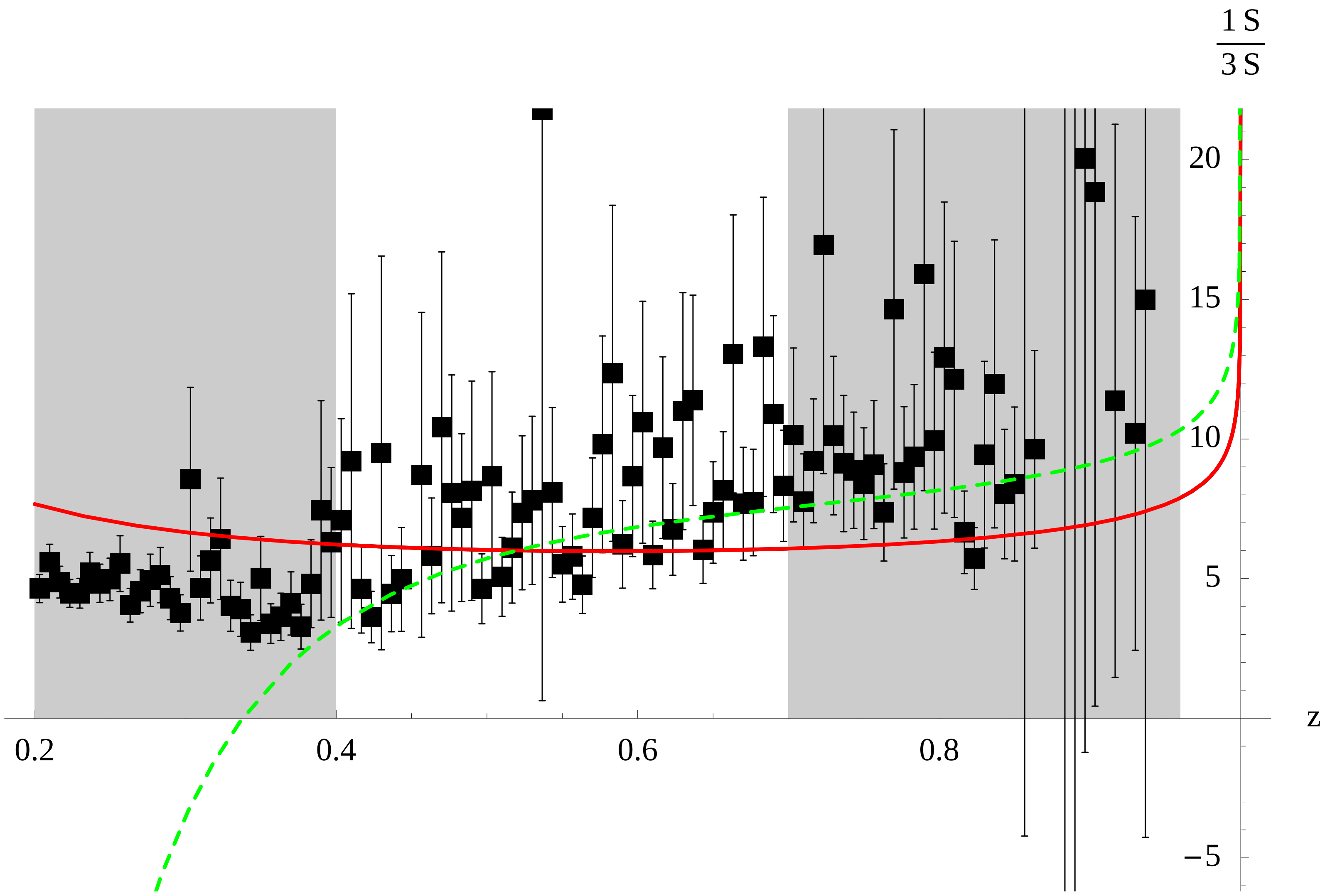
$$\frac{\frac{d\Gamma_n}{dz}}{\frac{d\Gamma_r}{dz}} = \frac{\langle \mathcal{O}_1(^3S_1)\rangle_n}{\langle \mathcal{O}_1(^3S_1)\rangle_r}\left(1+\frac{C_1'\left[^3S_1\right](z)}{C_1\left[^3S_1\right](z)}\frac{\mathcal{R}^{nr}_{\mathcal{P}_1(^3S_1)}}{m^2}\right.$$

$$\left.+\frac{f_{\mathcal{O}_8\left(^3S_1\right)}(z)}{C_1\left[^3S_1\right](z)}\mathcal{R}^{nr}_{\mathcal{O}_8(^3S_1)}+\frac{f_{\mathcal{O}_8\left(^1S_0\right)}(z)}{C_1\left[^3S_1\right](z)}\mathcal{R}^{nr}_{\mathcal{O}_8(^1S_0)}+\frac{f_{\mathcal{O}_8\left(^3P_J\right)}(z)}{C_1\left[^3S_1\right](z)}\frac{\mathcal{R}^{nr}_{\mathcal{O}_8(^3P_0)}}{m^2}\right)$$

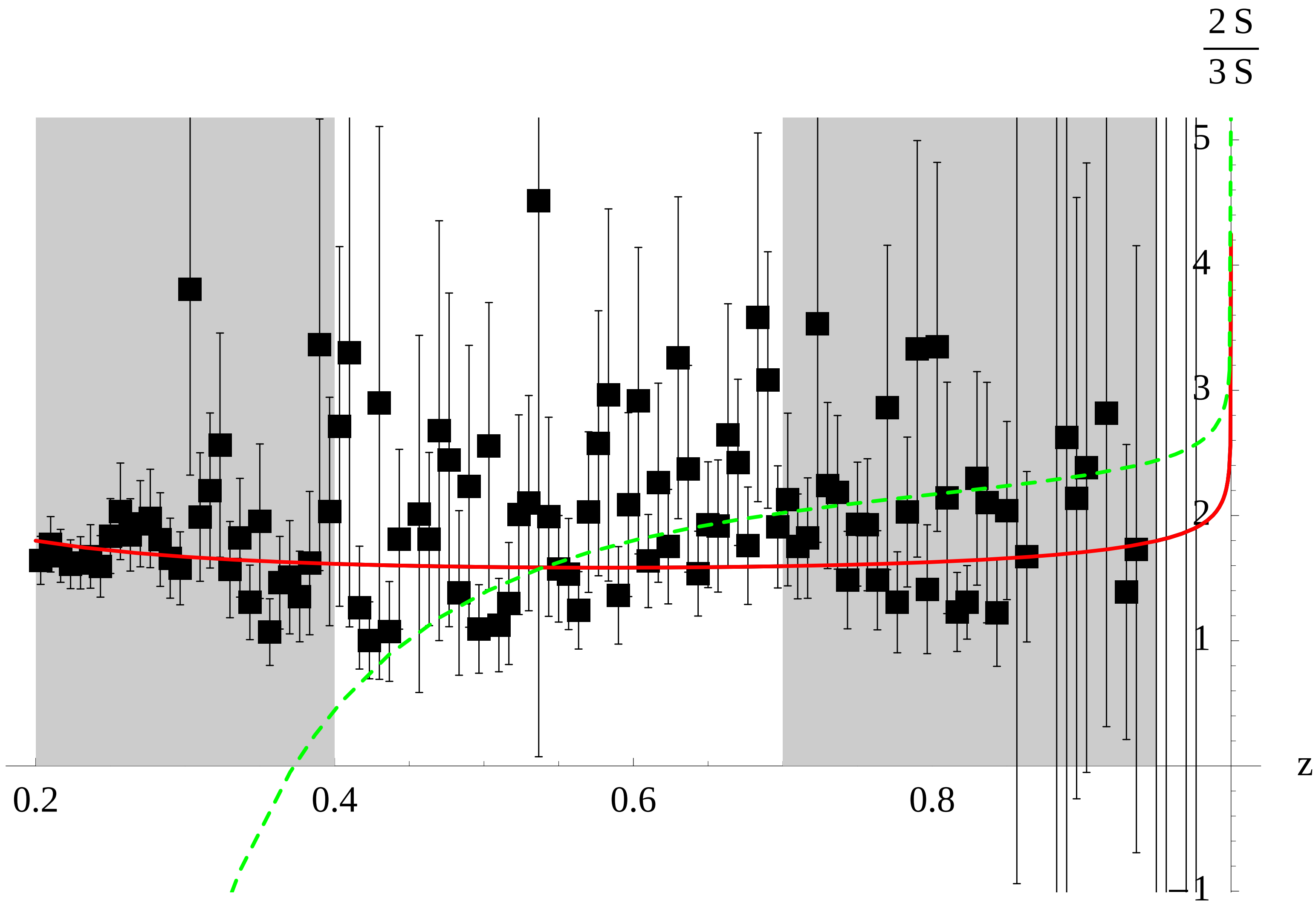
X.G.T. and Soto '05



$$\chi^2 = 1.2 \rightarrow 18\%\mathrm{CL}$$

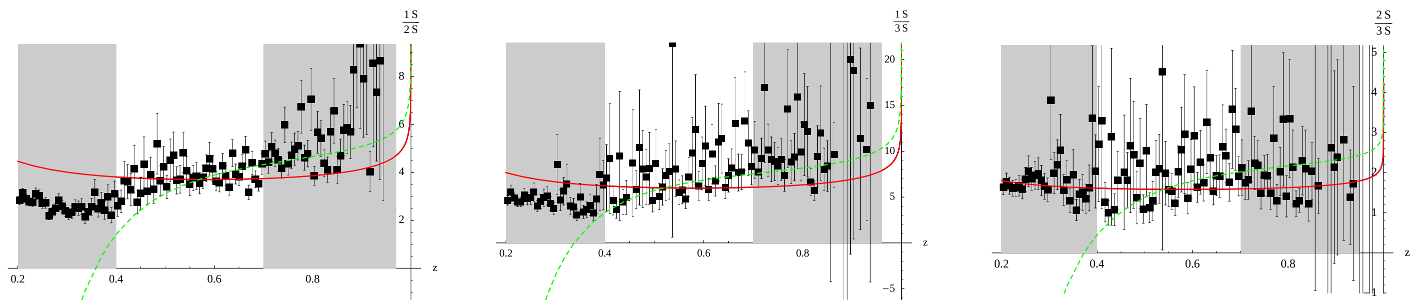


$$\chi^2 = 0.9 \rightarrow 68\%\mathrm{CL}$$



$$\chi^2 = 0.75 \rightarrow 89\%\mathrm{CL}$$

- Errors are large
  - Data disfavors $\Upsilon(1S)$ in the strong coupling regime (as expected)



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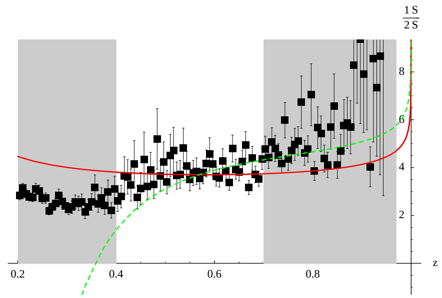
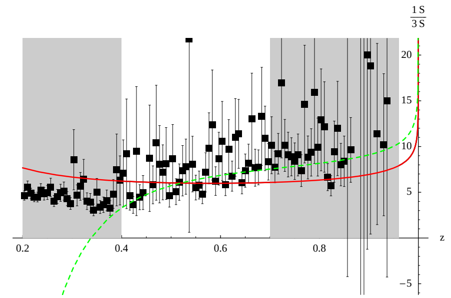
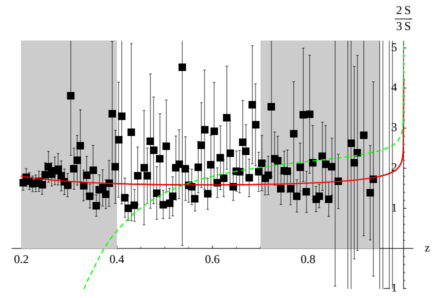
  - Consistent with $\Upsilon(2S)$ and $\Upsilon(3S)$ in it







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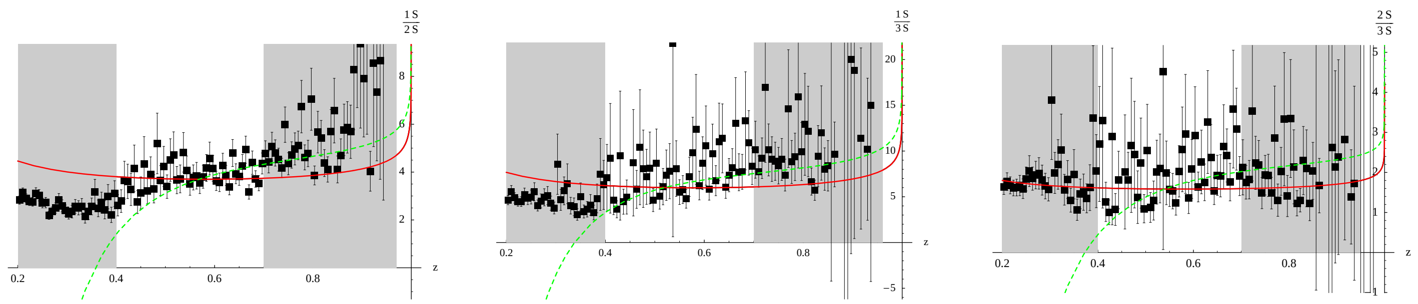
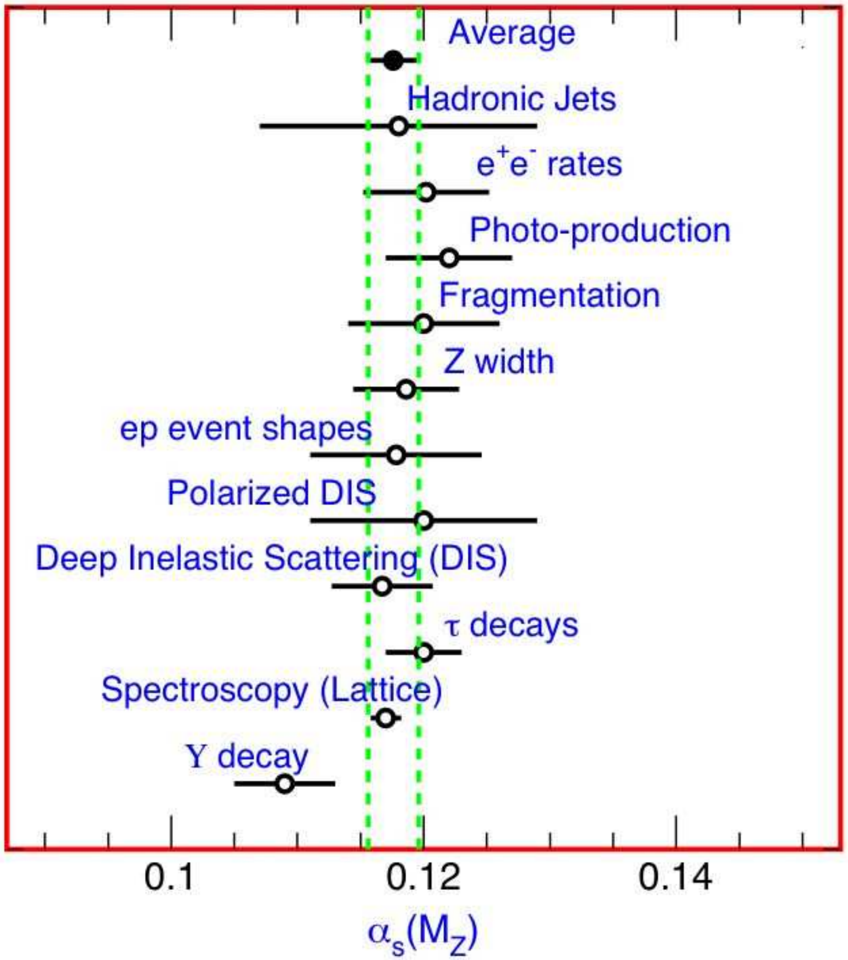


- Same analysis can be performed for charmonium (J/Ψ and Ψ')

The α_s extraction

The α_s extraction



From W.-M. Yao et al., J. Phys. G 33, 1 (2006)

- Extract α_{s} from the ratio

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- Compare then with a theoretical expression for R_γ to extract $\alpha_{\rm s}$

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  - Field's model and our calculation to extrapolate down to $z = 0$
  - Old expressions (prior to NRQCD) for the widths $\Gamma gg\gamma$ and Γggg (to extract α_{s}) Lepage and Mackenzie '81; Bardeen '78
- The idea is to use the same counting employed for the calculation of the spectrum to extract α_{s}

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- Inclusion of all those terms well known to be potentially important
- Data is now very precise and we have all the necessary theoretical ingredients to include all the pieces

Schematically

$$\frac{\Gamma gg\gamma}{\Gamma ggg} = \frac{C_{\gamma O_1(^3S_1)} O_1(^3S_1) + C_{\mathcal{P}_1(^3S_1)} \mathcal{P}_1(^3S_1) + C_{\gamma O_8(^1S_0,^3P_0)} O_8(^1S_0,^3P_0)}{C_{O_1(^3S_1)} O_1(^3S_1) + C_{\mathcal{P}_1(^3S_1)} \mathcal{P}_1(^3S_1) + C_{O_8(^1S_0,^3P_0,^3S_1)} O_8(^1S_0,^3P_0,^3S_1)}$$

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- $\mathcal{O}_8(^1S_0)$ and $\mathcal{O}_8(^3P_0)$ have been estimated in the continuum (weak coupling)

X.G.T. and Soto '04

- $\mathcal{O}_8(^3S_1)$ and $\mathcal{O}_8(^1S_0)$ have been calculated on the lattice

Bodwin, Lee and Sinclair '05

Two different extractions

- C (for continuum)

- L (for lattice)

Two different extractions

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  - Uses all the weak-coupling expressions available and lattice calculation for $\mathcal{O}_8({}^3S_1)$
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The two procedures give very similar results. We take the average as the final value

Error estimation

- C (for continuum)

- L (for lattice)

Error estimation

- C (for continuum)

$$
\begin{aligned}
0.18 \leq \quad & \alpha_{\rm s}(m_b v) & \leq 0.38 \\
0.32 \leq \quad & \alpha_{\rm s}(m_b v^2) & \leq 1.3 \\
0 \leq \quad & \mathcal{R}_{O_8(^3S_1)} & \leq 1.6 \times 10^{-4}
\end{aligned}
$$

- L (for lattice)

$$
\begin{aligned}
0 \leq \quad & \mathcal{R}_{O_8(^1S_0)} & \leq 4.8 \times 10^{-3} \\
0 \leq \quad & \mathcal{R}_{O_8(^3S_1)} & \leq 1.6 \times 10^{-4} \\
-2.4 \times 10^{-4} \leq \quad & \mathcal{R}_{O_8(^3P_0)} & \leq 2.4 \times 10^{-4} \\
-0.052 \leq \quad & \mathcal{R}_{\mathcal{P}_1(^3S_1)} & \leq -0.035
\end{aligned}
$$

Plus errors associated to higher order terms (v^3) and experimental errors

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- Much more sensitive to $O_8({}^3S_1)$ (but this matrix element is small according to the lattice)
- The error in the final result is taken as the full range of the two determinations

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N.Brambilla, X.G.T., J.Soto, A.Vairo '07

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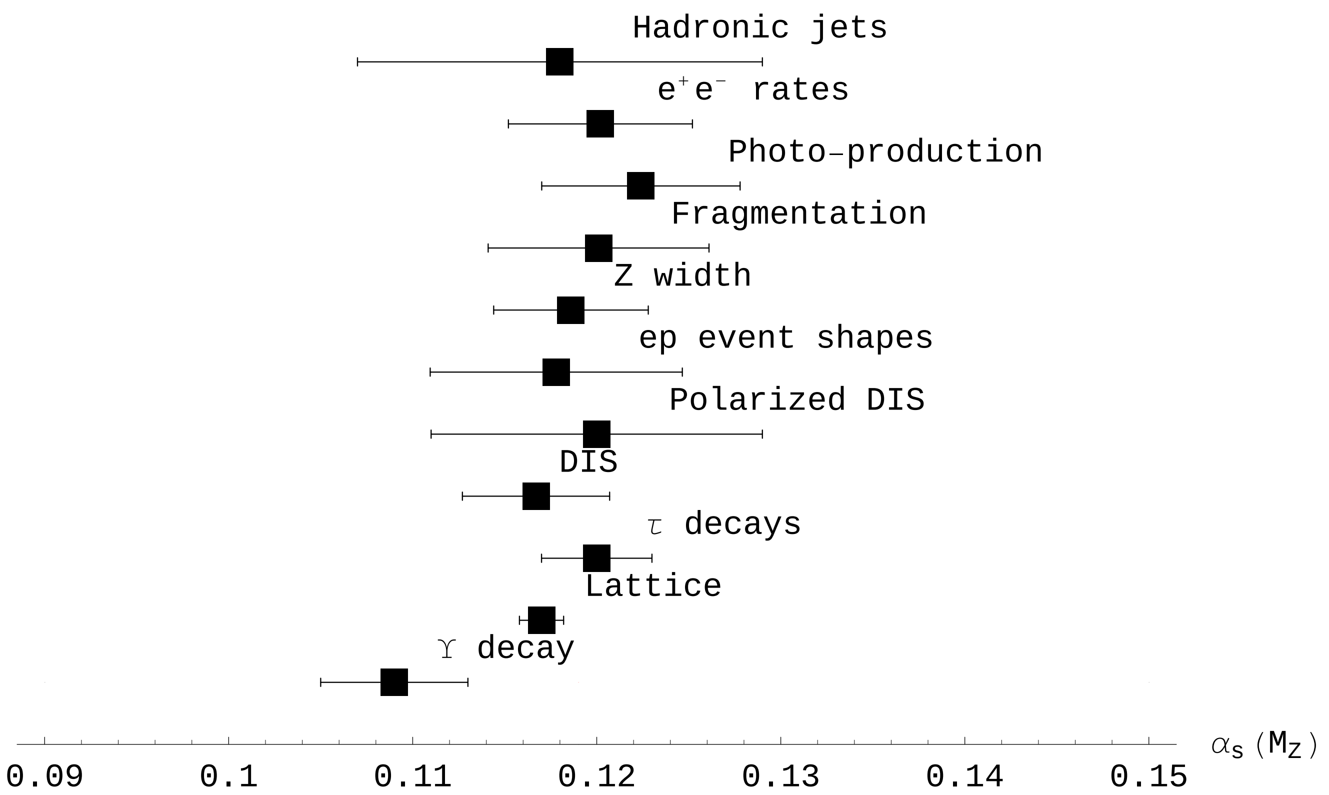
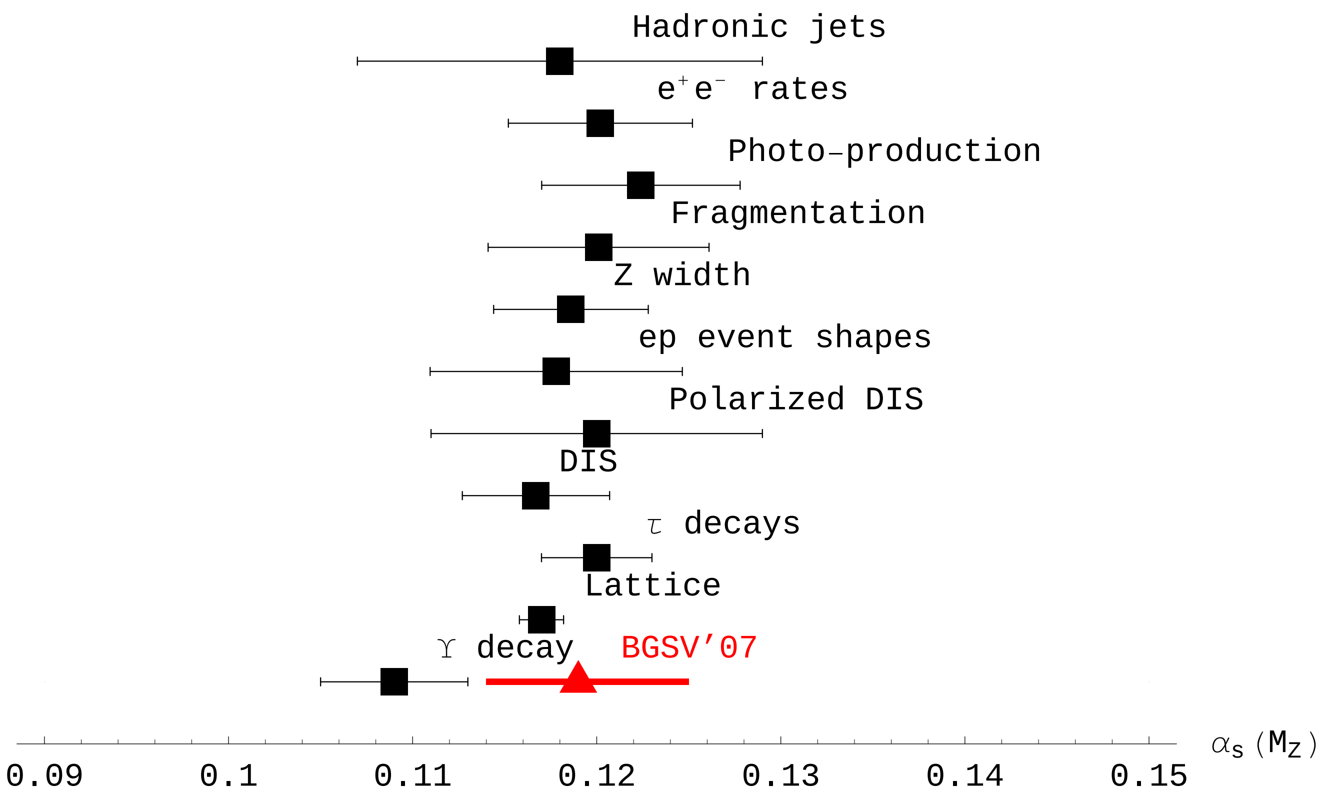
Recall, PDG average $\quad \longrightarrow \quad \alpha_s(M_Z) = 0.1176 \pm 0.0020$



Hadronic jets
e⁺e⁻ rates
Photo-production
Fragmentation
Z width
ep event shapes
Polarized DIS
DIS
τ decays
Lattice
Υ decay
0.09
0.1
0.11
0.12
0.13
0.14
0.15
$\alpha_s(M_Z)$



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Conclusions

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What can we learn from semi-inclusive radiative decays of heavy quarkonium?

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- We can obtain a consistent and precise determination of α_s from the the radiative decay width
- Much to be learned from the photon spectrum!