Lattice determination of $B^0 - \bar{B}^0$ mixing parameters in the Standard Model and beyond

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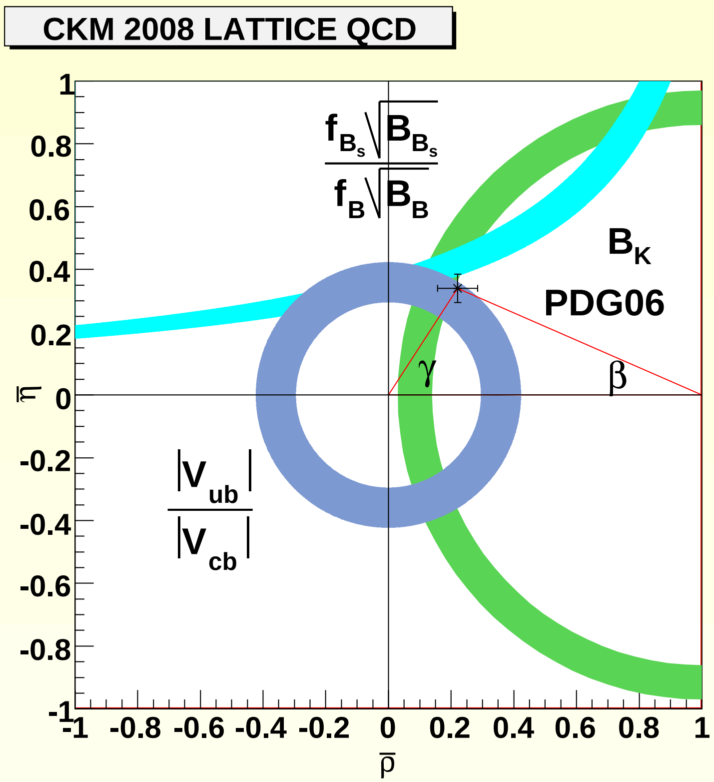
Theory seminar

· Fermilab, 20 November 2008 ·

1. Introduction: Why a precision calculation of $B^0 - \bar{B}^0$ parameters?



C. Davies

\# Determination of fundamental parameters of the SM

  * CKM matrix elements: $|V_{ub}|$, $|V_{cs}|$, $|V_{cd}|$, $|V_{cb}|$

  * heavy quark masses: m_b, m_c

\# Unveiling New Physics effects.

\# Constraining NP models.

\# In conjunction with experimental measurements ...

  * CDF and DØ tagged angular analysis of $B_0^s \to J/\Psi\phi$

In conjunction with experimental measurements ...

* Leptonic decays branching fractions **CLEO-c**, 0806.2112

Observable	% error in corresponding decay constant
$Br(D_s \to \mu\nu)/\ Br(D_s \to \tau\nu)$	3/6.5
$Br(D \to \mu\nu)$	4

* Semileptonic decays branching ratios **BaBar, Belle, CLEO-c**

Observable	% error in corresponding CKM element
$Br(D \to K(\pi)e\nu)$	1.5/4.5
$Br(B \to \pi l\nu)$	6
$Br(B \to D^* l\nu)$	1.5

* $B^0 - \bar{B}^0$ mixing observables

Observable	source	% error
ΔM_s	**CDF**	$<$1
ΔM_d	**PDG07**	$<$1

$\boxed{\text{Non-perturbative theory inputs still main source of error}}$

$\rightarrow$ Need to reduce lattice errors to $\leq 5\%$

Forget quenched approximation: $N_f = 2+1$ calculations

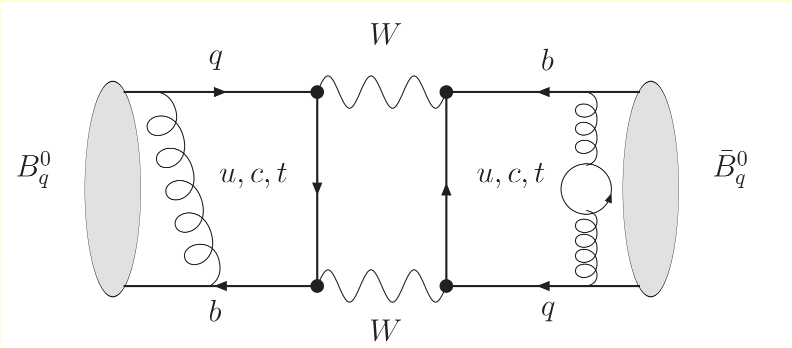
All the sources of systematic errors analyzed:

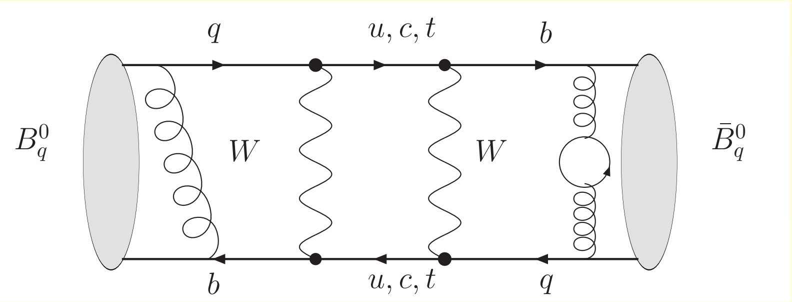
* Discretization (continuum limit): simulations at several lattice spacings.

* Finite volume: simulations at several volumes and/or χPT.

* Results relevant for phenomenology rely on $\chi\mathtt{PT}$ to go to physical masses $\rightarrow$ validity of $\chi\mathtt{PT}$ techniques to have accurate results.

New Physics effects on $B^0 - \bar{B}^0$ mixing





- B_0 mixing parameters determined by the off diagonal elements of the mixing matrix

$$i\frac{d}{dt}\begin{pmatrix} |B_{s/d}(t)\rangle \\ |\bar{B}_{s/d}(t)\rangle \end{pmatrix} = \left(M^{s/d} - \frac{i}{2}\Gamma^{s/d}\right)\begin{pmatrix} |B_{s/d}(t)\rangle \\ |\bar{B}_{s/d}(t)\rangle \end{pmatrix}$$

$$\Delta M_{s/d} \propto |M_{12}^{s/d}| \qquad \Delta\Gamma_{s/d} \propto |\Gamma_{12}^{s/d}|$$

New physics can significantly affect $M_{12}^{s/d} \propto \Delta M_{s/d}$

* Γ_{12} dominated by `CKM-favoured` $b \to c\bar{c}s$ tree-level decays.

Hints of discrepancies between SM expectations and some flavour observables (see, for example, **E. Lunghi**, talk at BEACH08)

* $sin(2\beta)$ **E. Lunghi and A. Soni**, arXiv:0803.0512

SM prediction for $\sin(2\beta)$ using $\Delta F = 2$ inputs (ξ and $\hat{B}_K$) disagrees by $\sim 2\sigma$ with direct experimental measurements via tree-level $B_d \rightarrow \psi K_s$ and penguin dominated modes in $b \rightarrow s$ decays

** Independent of (controversial) $|V_{ub}|$

** It would imply the existence of a BSM CP-odd phase

* B_s^0 mixing phase **UTfit coll.**, arXiv:0803.0659

** Combined fit to the time dependent tagger analyses of $B_s \rightarrow \psi\phi$ from **CDF** and **DØ**

** New phases have effects in $\Delta B = 2$ processes and $b \rightarrow s$ decays

* CP violating effects $B_d^0 - \bar{B}_d^0$, **Buras and Guadagnoli**, arXiv:0805.3887

** Compare theoretical prediction (based on $\Delta F = 2$ data with $B_d \to \psi K_s$ experimenal data.

** Need: new CP phase in B_d^0 system and/or in K system.

** Especially interesting: new CP phase in B_d^0 system equal to new CP phase in B_s^0

These analyses depend on several theoretical inputs:

V_{cb}, V_{ub}, $\hat{B}_K$ and `the SU(3) breaking mixing parameter` ξ:

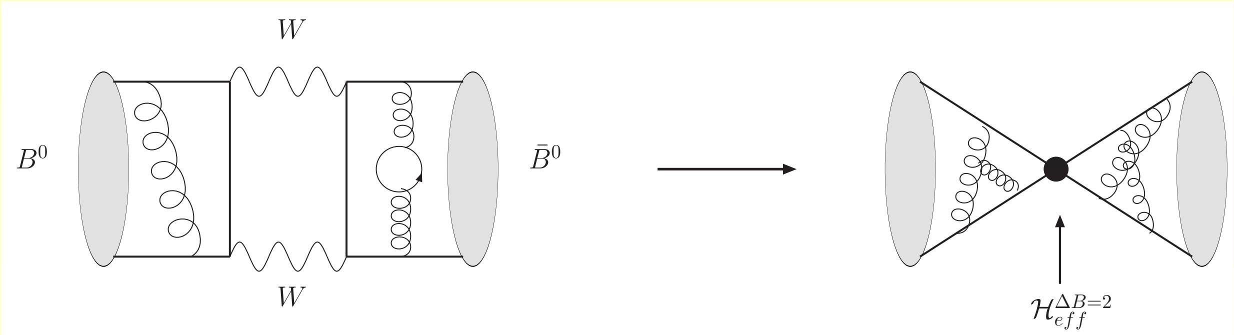
$$\left|\frac{V_{td}}{V_{ts}}\right| = \xi \sqrt{\frac{\Delta M_d M_{B_s}}{\Delta M_s M_{B_d}}}$$

* Comparison of ΔM and $\Delta\Gamma$ with experiment also provides bounds for **NP** effects

Improvement in $B^0 - \bar{B}^0$ mixing parameters which
enter on those analyses is crucial.

2. Mixing parameters in the Standard Model

In the Standard Model



$$\Delta M_q|_{theor.} = \frac{G_F^2 M_W^2}{6\pi^2}|V_{tq}^* V_{tb}|^2 \eta_2^B S_0(x_t) M_{B_s} f_{B_q}^2 \hat{B}_{B_q}$$

* Non-perturbative input

$$\frac{8}{3} f_{B_q}^2 B_{B_q}(\mu) M_{B_q}^2 = \langle \bar{B}_q^0 | O_L | B_q^0 \rangle(\mu) \quad \text{with} \quad O_L \equiv [\overline{b^i}\, q^i]_{V-A}[\overline{b^j}\, q^j]_{V-A}$$

$$\left(\frac{\Delta\Gamma}{\Gamma}\right) = \left(\frac{1}{245\text{MeV}}\right)^2 \left[0.170\left(f_{B_q}^2 B_{B_q}\right) + 0.059 R^2 \left(f_{B_q}^2 \tilde{B}_S R^2\right) - 0.044\, f_{B_q}^2\right]$$

* Non-perturbative input (NLO), **Lenz & Nierste**

$$\frac{1}{3}\frac{f_{B_s}^2 \tilde{B}_S(\mu)}{R^2} M_{B_s}^2 = \langle \bar{B}_q^0 | O_3 | B_q^0 \rangle(\mu) \quad \text{with} \quad O_3 \equiv [\overline{b^i}\, s^j]_{S-P}[\overline{b^j}\, s^i]_{S-P}$$

ξ is an important input for SM tests

In terms of decay constants and bag parameters

$$\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$$

* Many uncertainties in the theoretical (lattice) determination cancel totally or partially in the ratio

$\Longrightarrow$ **very accurate calculation**

3. Some details of the lattice formulations and simulations

Unquenched: Fully incorporate vacuum polarization effects

MILC $N_f^{sea} = 2 + 1$

	HPQCD	Fermilab/MILC
Light fermions	Asqtad	Asqtad
Heavy fermions	NRQCD	Fermilab
Matching	Perturbative: one-loop	Perturbative: one-loop

- **Asqtad** action: improved staggered quarks $\Longrightarrow$ errors $\mathcal{O}(a^2\alpha_s)$, $\mathcal{O}(a^4)$
  * good chiral properties
  * accessible dynamical simulations
- **NRQCD**: Non-relativistic QCD improved through $\mathcal{O}(1/M^2)$, $\mathcal{O}(a^2)$ and leading relativistic $\mathcal{O}(1/M^3)$
  * Simpler and faster algorithms to calculate b propagator

Unquenched: Fully incorporate vacuum polarization effects

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- **Fermilab** action: clover action with Fermilab interpretation

  (El-Khadra, Kronfeld, Mackenzie)

  * Errors: $\mathcal{O}(\alpha_s \Lambda_{QCD}/M)\,,\mathcal{O}(\left(\Lambda_{QCD}/M\right)^2)$

  * It can be efficiently used for both b and c quarks.

- **Improved gluon action**

  * For further reduction of discretization errors

4. Correlation functions and fitting

$$\left.\begin{aligned} O_L &\equiv [\overline{b^i}\, q^i]_{V-A}[\overline{b^j}\, q^j]_{V-A} \\ O_S &\equiv [\overline{b^i}\, q^i]_{S-P}[\overline{b^j}\, q^j]_{S-P} \\ O_3 &\equiv [\overline{b^i}\, q^j]_{S-P}[\overline{b^j}\, q^i]_{S-P} \end{aligned}\right\} \text{lowest order in } 1/M$$

$$O_L^{j1} \equiv \frac{1}{2am_b}\left\{[\vec{D}\overline{b^i}\cdot\vec{\gamma}\, q^i]_{V-A}[\overline{b^j}\, q^j]_{V-A} + [\overline{b^i}\, q^i]_{V-A}[\vec{D}\overline{b^j}\cdot\vec{\gamma}\, q^j]_{V-A}\right\}$$

$$O_S^{j1} \equiv \frac{1}{2am_b}\left\{[\vec{D}\overline{b^i}\cdot\vec{\gamma}\, q^i]_{S-P}[\overline{b^j}\, q^j]_{S-P} + [\overline{b^i}\, q^i]_{S-P}[\vec{D}\overline{b^j}\cdot\vec{\gamma}\, q^j]_{S-P}\right\}$$

$$O_3^{j1} \equiv \frac{1}{2am_b}\left\{[\vec{D}\overline{b^i}\cdot\vec{\gamma}\, q^j]_{S-P}[\overline{b^j}\, q^i]_{S-P} + [\overline{b^i}\, q^j]_{S-P}[\vec{D}\overline{b^j}\cdot\vec{\gamma}\, q^i]_{S-P}\right\}$$

with `i`, `j` colour indices and am_b the bare b mass in lattice units.

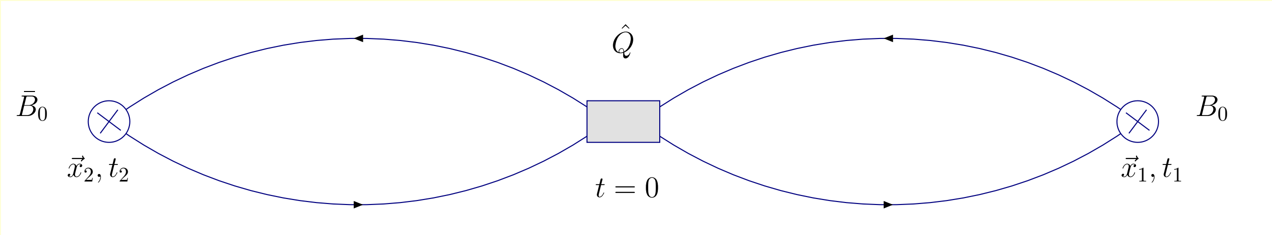
* Dimension 7 operators O_X^{j1} required at $\mathcal{O}(\Lambda_{QCD}/M)$

* **FNAL/MILC** includes dimension 7 operators by rotating b-fields

$$b(x) \to \left(1 + a\mathbf{d_1}\vec{\gamma}\cdot\vec{D}\right)\, b(x)$$

where $\mathbf{d_1}$ is a function of am_b, $\mathcal{O}(1/am_b)$ when am_b is large, and known at tree level (universal value)

Need 3-point (for any $\hat{Q} = Q_X, Q_X^{1j}$) and 2-point correlators



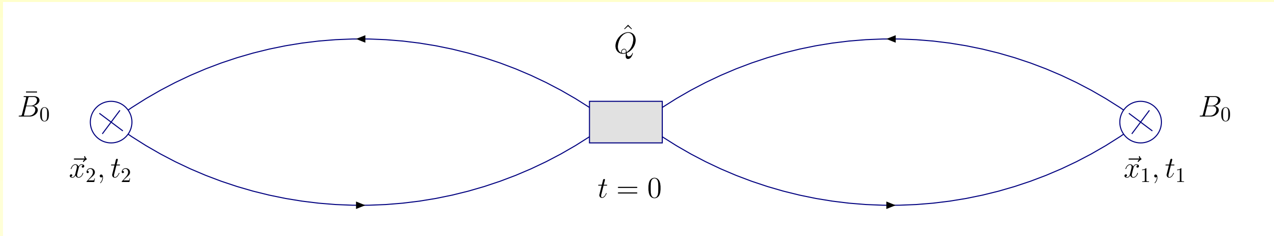
$$C^{(4f)}(t_1, t_2) = \sum_{\vec{x}_1, \vec{x}_2} \langle 0 | \Phi_{\bar{B}_q}(\vec{x}_1, t_1) \left[\hat{Q} \right] (0) \Phi^\dagger_{\bar{B}_q}(\vec{x}_2, -t_2) | 0 \rangle$$

$$C^{(B)}(t) = \sum_{\vec{x}} \langle 0 | \Phi_{\bar{B}_q}(\vec{x}, t) \Phi^\dagger_{\bar{B}_q}(\vec{0}, 0) | 0 \rangle$$

* In order to also extract the value of f_B, f_{B_s} (same function with NRQCD b-quarks)

$$C^{(A_4)}(t) = \sum_{\vec{x}} \langle 0 | \Phi_{\bar{B}_q}(\vec{x}, t) \bar{q}(0) \gamma_0 \gamma_5 b(0) | 0 \rangle$$

- $\Phi_{\bar{B}_q}(\vec{x}, t) = \bar{b}(\vec{x}, t) \gamma_5 q(\vec{x}, t)$ is an interpolating operator for the B_q^0 meson.



Open meson propagator: Basic objects to build all 3-point and 2-point functions

$$E^{ab}_{ij}(t) = \gamma_5^{ac} b^{*dc}_{ki}(t,0) q^{db}_{kj}(t,0)$$

where a, b, c, d are Dirac indices and i, j, k are colour indices.

* Most simulation time is employed in generating the open meson propagator

* **FNAL/MILC** is storing the open meson propagators $\rightarrow$ other four-fermion Dirac structure can be calculated from them without machine work

Fitting

We carried out **simultaneous** fits of the 3-point and 2-point correlators using bayesian statistics to the forms

$$C^{(4f)}(t_1, t_2) = \sum_{j,k=0}^{N_{exp}-1} A_{jk}\, \zeta_i\, \zeta_j\, (-1)^{j\cdot t_1}\, (-1)^{k\cdot t_2}\, e^{-E_B^{(j)}(t_1-1)}\, e^{-E_B^{(k)}(t_2-1)}$$

$$C^B(t) = \sum_{j=0}^{N_{exp}-1} \zeta_j\, (-1)^{j\cdot t}\, e^{-E_B^{(j)}(t-1)}$$

* The hadronic matrix element of any 4-fermion operator $\hat{Q} = O_X, O_X^{1j}$ defined before is given by $\boxed{\langle \hat{Q} \rangle \equiv \langle \bar{B}_s | \hat{Q} | B_q \rangle = A_{00}}$

* **Decay constants** are extracted from 2-point function fits

$$C^{A_4}(t) = \sum_{j=0}^{N_{exp}-1} \mathbf{A_4}\, (-1)^{j\cdot t} e^{-E_B^{(j)}(t-1)} \quad \text{and} \quad A_4/\zeta \propto f_{B_q}$$

$$C^B(t) = \sum_{j=0}^{N_{exp}-1} \zeta_j\, (-1)^{j\cdot t}\, e^{-E_B^{(j)}(t-1)}$$

Byproduct of the calculation: f_B and f_{B_s}

Extraction of CKM matrix elements: $\underbrace{B(B^- \to \tau^- \bar{\nu}_\tau)}_{\textbf{experiment}} \propto \boxed{|V_{ub}|^2} \; \underbrace{f_B^2}_{\textbf{lattice}}$

Decay constants needed in the SM prediction for processes potentially very sensitive to BSM effects: for example, f_{B_S} for $B_s \to \mu^+\mu^-$

$B^- \to \tau^- \bar{\nu}_\tau$ is a sensitive probe of effects from charged Higgs bosons.

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FNAL/MILC Separate project (more complete analysis): **check**
Preliminary results 2008

$$f_B = (195 \pm 11)\mathrm{MeV} \quad f_{B_s} = (243 \pm 11)\mathrm{MeV} \quad f_{B_s}/f_B = 1.25 \pm 0.04$$

HPQCD Update of 2005 results

$$f_B = (216 \pm 22)\mathrm{MeV} \quad f_{B_s} = (260 \pm 26)\mathrm{MeV} \quad f_{B_s}/f_B = 1.20 \pm 0.03$$

* Improvements: Reduction of `statistical errors`, main result from `smaller lattice spacing` ...

... but still final error dominated by `renormalization uncertainty`

5. Going to the continuum: Renormalization

The input for the SM prediction for ΔM_s is

$$\langle O_L \rangle^{\overline{MS}}(\mu) \equiv \frac{8}{3} f_{B_s}^2 B_{B_s}^{\overline{MS}}(\mu) M_{B_s}^2$$

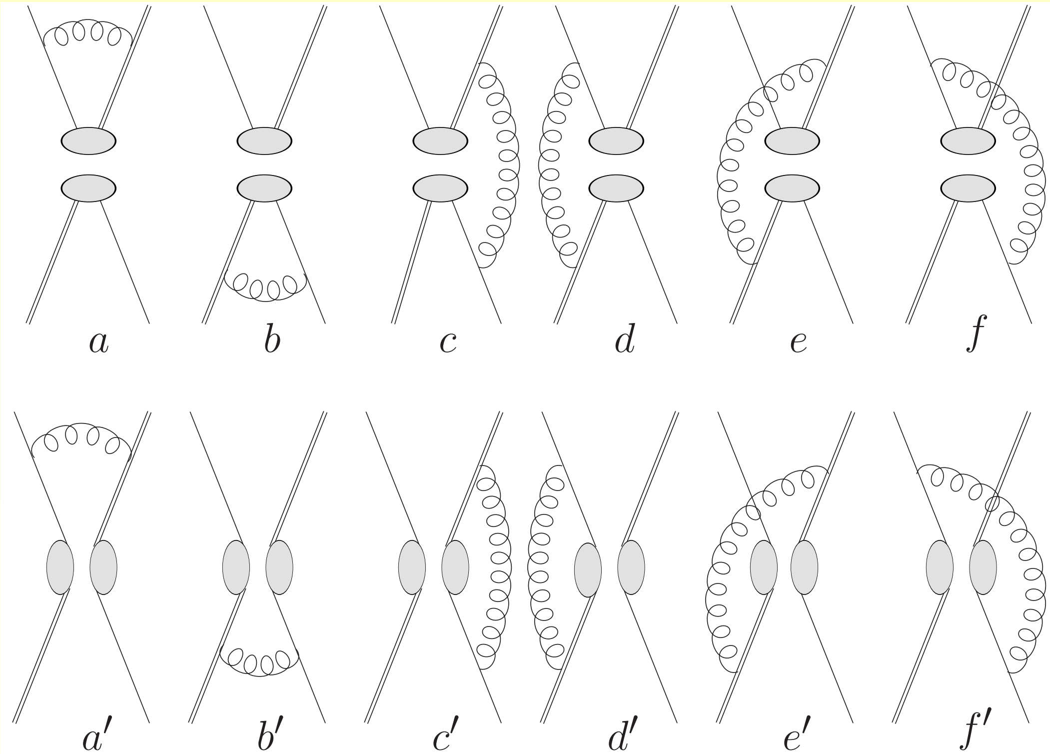
that is related to the lattice operators in a general way through

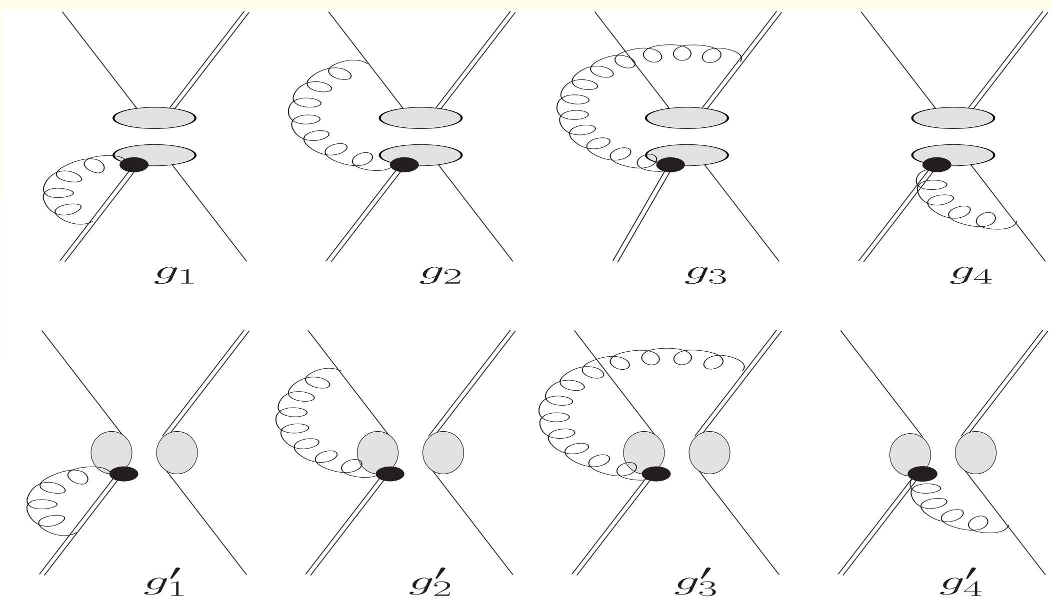
$$\frac{a^3}{2M_{B_s}} \langle O_L \rangle^{\overline{MS}}(\mu) \quad = \quad [1+\alpha_s \cdot \rho_{LL}] \langle O_L \rangle (a) + \alpha_s \cdot \rho_{LS} \langle O_S \rangle (a) + \underbrace{\langle O_L^{j1} \rangle (a)}_{HPQCD}$$

* $\langle O_X \rangle$: operator's matrix elements in the lattice theory

* One-loop renormalization coefficients $\rho_{XY} = \rho_{XY}^{\overline{MS}}(\mu, m_b) - \rho_{XY}^{latt.}(am_b)$, $\rho_{XY}^{latt.}(am_b)$ depends on the exact lattice action used

* $\alpha_s = \alpha_V(q^*) \rightarrow q^* = 2/a$, very close to q^*s for heavy-light currents





To what extent is four-fermion operator renormalization dominated by current-like diagrams?

Non-perturbative or partially non-perturbative matching for currents

FNAL/MILC: Rewrite the renormalization factor for any current J^{ac} as $Z_{J^{ac}} = \sqrt{Z_{V_4}^{aa} Z_{V_4}^{cc}}\rho_{J^{ac}}$

* For Fermilab currents and Fermilab-Asqtad currents

** $Z_{V_4}^{aa}$ and $Z_{V_4}^{cc}$ calculated nonperturbatively

** $\rho_{J^{ac}}$ calculated perturbatively $\rightarrow$ **very close to 1 at one-loop**

Important **reduction** of **matching uncertainties**

Non-perturbative or partially nonperturbative matching for currents

\# **FNAL/MILC**: Rewrite the renormalization factor for any current J^{ac} as $Z_{J^{ac}} = \sqrt{Z^{aa}_{V_4} Z^{cc}_{V_4}} \rho_{J^{ac}}$

* For Fermilab currents and Fermilab-Asqtad currents

** $Z^{aa}_{V_4}$ and $Z^{cc}_{V_4}$ calculated nonperturbatively

** $\rho_{J^{ac}}$ calculated perturbatively $\rightarrow$ **very close to 1 at one-loop**

\# **HPQCD** Determination of current renormalization Z_i from current-current correlators.

HPQCD determination of m_c from current-current correlators
0805.2999

Method analogous to the extraction of m_c from dispersion relations using `perturbative` **determination of zero-momentum** `moments of current-current correlators` **and** `experimental` **data from $e^+e^- \rightarrow hadrons$.**

Non-perturbative or partially nonperturbative matching for currents

m_c extracted from

* moments of charm-quark P, V and A correlators
* 4-loop continuum perturbation theory to determine $g_n(\alpha_{\overline{MS}}(\mu), \mu/m_c)$

* HISQ action used to determine moments G_n $(j_5 \equiv \bar{\psi}_c \gamma_5 \psi_c)$

$$G_n \equiv \sum_t (t/a)^n G(t) \quad \text{with} \quad G(t) \equiv a^6 \sum_{\vec{x}} (am_{0c})^2 \langle 0 | j_5(\vec{x}, t) j_5(0,0) | 0 \rangle$$

$$G_n = \frac{g_n(\alpha_{\overline{MS}}(\mu), \mu/m_c)}{\left(am_c^{\overline{MS}}(\mu)\right)^{n-4}}$$

* If current in correlation function is not conserved (`NRQCD-Asqtad` currents) $\rightarrow G_n^{cont} \propto Z_i^2 G_n^{latt.}$

  ** Extraction of m_c: Taking ratios of moments to cancel renormalization or ...

  ** extract value of Z_i with m_c from elsewhere.

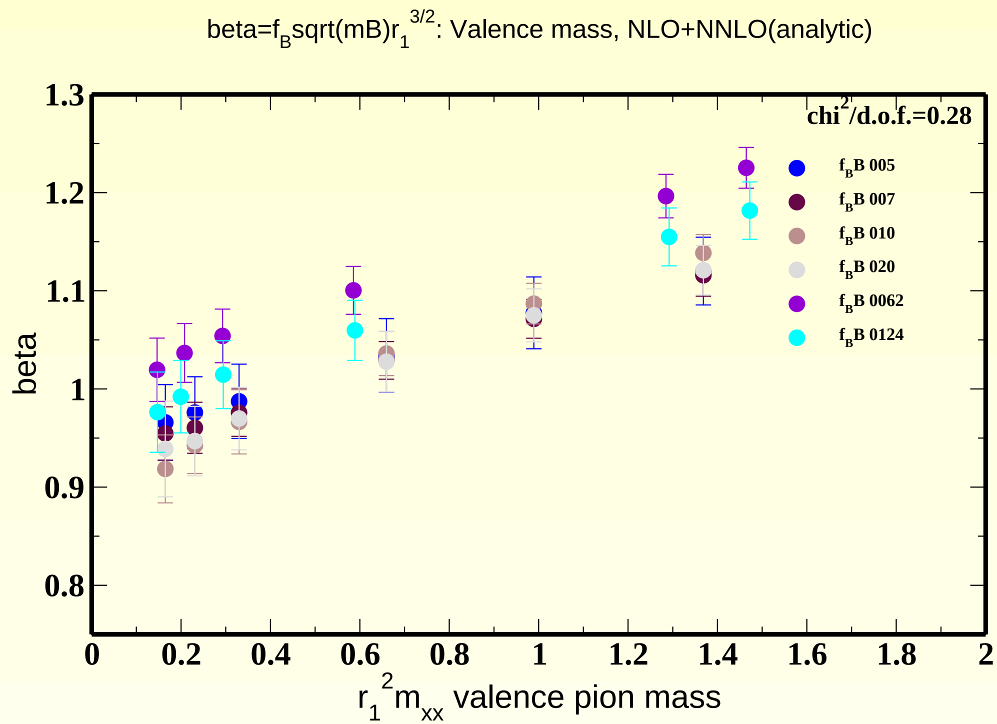
6. Preliminary results

	FNAL/MILC	HPQCD
# Calculation of all the matrix elements needed to determine $\Delta M_{d,s}$, $\Delta\Gamma_{d,s}$ and ξ		
# MILC configurations: Asqtad for light sea (and valence) quarks ($m_\pi^{min.} \simeq 230\mathrm{MeV} \rightarrow$ $\boxed{\text{chiral regime}}$)		
b quarks	Fermilab	NRQCD
a(fm)	0.15, 0.12, 0.09	0.12, 0.09
light sea masses	$3+4+2$	$4+2$
light valence masses	6 for each sea mass	full QCD
# Simultaneous fits of the 2-pt and 3-pt correlators for any four-fermion operator		
# Perturbative renormalization: one loop.		

$\Longrightarrow$ Valence m_b fixed to its physical value. Sea and valence m_s close to its physical value.

$B^0 - \bar{B}^0$ **mixing:** $N_f = 2+1$ **Preliminary results for** $f_{B_q}\sqrt{M_{B_q}B_{B_q}}$

FNAL/MILC



$$\beta = f_{B_q}\sqrt{M_{B_q}B_{B_q}}$$

Renormalization not applied yet.

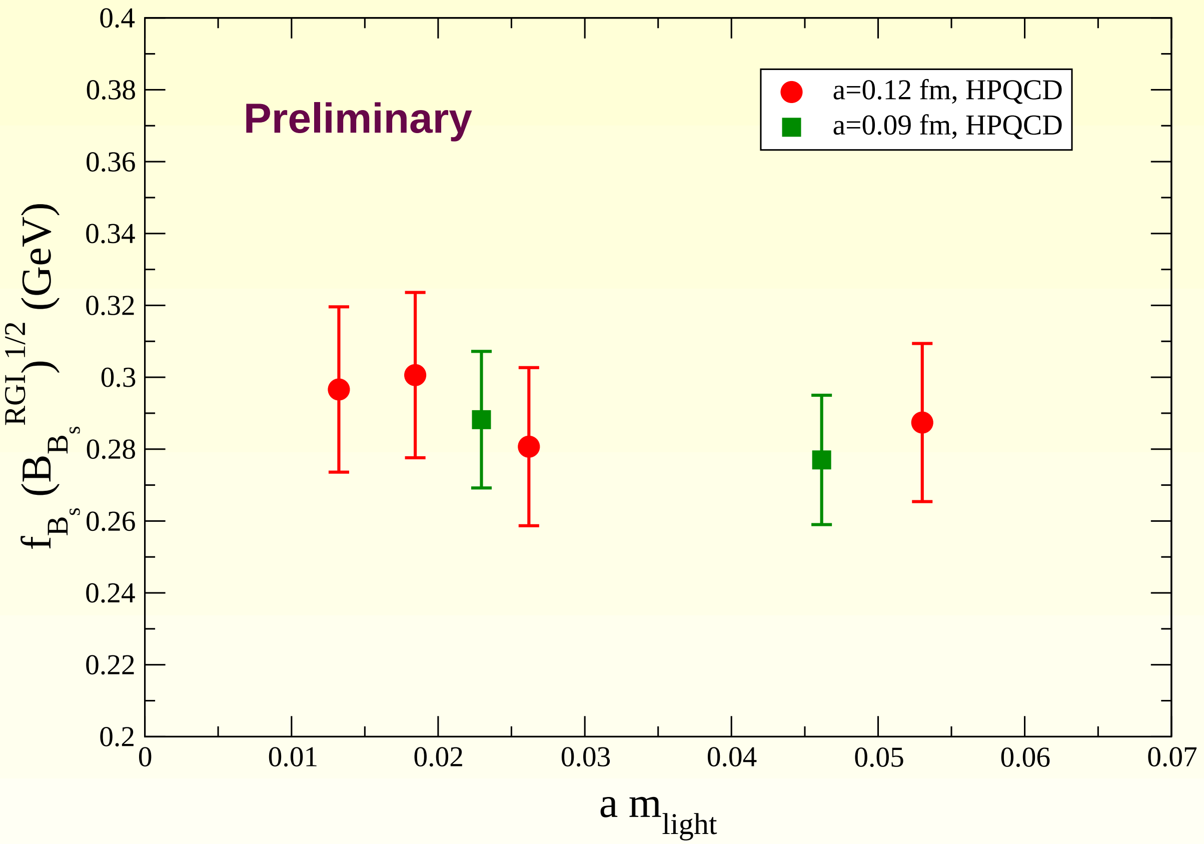
Statistics+fitting errors: $2-5\%$ (B_s^0-B_d^0)

Very mild dependence on light sea quark masses.

Small difference between `fine` and `coarse` points
$\longrightarrow$ discretization errors under control.

$B^0 - \bar{B}^0$ mixing: $N_f = 2+1$ Preliminary results for $f_{B_s}\sqrt{\hat{B}_{B_s}}$

HPQCD



All systematic sources included in error bars.

Full QCD points

Statistics+fitting errors: $1-4\%$ (B_s^0-B_d^0)

Very mild dependence on light sea quark masses.

Fine lattice points fall on the `coarse line` $\rightarrow$ small discretization errors.

* `Relativistic` corrections are $\sim 5-6\%$ for coarse and $\sim 3-4\%$ for fine.

Chiral extrapolation

Extrapolation to $m_u = m_d$ and m_s physical masses ($m_{sea} = m_{valence}$)
$+$ continuum extrapolation

* Using NLO Staggered χPT (J. Laiho and R. Van de Water)

HMChPT: Detmold and Lin, Becirevic et al.

** Accounts for NLO quark mass dependence.
** Accounts for dominant $\mathcal{O}(a^2)$ taste violations
$\rightarrow$ remove the dominant light discretization errors (remain $\mathcal{O}(a^2\alpha_s^2)$)

$$\langle \bar{B}_q | Q_1^q | B_q \rangle = m_{B_q} \beta \left[1 + NLO\,\mathrm{logs} + L_v m_q + L_s (2m_L + m_S) + L_a a^2 \right] + NNLO\,\mathrm{analytic}$$

* Logs terms depend on $M^2_{ij,\Xi} = \mu(m_i + m_j) + a^2 \Delta_\Xi$ with

** m_i, m_j valence or sea quark masses

** μ and Δ_Ξ (as well as δ'_I) come from simulations of light quantities

** Experimental values of f_π and $g_{BB^*\pi}$ (fits quite insensitive to exact value)

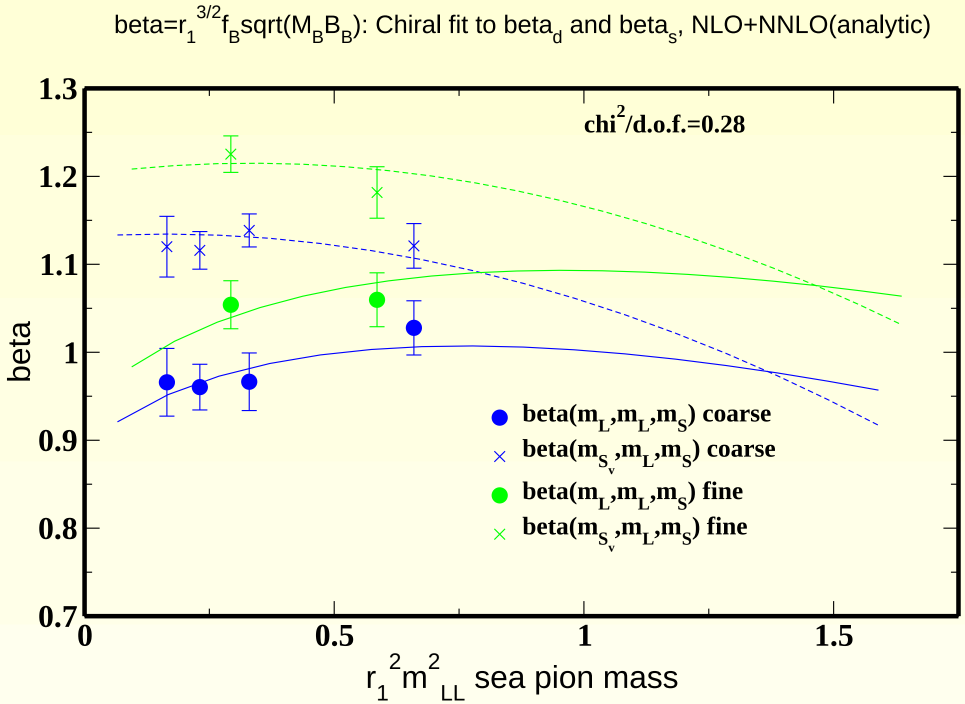
Chiral extrapolation

$$\langle \bar{B}_q | Q_1^q | B_q \rangle \quad = \quad m_{B_q} \beta \left[1 + NLO\,\mathrm{logs} + L_v m_q + L_s (2m_L + m_S) + L_a a^2 \right] + NNLO\,\mathrm{analytic}$$

* To extrapolate: $a \to 0$, $m_L \to \frac{m_u + m_d}{2}$, $m_S \to m_s$, and $m_q \to m_s, m_d$
* **FNAL/MILC** Central values including `NNLO` analytic terms.
  ** Light quark + (systematic) fit errors estimated by excluding/including `NNLO` terms.
  ** Errors associated with uncertainty in the inputs used: `light quark masses`, scale (r_1), Δ_Ξ, μ, ...
  ** $g_{BB^*\pi}$, δ'_I: Introduce uncertainty in the priors
  ** Finite volume effects

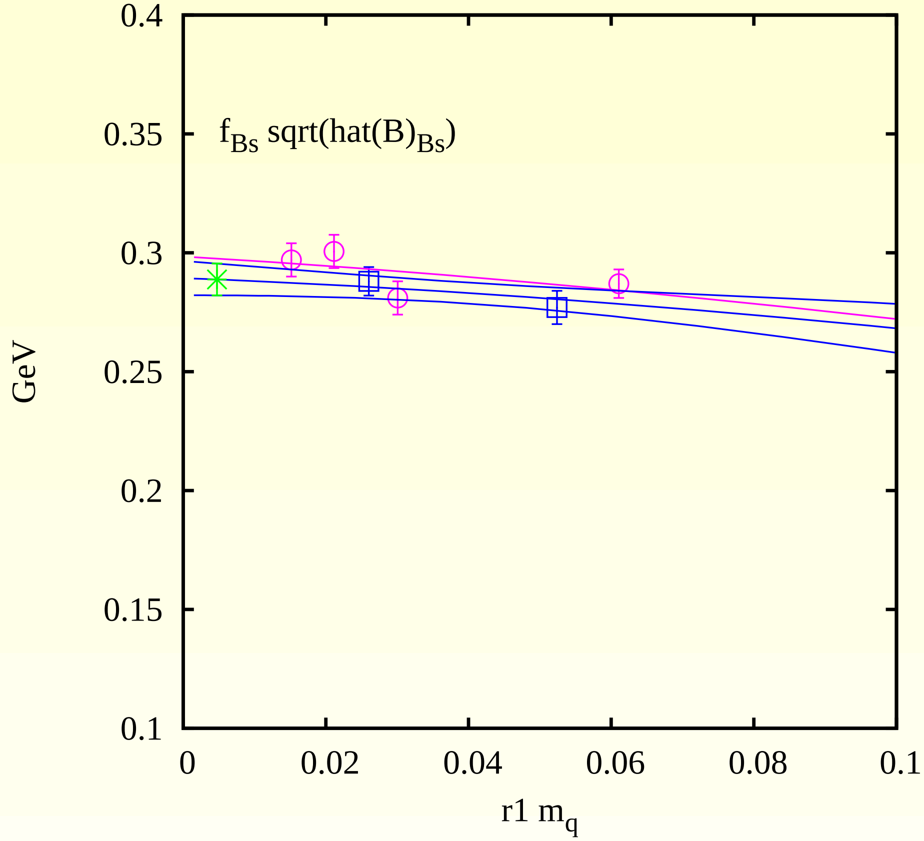
Preliminary results for $f_{B_q}\sqrt{B_{B_q}}$: Extrapolation

FNAL/MILC



Renormalization not applied yet.

HPQCD ($f_{B_s}\sqrt{\hat{B}_{B_s}}$)



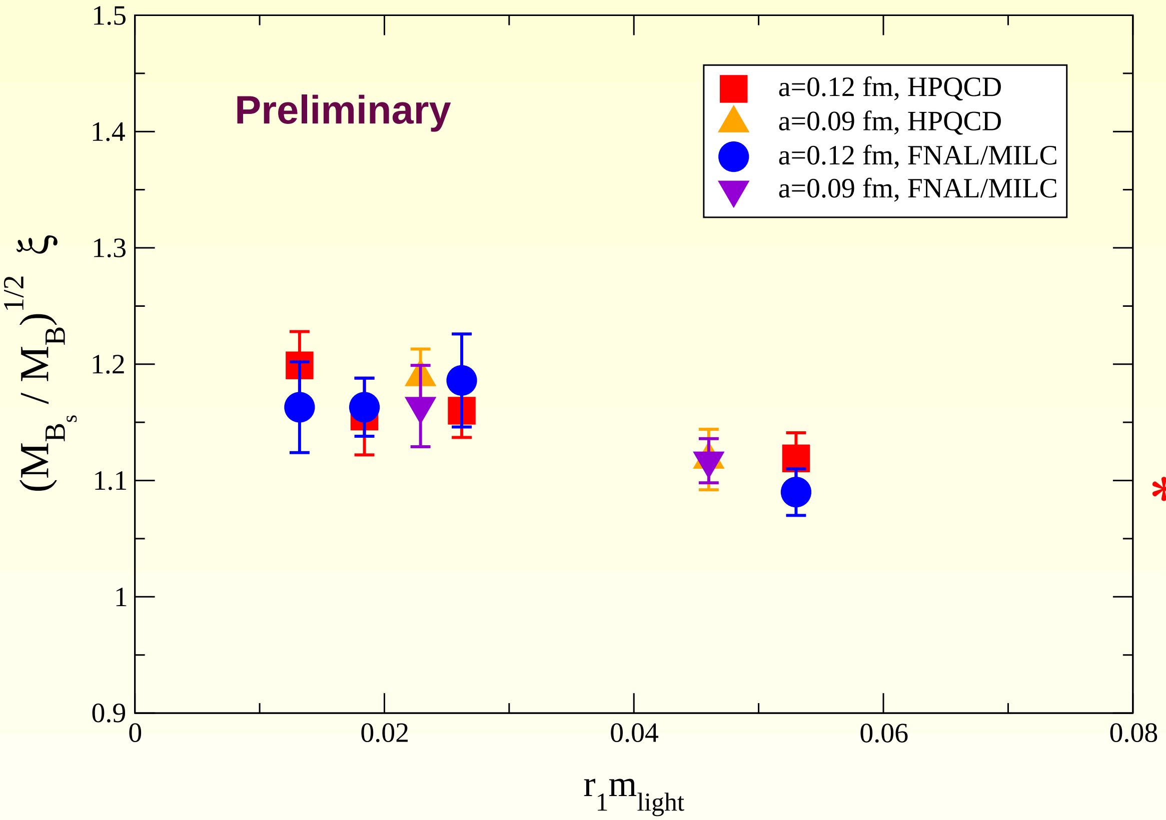
\# Examples shown for one lattice spacing fits.

\# For $f_{B_s}\sqrt{M_{B_s}B_{B_s}}$, results are very flat with m_{sea}^{light}

$\rightarrow$ small error associated to the extrapolation

$B^0 - \bar{B}^0$ **mixing:** $N_f = 2+1$ **Preliminary results for** ξ

(**FNAL/MILC** & **HPQCD**)



* **Only full QCD for FNAL/MILC shown.**

Statistical errors: $1 - 3\%$

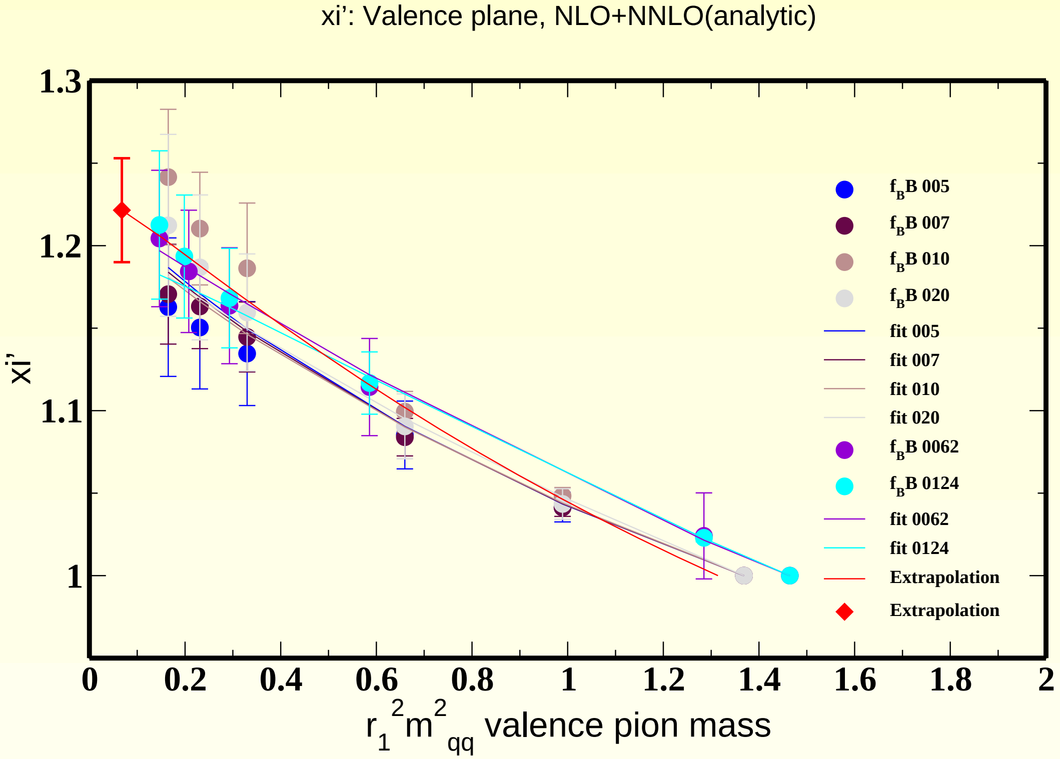
* **Very small discretization errors and very mild light quark mass dependence.**

\# Very good agreement between both coll. $\rightarrow$ small systematic associated with heavy quark discretization.

\# **Extrapolation**: some terms cancel in the ratio $\rightarrow$ less parameters

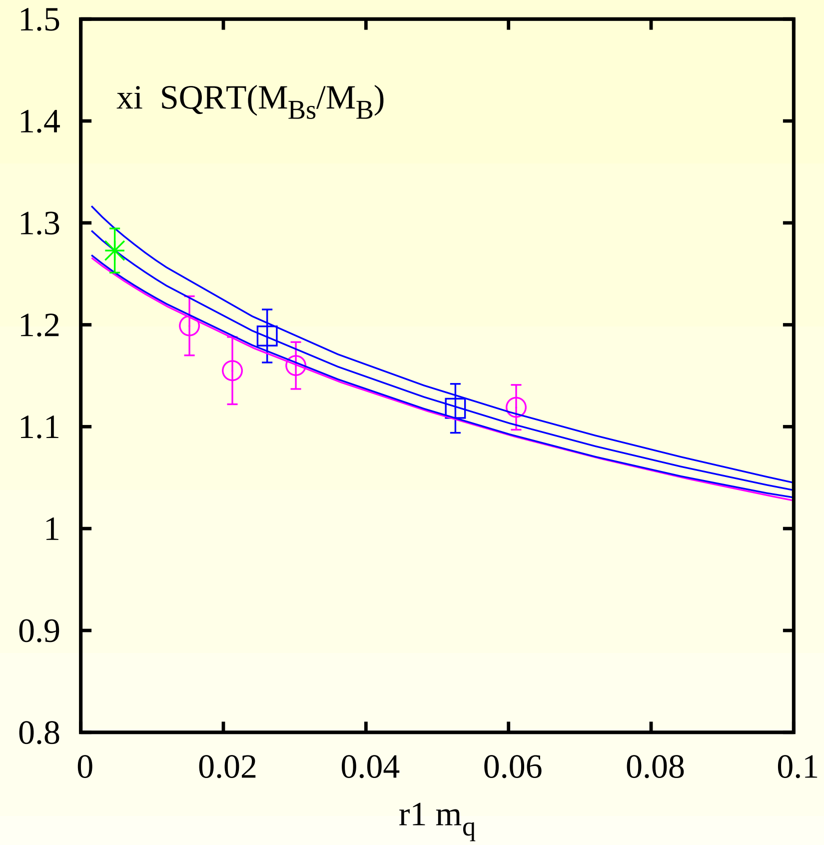
Preliminary results for ξ: Extrapolation

FNAL/MILC



Renormalization not applied yet.

HPQCD $(f_{B_s}\sqrt{\hat{B}_{B_s}})$



Preliminary: only fine data and NLO terms

FNAL/MILC: Simultaneous chiral and continuum extrapolation with $S\chi PT$ at NLO + NNLO analytic terms: (all data included)

$$\boxed{\xi = 1.211 \pm 0.038 \pm 0.024_{estimate}}$$

Error budget (in % for FNAL/MILC)

	ξ	β_d	β_s
Statistical	2.5	4	2.7
Matching	$\sim$ 0.5	$\sim$ 3	$\sim$ 3
Heavy quark discretization	0.2	2	2
Light quark discretization + chiral fits	2.5	4.3	1.3
scale error (r_1)	0.2	3.1	3.0
$g_{BB^*\pi}$	0.8	1.4	2.3
input parameters: $\hat{m}$, m_d, m_s	0.7	0.5	0.3
Estimated from FNAL/MILC calculation of f_B and f_{B_s}			
κ_b	$\leq$0.1	1.1	1.1
Finite Volume	0.6	0.6	0.2
Total	3.8	7.8	6.1

with $\beta_q = f_{B_q}\sqrt{M_{B_q} B_{B_q}}$.

Similar errors for HPQCD calculation.

7. B_0 mixing beyond the SM

New physics can significantly affect $M_{12}^s \propto \Delta M_s$

A general parametrization of **NP** effects in the mass difference and the mixing phase is

$$\Delta M_q = \Delta M_q^{SM}\left(1+\kappa_q e^{i\sigma_q}\right)$$
$$\phi_q = \phi_q^{SM} + arg\left(1+\kappa_q e^{i\sigma_q}\right)$$

with $\phi_q^{SM} \equiv 2arg\left(V_{tq}^* V_{tb}\right) = \begin{cases} +2\beta & (q=d) \\ -2\delta\gamma & (q=s) \end{cases}$

* This phase will also governs `mixing-induced CP` violation in exclusive channels like $B_s \to J/\psi\phi$.

To compare these expressions with experiment and get information about **NP** one needs

* A precise determination of **SM** contributions

* A prediction of the **NP** effects for a particular theory

Comparison of experimental measurements and theoretical predictions can constraint some BSM parameters and help to understand BSM physics. Examples:

F. Gabbiani et al, Nucl.Phys.B477 (1996), **D. Bećirević et al**, Nucl.Phys.B634 (2002); `general SUSY models`

P. Ball and R. Fleischer, Eur.Phys.J. C48(2006); `extra Z' boson`; `SUSY`

Help to constrain the soft SUSY breaking terms and the mechanism of SUSY breaking.

M. Ciuchini and L. Silvestrini, PRL 97 (2006) 021803; `SUSY`

Constraints on the mass insertions ($|Re(\delta^d_{23})_{RR}| < 0.4$, $|(\delta^d_{23})_{LL}| < 0.1$,...)

M. Blanke et al, JHEP 12(2006) 003; `Little Higgs model with T-parity`

ΔM_q can be used to test viability of the model. To constrain and test the model in detail $\Delta M_s/\Delta M_d$ and $\Delta\Gamma_q$.

Lunghi and Soni, 0707.0212; `Top Two Higgs Doublet Model`

Constraints on β_H (ratio of vev's of the two Higgs) and m_{H^+}

M. Blanke et al, 0809.1073; `Warped Extra Dimensional Models`

Constraints on the KK mass scale (it can be as low as $M_{KK} \simeq 3TeV$)

Description of $B^0 - \bar{B}^0$ effects in beyond the Standard Model theories

\# Effects of heavy new particles seen in the form of effective operators built with **SM** degrees of freedom

\# The most general `Effective Hamiltonian` describing $\Delta B = 2$ processes is

$$\mathcal{H}_{eff}^{\Delta B=2} = \sum_{i=1}^{5} C_i Q_i + \sum_{i=1}^{3} \widetilde{C}_i \widetilde{Q}_i \qquad \text{with}$$

$$Q_1^q = \left(\bar{\psi}_b^i \gamma^\nu (\mathrm{I} - \gamma_5) \psi_q^i\right) \left(\bar{\psi}_b^j \gamma^\nu (\mathrm{I} - \gamma_5) \psi_q^j\right) \qquad \textbf{SM}$$

$$Q_2^q = \left(\bar{\psi}_b^i (\mathrm{I} - \gamma_5) \psi_q^i\right) \left(\bar{\psi}_b^j (\mathrm{I} - \gamma_5) \psi_q^j\right) \qquad Q_3^q = \left(\bar{\psi}_b^i (\mathrm{I} - \gamma_5) \psi_q^j\right) \left(\bar{\psi}_b^j (\mathrm{I} - \gamma_5) \psi_q^i\right)$$

$$Q_4^q = \left(\bar{\psi}_b^i (\mathrm{I} - \gamma_5) \psi_q^i\right) \left(\bar{\psi}_b^j (\mathrm{I} + \gamma_5) \psi_q^j\right) \qquad Q_5^q = \left(\bar{\psi}_b^i (\mathrm{I} - \gamma_5) \psi_q^j\right) \left(\bar{\psi}_b^j (\mathrm{I} + \gamma_5) \psi_q^i\right)$$

$$\tilde{Q}_{1,2,3}^q = Q_{1,2,3}^q \text{ with the replacement } (\mathrm{I} \pm \gamma_5) \rightarrow (\mathrm{I} \mp \gamma_5)$$

where ψ_b is a heavy b-fermion field and ψ_q a light $(q = u, d)$ fermion field.

- $C_i, \widetilde{C}_i$ Wilson coeff. calculated for a particular BSM theory
- $\langle \bar{B^0} | Q_i | B^0 \rangle$ calculated on the `lattice`

Description of $B^0 - \bar{B}^0$ effects in beyond the Standard Model theories

Quenched lattice calculation of matrix elements still the only ones available for these studies

Bećirević et al, JHEP 0204 (2002), Wilson fermions and static limit

Need an unquenched determination of the BSM matrix elements

Strong interactions conserve parity $\rightarrow \langle \widetilde{Q}_{i=1,2,3} \rangle = \langle Q_{i=1,2,3} \rangle$.

5 different matrix elements, $\langle \bar{B}^0{}_{d(s)} | Q_{i=1-5} | B^0_{d(s)} \rangle$.

Only Dirac structure of four-fermion operators change
$\rightarrow$ same programme can be applied

* Open meson propagators stored (**FNAL/MILC**): No significant extra computer time needed

Description of $B^0 - \bar{B}^0$ effects in beyond the Standard Model theories

\# One-loop renormalization coefficients

* Calculation completed by the **HPQCD** col., PRD 77 (2008) 114505
  ** For `NRQCD` heavy and (`staggered`) `Asqtad` light
  ** Needed to calculate some continuum renormalization coefficients for BSM. Quote results for two different schemes.
  ** Well behaved matching coefficients
  ** Matching coef. dominated by `current-like` contributions and `wave function` renormalization

\# Chiral perturbation theory

* Continuum `HMCHPT` expressions exist. Extra parameters appear for **BSM** operators

$$\langle \bar{B}_q | Q^q_{2-5} | B_q \rangle = m_{B_q} \beta \left[1 + NLO\,\mathrm{logs} + \beta_2 NLO\,\mathrm{logs}_2 + L_v m_q + L_s (2m_L + m_S) + L_a a^2 \right] + \mathcal{O}(NNLO)$$

Description of $B^0 - \bar{B}^0$ effects in beyond the Standard Model theories

* Same kind of chiral expressions for $Q_{2,3}$ needed for $\Delta\Gamma$
* Staggered HMCHPT expressions also exist.
  ** Do not expect complications due to the extra parameter

8. Conclusions and outlook

SM results for the B_s^0 and B_d^0 mixing parameters (ΔM and $\Delta\Gamma$) soon:

* **FNAL/MILC** and **HPQCD**: $f_B\sqrt{B_B}$ with $6-8\%$ error and ξ with $3-4\%$ error.

* Expected reduction of the errors by a factor of $\sim 1.5-2$ in a few years: finer lattice spacing, improved perturbation theory, more statistics, better fitting methods, improved actions ...

* An accurate calculation of $B^0-\bar{B}^0$ parameters will help to clarify the origin of the theory-experiment disagreements in some CP violating observables.

Same accuracy can be achieved for the matrix elements in the general $\Delta B=2$ effective hamiltonian **BSM**.

* The value of those matrix elements, together with experimental data, will help to constrain the parameter space in **BSM** theories

Short-distance contributions to $D^0-\bar{D}^0$ can also be calculated in the same way by FNAL/MILC. HPQCD will need to use something like **HISQ** for the c-fermions