

Superfiniteness of $\mathcal{N} = 8$ supergravity at three loops and beyond



Fermilab
Dec 4, 2008

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Bern, Carrasco, Dixon, HJ, Kosower, Roiban
[hep-th/0702112](#)

Bern, Carrasco, Dixon, HJ, Roiban
[arXiv:0808.4112 \[hep-th\]](#)

Bern, Carrasco, Forde, Ita, HJ
[arXiv:0707.1035 \[hep-th\]](#)

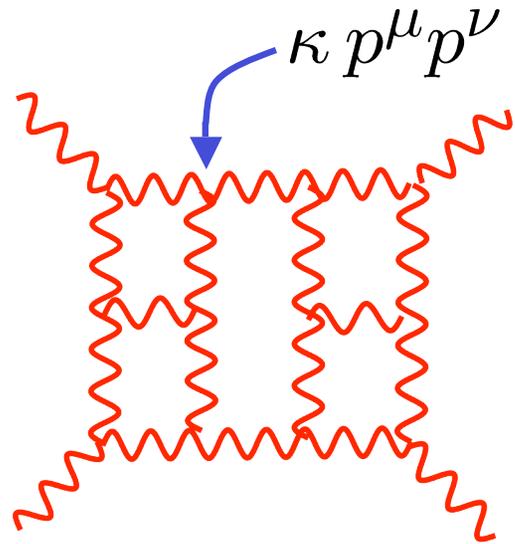
Outline

- Motivation & Introduction
- Tools
 - On-shell recursion
 - KLT, gravity \sim (Yang-Mills)²
 - Unitarity method
- Divergence opinions
- $\mathcal{N} = 8$ sugra 3-loop results
 - UV behavior
 - 4-loop preview
 - Origin of UV cancellations
- Conclusion

Motivation

- Is a 4-d point-like QFT of gravity possible ?
- Dim. of coupling $\kappa \sim m_{\text{Pl}}^{-1} \rightarrow$ gravity non-renormalizable
- Consistent theory should then be finite $\kappa = \sqrt{32\pi G_N}$
- Known (by calculation)
 - Pure Einstein gravity diverges at two loops *Goroff and Sagnotti*
 - with matter at one loop *'t Hooft and Veltman*
- Unknown
 - Maximal $\mathcal{N} = 8$ supergravity - no known divergence
 - \mathcal{N} -extended supergravity - no known div. for $\mathcal{N} > 0$

Non-Renormalizable by Power Counting



Gravity:

$$\int \prod_{i=1}^L \frac{d^D p_i}{(2\pi)^D} \frac{(\kappa p_j^\mu p_j^\nu) \cdots}{\text{propagators}}$$

Gauge theory:

$$\int \prod_{i=1}^L \frac{d^D p_i}{(2\pi)^D} \frac{(g p_j^\nu) \cdots}{\text{propagators}}$$

Extra powers of loop momenta in numerator means integrals are badly behaved in the UV

Key problem for any gravity theory

$\mathcal{N} = 8$ SUGRA

- $\mathcal{N} = 8$ supergravity ($D = 4$) by **Cremmer, Julia and Scherk** (1978, 1979)
- **8 susys** – maximum number of susys for a spin-2 theory
- One supermultiplet of **256 massless states** ($=2^8$)

# states	1	8	28	56	70	56	28	8	1
helicity	-2	-3/2	-1	-1/2	0	1/2	1	3/2	2
	h^-	ψ^-_i	ν^-_{ij}	χ^-_{ijk}	S_{ijkl}	χ^+_{ijk}	ν^+_{ij}	ψ^+_i	h^+

- Reasons to focus on this theory:
 - With more susy expect better UV properties.
 - High symmetry implies technical simplicity.
 - Recently conjectured by Arkani-Hamed, Cachazo and Kaplan to be “simplest” quantum field theory.

Focus on Basic Issue

- Our interest in $\mathcal{N}=8$ sugra as a UV finite theory is driven by the theoretical implications - explanation may call for a new symmetry, a non-trivial dynamical mechanism, or a dual formulation
- The discovery of either would have a fundamental impact on our understanding of gravity
- Non-perturbative issues and viable models of Nature are *not* the goal for now
- **Here we only focus on order-by-order UV finiteness and to identify the mechanism behind**

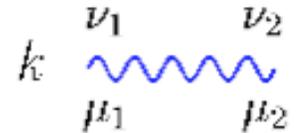
Perturbative Gravity Calculations

Pure Einstein Gravity

$$\mathcal{L} = \frac{2}{\kappa^2} \sqrt{g} R, \quad g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

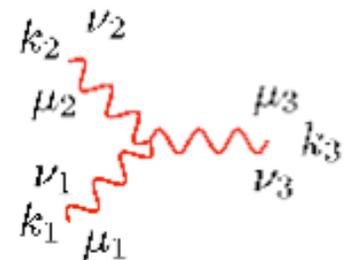
de Donder gauge propagator :

$$P_{\mu_1\nu_1\mu_2\nu_2}(k) = \frac{1}{2} \left[\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2} + \eta_{\mu_1\nu_2}\eta_{\mu_2\nu_1} - \frac{2}{D-2}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2} \right] \frac{i}{k^2 + i\epsilon}$$



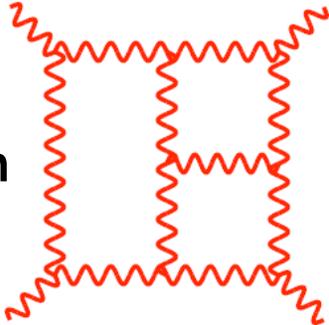
cubic vertex:

$$\begin{aligned} G_{3\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}(k_1, k_2, k_3) = & \\ & \text{sym} \left[-\frac{1}{2} P_3(k_1 \cdot k_2 \eta_{\mu_1\nu_1} \eta_{\mu_2\nu_2} \eta_{\mu_3\nu_3}) - \frac{1}{2} P_6(k_{1\mu_1} k_{1\nu_2} \eta_{\mu_1\nu_1} \eta_{\mu_3\nu_3}) + \frac{1}{2} P_3(k_1 \cdot k_2 \eta_{\mu_1\mu_2} \eta_{\nu_1\nu_2} \eta_{\mu_3\nu_3}) \right. \\ & + P_6(k_1 \cdot k_2 \eta_{\mu_1\nu_1} \eta_{\mu_2\mu_3} \eta_{\nu_2\nu_3}) + 2P_3(k_{1\mu_2} k_{1\nu_3} \eta_{\mu_1\nu_1} \eta_{\nu_2\mu_3}) - P_3(k_{1\nu_2} k_{2\mu_1} \eta_{\nu_1\mu_1} \eta_{\mu_3\nu_3}) \\ & + P_3(k_{1\mu_3} k_{2\nu_3} \eta_{\mu_1\mu_2} \eta_{\nu_1\nu_2}) + P_6(k_{1\mu_3} k_{1\nu_3} \eta_{\mu_1\mu_2} \eta_{\nu_1\nu_2}) + 2P_6(k_{1\mu_2} k_{2\nu_3} \eta_{\nu_2\mu_1} \eta_{\nu_1\mu_3}) \\ & \left. + 2P_3(k_{1\mu_2} k_{2\mu_1} \eta_{\nu_2\mu_3} \eta_{\nu_3\nu_1}) - 2P_3(k_1 \cdot k_2 \eta_{\nu_1\mu_2} \eta_{\nu_2\mu_3} \eta_{\nu_3\mu_1}) \right] \end{aligned}$$

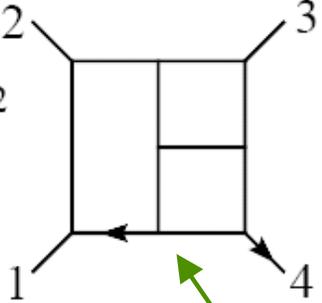


After symmetrization ~ 100 terms !

Feynman Diagrams not optimal

one Feynman diagram  = $100^8 \times 3^{10} \sim \mathbf{10^{21}}$ terms in pure gravity

$\mathcal{N} = 8$ supergravity – add contributions from 256 states in each loop...impossible to compute!

But answer is simple...**only one term** $\sim s^2 [(l + k_4)^2]^2$ 

Gravity contains **infinite number of vertices** beyond cubic level, however in new *on-shell* formalism **cubic vertex contains all information**

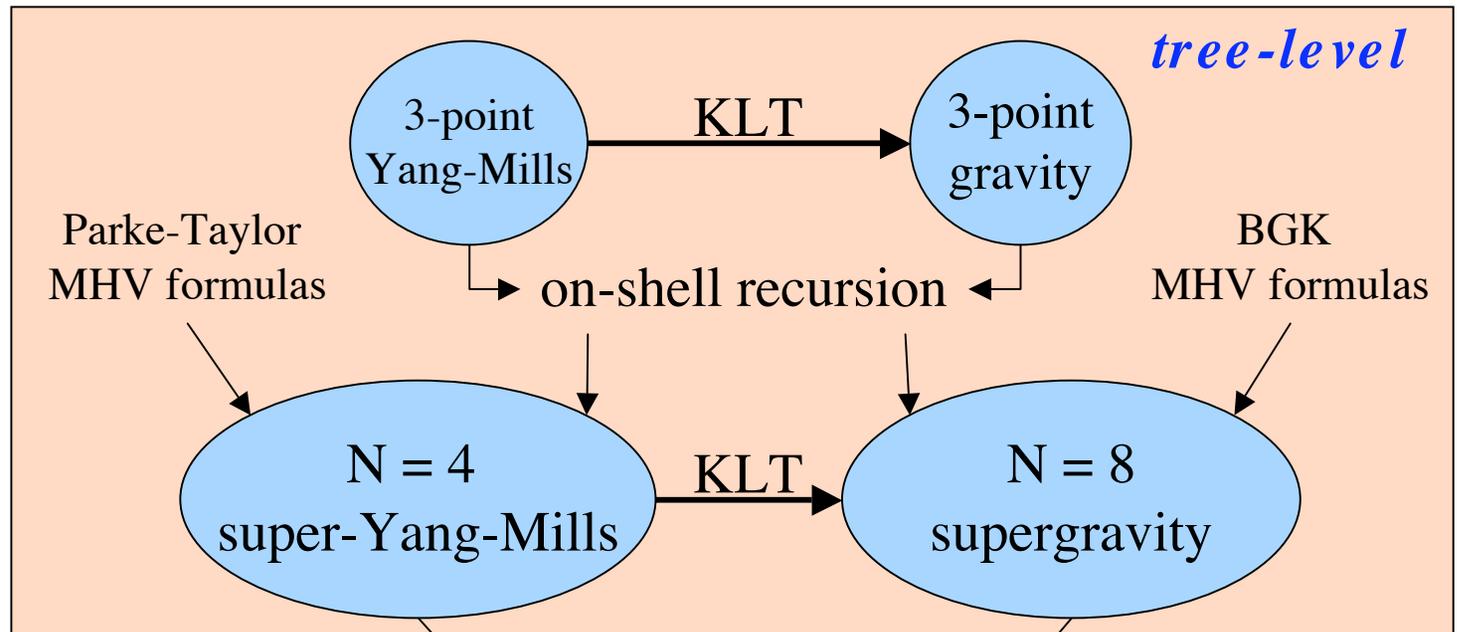
Britto, Cachazo, Feng and Witten; Bedford, Brandhuber, Spence and Travaglini
Cachazo and Svrcek; Benincasa, Boucher-Veronneau and Cachazo

Our Toolbox

On-shell

KLT

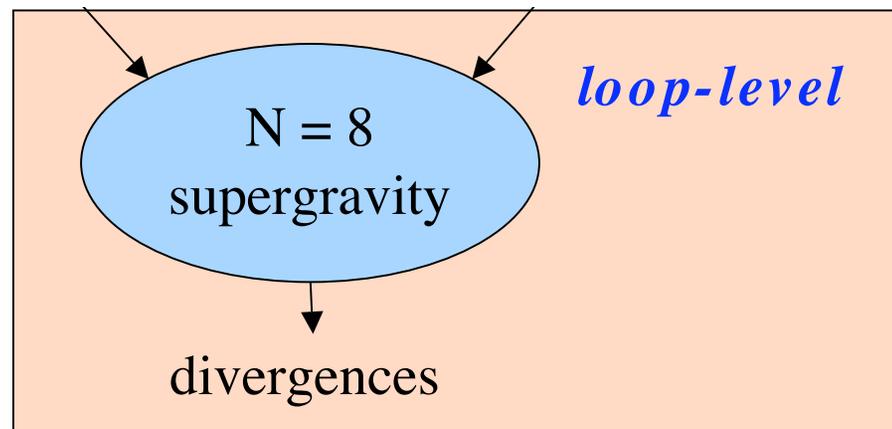
Unitarity



Unitarity + KLT

Unitarity

- Lagrangian not needed
- No Feynman diagrams
- No gauge fixing required
- No unphysical off-shell states
- Many cross-checks
- Results are very compact



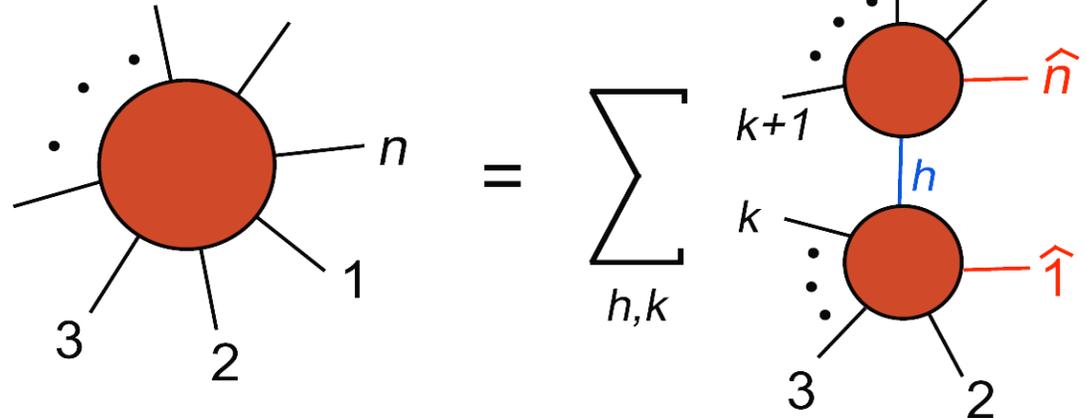
Tree-Level Amplitudes

On-shell BCFW recursion

Britto, Cachazo, Feng and Witten

$$p_1^\mu(z) = p_1^\mu - \frac{z}{2} \langle 1^- | \gamma^\mu | n^- \rangle$$

$$p_n^\mu(z) = p_n^\mu + \frac{z}{2} \langle 1^- | \gamma^\mu | n^- \rangle$$



Cauchy's theorem gives amplitude in terms of poles in z

MHV amplitudes + susy Ward identities \Rightarrow gravitino, graviphoton, fermion and scalar amplitudes

Also CSW rules from twistor string theory \Rightarrow $\mathcal{N}=4$ SYM tree amplitudes

Cachazo-Svrcek-Witten

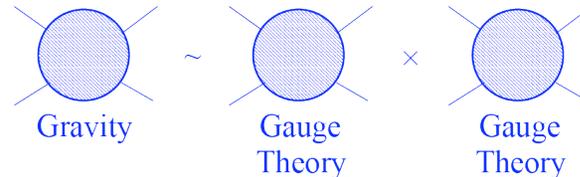
Recent proof by
Kiermaier, Elvang, Freedman

Kawai-Lewellen-Tye Relations

Originally string theory
tree level identity:

closed string \sim (left open string) \times (right open string)

Field theory limit \Rightarrow gravity theory \sim (gauge theory) \times (gauge theory)



$$M_4^{\text{tree}}(1, 2, 3, 4) = -is_{12}A_4^{\text{tree}}(1, 2, 3, 4)\tilde{A}_4^{\text{tree}}(1, 2, 4, 3)$$

color-stripped amplitudes

$$M_5^{\text{tree}}(1, 2, 3, 4, 5) = is_{12}s_{34}A_5^{\text{tree}}(1, 2, 3, 4, 5)\tilde{A}_5^{\text{tree}}(2, 1, 4, 3, 5) \\ + is_{13}s_{24}A_5^{\text{tree}}(1, 3, 2, 4, 5)\tilde{A}_5^{\text{tree}}(3, 1, 4, 2, 5)$$

Particularly useful in sugra cuts: $[\mathcal{N}=8] = [\mathcal{N}=4] \times [\mathcal{N}=4]$ $256 = 16 \times 16$

From Lagrangian point of view relations are very obscure

Unitarity

Optical theorem:

$$1 = S^\dagger S = (1 - iT^\dagger)(1 + iT)$$

$$2\text{Im } T = T^\dagger T$$

$$2\text{Im} \left[\text{Diagram: a square loop with four external lines and a vertical dashed green line through the center} \right] = \int_{d\text{LIPS}} \left[\text{Diagram: two tree-level diagrams with two external lines each, connected at a vertex} \right]$$

on-shell

Cutting rules by **Cutkosky**

$$\begin{array}{ccc} \text{---} & \xrightarrow{\text{blue arrow}} & \text{---} \\ \frac{i}{p^2} & \implies & 2\pi i \delta(p^2) \end{array}$$

Unitarity method reverses "cutting" avoiding dispersion relations

Bern, Dixon, Dunbar and Kosower (1994)

\Rightarrow efficient perturbative quantization of gauge and gravity theories

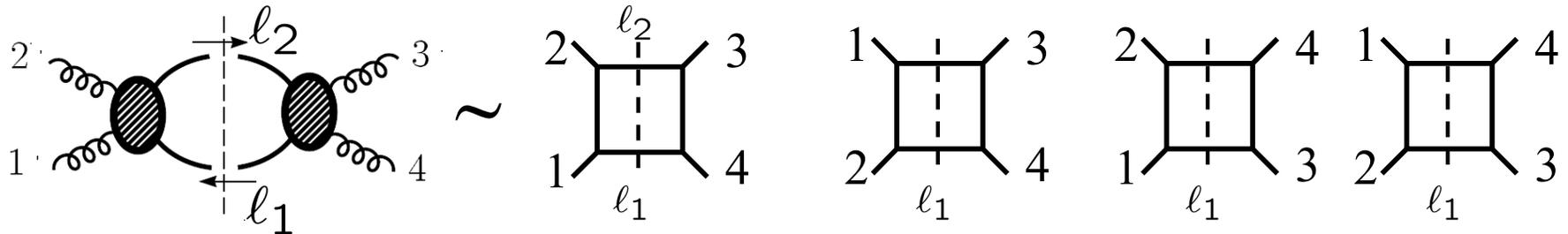
Unitarity Method

Bern, Dixon, Dunbar and Kosower (1994)

Sewing one-loop $N = 8$ sugra:

$$\sum_{N=8 \text{ states}} M_4^{\text{tree}}(-l_1, 1, 2, l_2) \times M_4^{\text{tree}}(-l_2, l_3, -l_4, l_1)$$

$$= \text{istu } M_4^{\text{tree}}(1, 2, 3, 4) \left[\frac{1}{(l_1 - k_1)^2} + \frac{1}{(l_1 - k_2)^2} \right] \left[\frac{1}{(l_2 - k_3)^2} + \frac{1}{(l_2 - k_4)^2} \right]$$



s-channel cut \Rightarrow one-loop box integrals

Any massless amplitude can be constructed from D -dimensional cuts

Unitarity Method

T
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M
E

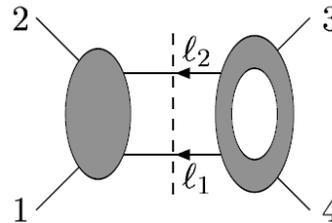
optical theorem

$$2 \operatorname{Im} \left[\text{Diagram: square loop with external lines} \right] = \int d\text{LIPS} \left[\text{Diagram: two tree-level diagrams} \right]$$

on-shell

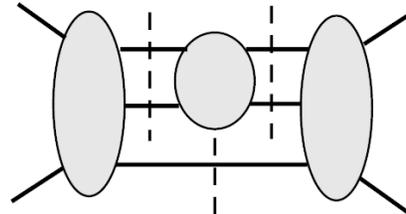
unitarity method

Bern, Dixon, Dunbar and Kosower (1994)



generalized unitarity

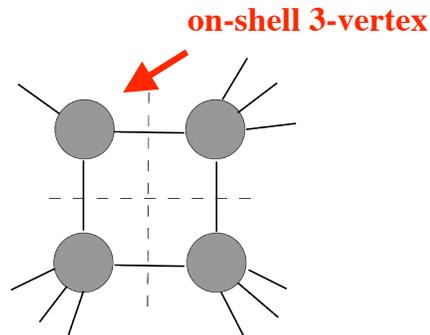
Bern, Dixon and Kosower



quadruple cut
(leading singularity)

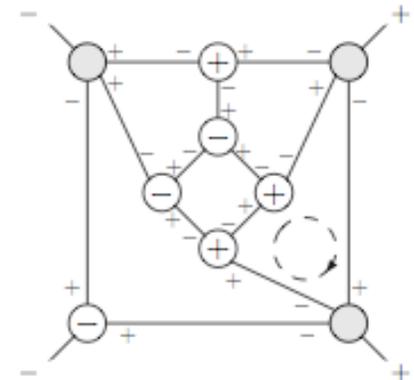
Britto, Cachazo, Feng;
Buchbinder, Cachazo (2004)

Cachazo and Skinner
Cachazo, Spradlin, Volovich
(2008)



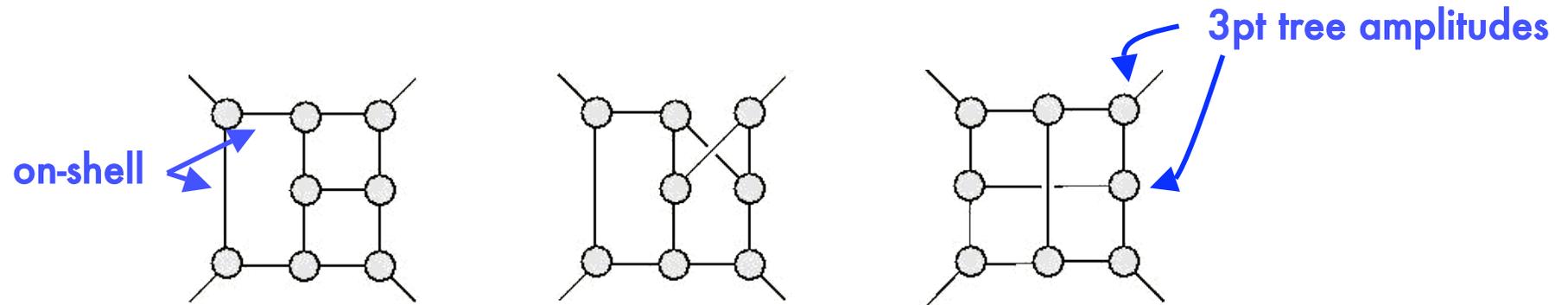
maximal cut

Bern, Carrasco, HJ
and Kosower (2007)

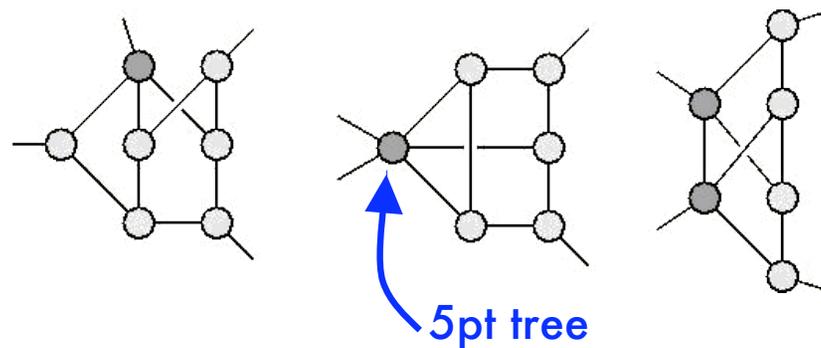


Maximal cuts - a systematic approach for any theory

- put maximum number of propagator on-shell \rightarrow simplifies calculation



- systematically release cut conditions \rightarrow great control of missing terms



Preferred technique for 4-loop calculation

Further Motivation

Reasons to Reexamine $\mathcal{N} = 8$

- Number of established counterterms in any supergravity is **zero**
- Every explicit calculation to date finds $\mathcal{N} = 8$ supergravity has identical power counting as $\mathcal{N} = 4$ super-Yang-Mills theory, which is UV finite
- Hints from string dualities (non-renormalization) **Green, Russo, Vanhove**
- Gravity amplitudes have amazingly simple structures in **twistor space**
⇒ restrictions on the theory **Witten**
- Discovery of remarkable cancellations at 1 loop – the “no-triangle hypothesis” – unitarity implies higher loop cancellations
Bern, Bjerrum-Bohr, Dixon, Dunbar, Ita, Perkins, Risager, Roiban
- **We now have the technical tools to perform the needed computations**

Where is the First $\mathcal{N} = 8$ Divergence?

3 loops	Conventional superspace power counting	Green, Schwarz, Brink (1982) Howe and Stelle (1989) Marcus and Sagnotti (1985)
5 loops	Partial analysis of unitarity cuts <i>If</i> harmonic superspace with $\mathcal{N} = 6$ susy manifest exists	Bern, Dixon, Dunbar, Perelstein, Rozowsky (1998) Howe and Stelle (2003)
6 loops	<i>If</i> harmonic superspace with $\mathcal{N} = 7$ susy manifest exists	Howe and Stelle (2003)
7 loops	<i>If</i> harmonic superspace with $\mathcal{N} = 8$ susy manifest were to exist	Grisaru and Siegel (1982)
8 loops	Explicit identification of potential susy invariant counterterm with full non-linear susy	Kallos; Howe and Lindström (1981)
9 loops	Assume Berkovits' superstring non-renormalization theorems can be carried over to $\mathcal{N} = 8$ supergravity	Green, Russo, Vanhove (2006)

Note: no divergence demonstrated above. Arguments based on lack of susy protection!

Opinions from the 80's

“If certain patterns that emerge should persist in the higher orders of perturbation theory, then ... $N = 8$ supergravity in four dimensions would have ultraviolet divergences starting at three loops.”

Green, Schwarz, Brink, (1982)

“There are no miracles... It is therefore very likely that all supergravity theories will diverge at three loops in four dimensions. ... The final word on these issues may have to await further explicit calculations.”

Marcus, Sagnotti (1985)

The idea that all supergravity theories diverge at 3 loops has been the accepted status for over 20 years

Results

Known 4-pt Amplitudes

1-loop: $K^2 \left(\begin{array}{c} 2 \quad 3 \\ \diagdown \quad / \\ \square \\ / \quad \diagdown \\ 1 \quad 4 \end{array} + \begin{array}{c} 3 \quad 4 \\ \diagdown \quad / \\ \square \\ / \quad \diagdown \\ 1 \quad 2 \end{array} + \begin{array}{c} 4 \quad 2 \\ \diagdown \quad / \\ \square \\ / \quad \diagdown \\ 1 \quad 3 \end{array} \right)$

Green, Schwarz,
Brink (1982)

2-loop: $K^2 \left(s^2 \begin{array}{c} 2 \quad 3 \\ \diagdown \quad / \\ \square \quad \square \\ / \quad \diagdown \\ 1 \quad 4 \end{array} + s^2 \begin{array}{c} 3 \\ \diagdown \quad / \\ \square \quad \square \\ / \quad \diagdown \\ 1 \quad 2 \quad 4 \end{array} + \text{perms} \right)$

Bern, Dixon,
Dunbar, Perelstein
and Rozowsky
(1998)

prefactor contains
helicity structure:

$$K^2 = stuM_4^{tree}$$

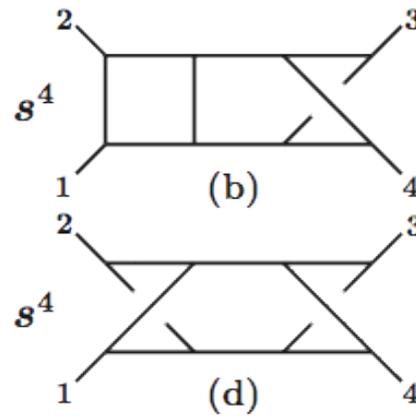
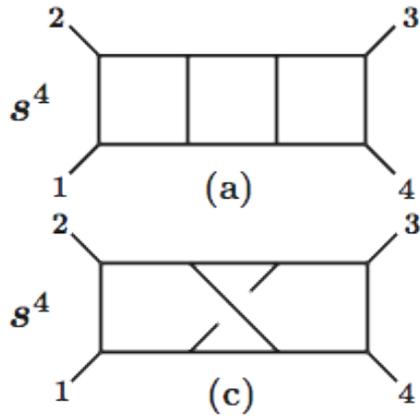
$$K = stA_4^{tree}$$

$\mathcal{N}=4$ SYM

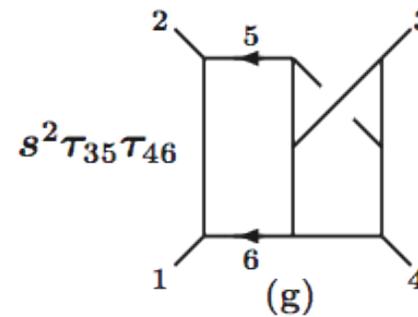
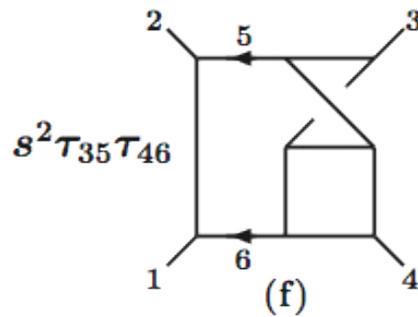
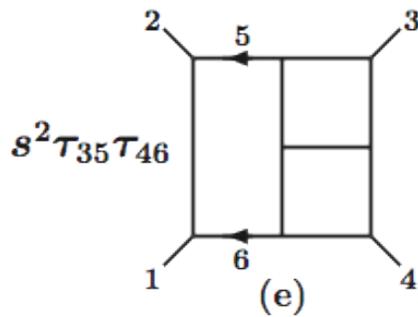
$\mathcal{N}=4$ super-Yang-Mills is obtained if $2 \rightarrow 1$ and vertices are dressed with f^{abc}

Same integrals appear in $\mathcal{N}=8$ sugra and $\mathcal{N}=4$ SYM
 \Rightarrow UV behavior same through 2 loops

Three-Loop Integrals

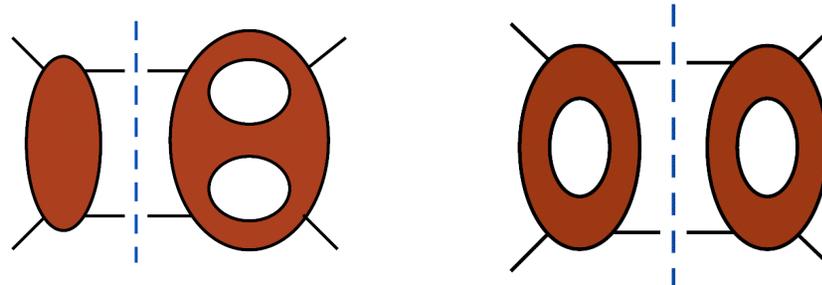


Bern, Carrasco, Dixon,
HJ, Kosower, Roiban,
Bern, Carrasco, Dixon,
HJ, Roiban



$$\tau_{ij} = 2k_i \cdot l_j$$

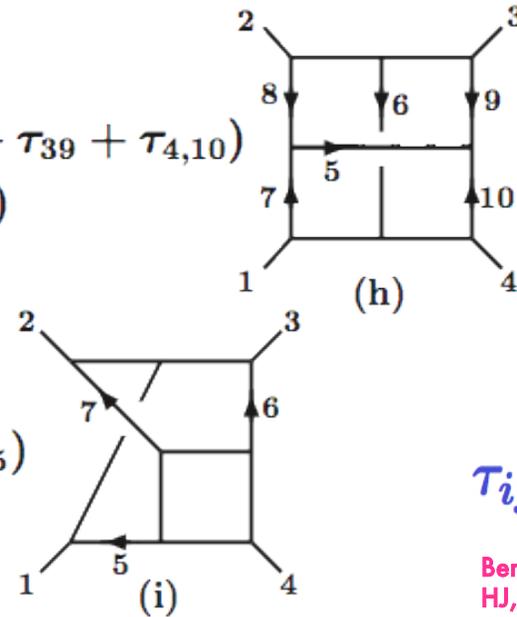
These integrals have
correct 2-particle cuts



Three-Loop Integrals

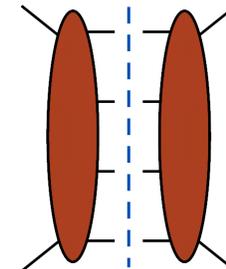
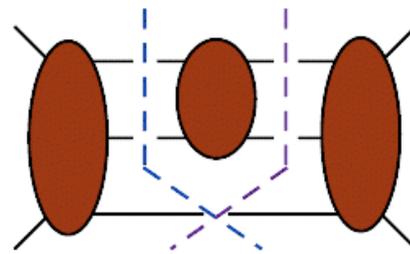
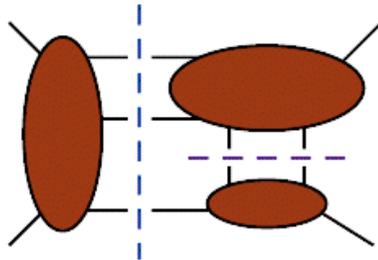
$$\begin{aligned}
 & (s(\tau_{26} + \tau_{36}) + t(\tau_{15} + \tau_{25}) + st)^2 \\
 & + (s^2(\tau_{26} + \tau_{36}) - t^2(\tau_{15} + \tau_{25}))(\tau_{17} + \tau_{28} + \tau_{39} + \tau_{4,10}) \\
 & + s^2(\tau_{17}\tau_{28} + \tau_{39}\tau_{4,10}) + t^2(\tau_{28}\tau_{39} + \tau_{17}\tau_{4,10}) \\
 & + u^2(\tau_{17}\tau_{39} + \tau_{28}\tau_{4,10})
 \end{aligned}$$

$$\begin{aligned}
 & (s\tau_{45} - t\tau_{46})^2 - \tau_{27}(s^2\tau_{45} + t^2\tau_{46}) \\
 & - \tau_{15}(s^2\tau_{47} + u^2\tau_{46}) - \tau_{36}(t^2\tau_{47} + u^2\tau_{45}) \\
 & + l_5^2 s^2 t + l_6^2 st^2 - \frac{1}{3} l_7^2 stu
 \end{aligned}$$



$$\tau_{ij} = 2k_i \cdot l_j$$

Bern, Carrasco, Dixon,
HJ, Kosower, Roiban,
Bern, Carrasco, Dixon,
HJ, Roiban



Complete Three-Loop amplitude

$$M_4^{(3)} = \left(\frac{\kappa}{2}\right)^8 stu M_4^{\text{tree}} \sum_{S_3} \left[I^{(a)} + I^{(b)} + \frac{1}{2}I^{(c)} + \frac{1}{4}I^{(d)} + 2I^{(e)} + 2I^{(f)} + 4I^{(g)} + \frac{1}{2}I^{(h)} + 2I^{(i)} \right]$$

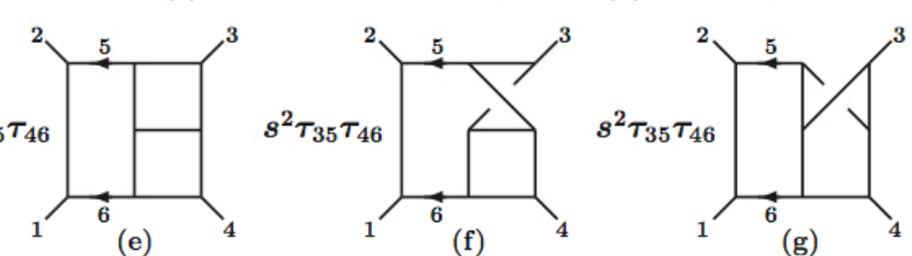
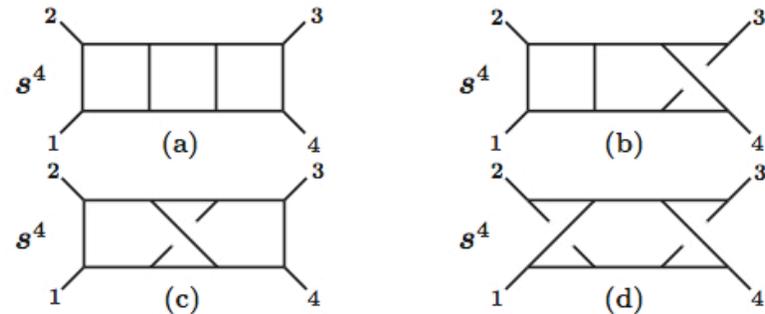
\uparrow K^2 \uparrow leg perms \uparrow symmetry factors

UV "superfinite" for $D = 4$

After integration

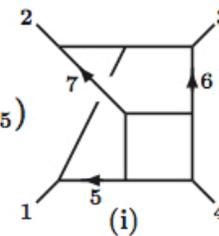
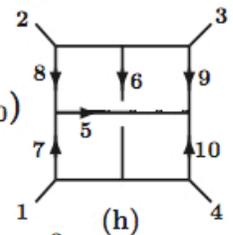
$$\text{UV pole}_{6-2\epsilon} = \frac{1}{\epsilon} \frac{5\zeta_3}{(4\pi)^9} \left(\frac{\kappa}{2}\right)^8 (stu)^2 M_4^{\text{tree}}$$

$D = 6$ div. demonstrated



$$\begin{aligned} & (s(\tau_{26} + \tau_{36}) + t(\tau_{15} + \tau_{25}) + st)^2 \\ & + (s^2(\tau_{26} + \tau_{36}) - t^2(\tau_{15} + \tau_{25}))(\tau_{17} + \tau_{28} + \tau_{39} + \tau_{4,10}) \\ & + s^2(\tau_{17}\tau_{28} + \tau_{39}\tau_{4,10}) + t^2(\tau_{28}\tau_{39} + \tau_{17}\tau_{4,10}) \\ & + u^2(\tau_{17}\tau_{39} + \tau_{28}\tau_{4,10}) \end{aligned}$$

$$\begin{aligned} & (s\tau_{45} - t\tau_{46})^2 - \tau_{27}(s^2\tau_{45} + t^2\tau_{46}) \\ & - \tau_{15}(s^2\tau_{47} + u^2\tau_{46}) - \tau_{36}(t^2\tau_{47} + u^2\tau_{45}) \\ & + l_5^2 s^2 t + l_6^2 st^2 - \frac{1}{3} l_7^2 stu \end{aligned}$$



Bern, Carrasco, Dixon,
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UV behavior

In [hep-th/0702112](https://arxiv.org/abs/hep-th/0702112) individual diagrams had worse power counting than 3-loop $\mathcal{N} = 4$ super-Yang-Mills - **now they have same manifest power count**

Integral $I^{(x)}$	$\mathcal{N} = 4$ Super-Yang-Mills	$\mathcal{N} = 8$ Supergravity	$\tau_{ij} = 2k_i \cdot l_j$ $l_{i,j}^2 = (k_i + l_j)^2$
(a)–(d)	s^2	$[s^2]^2$	
(e)–(g)	$s l_{4,6}^2$	$s^2 \tau_{35} \tau_{46}$	
(h)	$s(\tau_{26} + \tau_{36}) + t(\tau_{15} + \tau_{25}) + st$	$(s(\tau_{26} + \tau_{36}) + t(\tau_{15} + \tau_{25}) + st)^2$ $+ (s^2(\tau_{26} + \tau_{36}) - t^2(\tau_{15} + \tau_{25}))(\tau_{17} + \tau_{28} + \tau_{39} + \tau_{4,10})$ $+ s^2(\tau_{17} \tau_{28} + \tau_{39} \tau_{4,10}) + t^2(\tau_{28} \tau_{39} + \tau_{17} \tau_{4,10}) + u^2(\tau_{17} \tau_{39} + \tau_{28} \tau_{4,10})$	
(i)	$s l_{4,5}^2 - t l_{4,6}^2 - \frac{1}{3}(s - t)l_7^2$	$(s\tau_{45} - t\tau_{46})^2 - \tau_{27}(s^2\tau_{45} + t^2\tau_{46}) - \tau_{15}(s^2\tau_{47} + u^2\tau_{46})$ $- \tau_{36}(t^2\tau_{47} + u^2\tau_{45}) + l_5^2 s^2 t + l_6^2 st^2 - \frac{1}{3}l_7^2 stu$	

Critical dimension:

$$L > 1$$

$$D_c = 4 + \frac{6}{L} \quad \mathcal{N}=4 \text{ SYM \& } \mathcal{N}=8 \text{ Sugra} \quad \text{new trend}$$

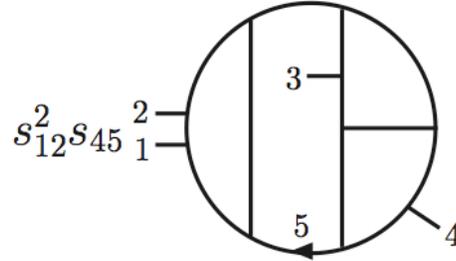
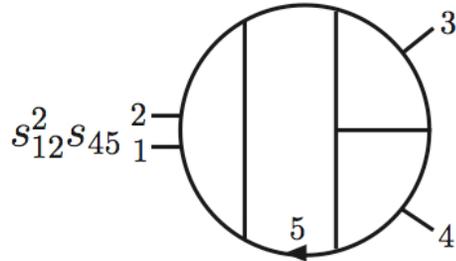
$$D_c = 2 + \frac{10}{L} \quad \text{old trend } \mathcal{N}=8 \quad \text{Bern, Dixon, Dunbar, Perelstein and Rozowsky (1998)}$$

Four-loop calculation in progress

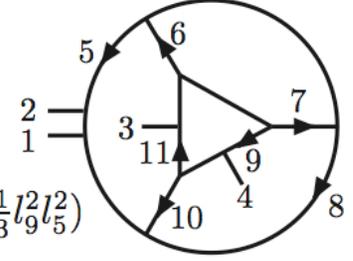
- $\mathcal{N} = 4$ super-Yang-Mills case is complete
- $\mathcal{N} = 8$ supergravity still in progress

Bern, Carrasco,
Dixon, HJ, Roiban
(2009)

Some $\mathcal{N} = 4$ SYM contributions...



$$s_{12}^2 s_{98} - s_{12} s_{35} s_{67} \\ + \frac{1}{3} l_9^2 s_{12} (s_{35} - s_{12}) \\ + s_{12} (l_5^2 s_{4,10} - l_5^2 l_{11}^2 - \frac{1}{3} l_9^2 l_5^2)$$



...50 distinct diagrammatic topologies

Motivation:

1. Direct challenge to a potential $\mathcal{N} = 6$ superspace explanation suggested by Stelle.
2. Study of non-trivial cancellations gives better understanding.
3. Need 16 not 14 powers of loop momenta to come out of integrals to get power counting of $\mathcal{N} = 4$ SYM.

Mechanism

Origin of UV Cancellations

- Observed 1, 2 and 3-loop UV cancellations do not appear to fit smoothly into known susy schemes.
- A finite theory requires ∞ number of cancellations - supersymmetry only seem to be responsible for a finite number of these.
- If not supersymmetry, what then ?
- **UV cancellations may be generic to gravity theories**
- Hints:
 - Gravity tree-amplitudes well behaved under large momenta deformations $A(z) \sim 1/z^2$
Benincasa, Boucher-Veronneau and Cachazo; Arkani-Hamed, Kaplan
 - Surprising one-loop cancellations present in pure gravity
Bern, Carrasco, Forde, Ita, HJ
 - No triangle property $\mathcal{N} = 8$ sugra
Bern, Bjerrum-Bohr and Dunbar; Bjerrum-Bohr, Dunbar, Ita, Perkins and Risager

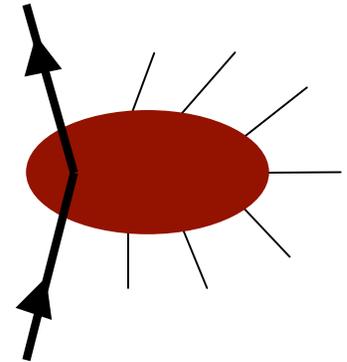
Generic Gravity UV Cancellations

tree-level
large complex momenta
 $z \rightarrow \infty$

BCFW shift:

$$p_1^\mu(z) = p_1^\mu - \frac{z}{2} \langle 1^- | \gamma^\mu | n^- \rangle$$

$$p_n^\mu(z) = p_n^\mu + \frac{z}{2} \langle 1^- | \gamma^\mu | n^- \rangle$$



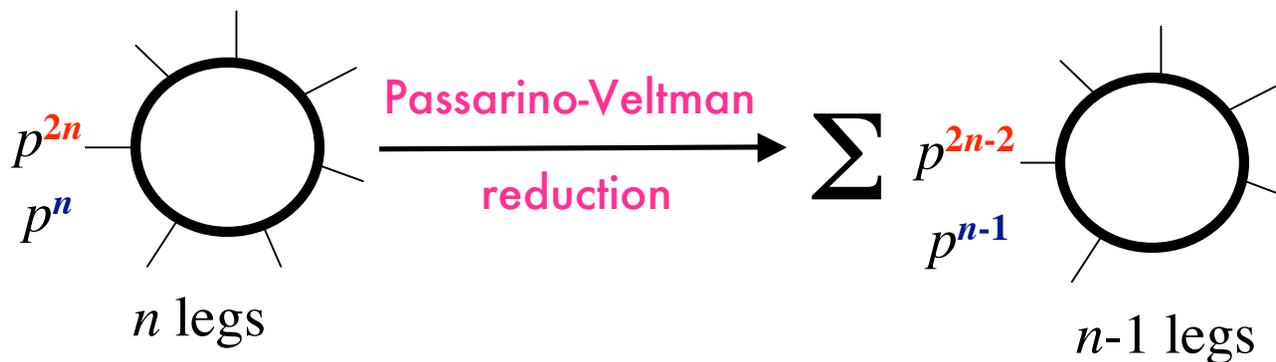
Gravity: $A(z) \sim 1/z^2$ (naive p. c. $A(z) \sim z^n$)

Gauge theory: $A(z) \sim 1/z$

Benincasa, Boucher-Veronneau
and Cachazo; Arkani-Hamed, Kaplan

one-loop cancellations in pure gravity

Bern, Carrasco, Forde, Ita, HJ



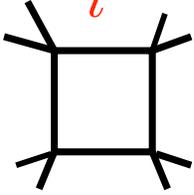
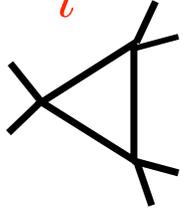
Gauge theory:
Individual pieces
worse UV behavior
in P-V reduction

Gravity unchanged
→ better than naive
power counting

No-Triangle Property

4-d theorem: massless one loop amplitudes are linear combinations of scalar box, triangle and bubble integrals plus a rational term

$$A_n^{1\text{-loop}} = \sum_i d_i I_4^{(i)} + \sum_i c_i I_3^{(i)} + \sum_i b_i I_2^{(i)} + \text{Rational}$$

			
$\int \frac{d^4 p}{(p^2)^4}$	$\int \frac{d^4 p}{(p^2)^3}$	$\int \frac{d^4 p}{(p^2)^2}$	$\frac{N}{D}$

In $\mathcal{N} = 4$ SYM only boxes contribute. No triangle, bubbles or rational

the same holds true in $\mathcal{N} = 8$ sugra - recent proof

Bjerrum-Bohr and Vanhove;
Arkani-Hamed, Cachazo,
Kaplan

Outlook

- Soon results from 4-loop supergravity calculation
 - is power counting same as for $\mathcal{N}=4$ SYM?
- 5-loop calculation possible - but difficult
- L -loop evidence of finiteness would be ideal
 - perhaps by exploiting the recursive nature of unitarity
- Link observed cancellations to hidden symmetries or other structures – $E_{7(7)}$, Witten's twistor strings, etc. ?

Summary

- Modern computational **on-shell methods**, **unitarity** and **KLT** provide powerful new ways to study UV divergences of gravity theories.
- While naive power counting arguments disfavor UV-finite pointlike gravity theories, no explicit calculation have yet identified a divergence in any 4-d supergravity.
- All calculations to date show that maximal $\mathcal{N} = 8$ sugra have identical power counting to that of $\mathcal{N} = 4$ SYM, a finite theory.
- **Proving finiteness of $\mathcal{N} = 8$ sugra remains an open challenge !**