Proton and dark matter without $R$-parity

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Seminar at Fermilab (May 29, 2008)
Proton and dark matter without $R$-parity

: $U(1)'$ as an alternative to $R$-parity

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- Paul Langacker (UPenn, IAS)
- Christoph Luhn (Florida)
- Konstantin Matchev (Florida)
- Salah Nasri (UAE)
- . . .
Outline

- Companion symmetry of SUSY
  - $R$-parity
  - TeV scale $U(1)'$ gauge symmetry

- $R$-parity violating, $U(1)'$-extended SUSY model
  - Proton stability
  - Dark matter candidate
Supersymmetry
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General SUSY

\[
W = \mu H_u H_d \\
+ y_E H_d L E^c + y_D H_d Q D^c + y_U H_u Q U^c \\
+ \lambda L L E^c + \lambda' L Q D^c + \mu' L H_u + \lambda'' U^c D^c D^c \\
+ \frac{\eta_1}{M} Q Q Q L + \frac{\eta_2}{M} U^c U^c D^c E^c + \cdots
\]

1. $\mu$-problem: $\mu \sim \mathcal{O}$(EW) to avoid fine-tuning in the EWSB.
   
   (Kim, Nilles [1984])

2. lepton number ($\mathcal{L}$) and/or baryon number ($\mathcal{B}$) violating terms at renormalizable and non-renormalizable levels: one of the most general predictions of SUSY.
Proton decay

[Dim 4 $\mathcal{L}$ violation & Dim 4 $\mathcal{B}$ violation]

$$\lambda_{LLE}^c + \lambda'_{LQD}^c \, & \, \lambda''_{U^c D^c}$$

[Dim 5 $\mathcal{B}$&$\mathcal{L}$ violation]

$$\frac{n_1}{M} QQQ L + \frac{n_2}{M} U^c U^c D^c E^c$$

To satisfy $\tau_p \gtrsim 10^{29}$ years,

- **Dim 4:** $|\lambda_{LV} \cdot \lambda_{BV}| \lesssim 10^{-27} \quad$ (if one is 0, the other can be sizable)

- **Dim 5:** $|\eta| \lesssim 10^{-7} \quad$ (for $M = M_{Pl}$)
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**Lightest superparticle (LSP) decay**

\[ \Gamma = \frac{\lambda_{ijk}^2 \alpha}{128\pi^2} \frac{m_{\tilde{\chi}_1^0}^5}{m_{\tilde{\chi}_j^0}^4} \]  
(for $\chi_1^0 \sim$ photino)

To be a viable dark matter, $\tau_{LSP} \gtrsim 14 \times 10^9$ years (Universe age).

\[ |\lambda|, |\lambda'|, |\lambda''| \approx 10^{-20} \]
SUSY needs a companion mechanism or symmetry.
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Supersymmetry + $R$-parity
**Proton and dark matter without \( R \)-parity**

\( R \)-parity (or matter parity)

\[ R_p[SM] = \text{even}, \quad R_p[\text{superpartner}] = \text{odd} \]

\( R \)-parity is defined on component fields, and matter parity is defined on superfields. They are equivalent.

\[ R \text{-parity} : \quad R_p = (-1)^{3(B-L)+2s} \]

\[ \text{Matter parity} : \quad M_p = (-1)^{3(B-L)} \]

<table>
<thead>
<tr>
<th></th>
<th>( Q )</th>
<th>( U^c )</th>
<th>( D^c )</th>
<th>( L )</th>
<th>( E^c )</th>
<th>( H_u )</th>
<th>( H_d )</th>
</tr>
</thead>
<tbody>
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<td>Matter parity</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

- LSP is absolutely stable (dark matter candidate).
SUSY with $R$-parity

$$W_{R_p} = \mu H_u H_d$$

$$+ y_E H_d L E^c + y_D H_d Q D^c + y_U H_u Q U^c$$

$$+ \cdots$$

$$+ \frac{\eta_1}{M} QQQQL + \frac{\eta_2}{M} U^c U^c D^c E^c + \cdots$$

1. $\mu$-problem: Not addressed.

2. over-constraining of the $R$-parity: All renormalizable $L$ violating and $B$ violating terms are (unnecessarily) forbidden.

3. under-constraining of the $R$-parity: Dimension 5 $L&B$ violating terms still mediate too fast proton decay.

→ Look for an additional or alternative explanation (symmetry).
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Supersymmetry + $R$-parity + $U(1)'$ gauge symmetry
TeV scale $U(1)'$ gauge symmetry

Natural scale of $U(1)'$ in SUSY models is TeV (linked to sfermions scale).

→ provides a natural solution to the $\mu$-problem.

Two conditions to “solve the $\mu$-problem”. ($z[F]: U(1)'$ charge of $F'$)

- $\mu H_u H_d :$ forbidden \hspace{1cm} \( z[H_u] + z[H_d] \neq 0 \)
- $h S H_u H_d :$ allowed \hspace{1cm} \( z[S] + z[H_u] + z[H_d] = 0 \)

$S$ is a Higgs singlet that breaks the $U(1)'$ spontaneously.

$$\mu_{\text{eff}} = h \langle S \rangle \sim \mathcal{O}(\text{EW}/\text{TeV})$$
SUSY with $R$-parity and $U(1)'$

\[ W_{R_p+U(1)'} = h SH_u H_d \]
\[ + y_E H_d L E^c + y_D H_d Q D^c + y_U H_u Q U^c \]
\[ + \ldots \]
\[ + \left( \frac{\eta_1}{M} Q Q Q L + \frac{\eta_2}{M} U^c U^c D^c E^c + \ldots \right) \]

1. $\mu$-problem: Resolved by replacing $\mu$ with $\mu_{\text{eff}}$.
2. over-constraining of the $R$-parity: It forbids all renormalizable terms.
3. non-renormalizable terms: Maybe forbidden depending on charges.

$\rightarrow$ Usual set up of the $U(1)'$-extended MSSM (UMSSM).

In principle, the $U(1)'$ can embed the $R$-parity (matter parity), which is more economic than having 2 companion symmetries.
LSP dark matter candidates in the UMSSM (brief review)

A viable dark matter candidate should

1. be neutral, stable, cold

2. give right relic density
   \[ \Omega_{\text{DM}} h^2 = 0.1099 \pm 0.0124 \text{ from } 2\sigma \text{ WMAP} \]

3. avoid direct detection constraint
   \[ \sigma_n^{\text{SI}} \lesssim 10^{-7} \text{ pb from CDMS/XENON} \]

Cold dark matter candidates stable under $R$-parity:

- neutralino ($\chi^0$) LSP
- sneutrino ($\tilde{\nu}$) LSP
Neutralino LSP dark matter candidate

- **UMSSM**: $6 \times 6$ matrix, in the basis of $\{\tilde{B}, \tilde{W}_3, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{S}, \tilde{Z}'\}$

$$
\begin{pmatrix}
M_1 & 0 & -g_1 v_d/2 & g_1 v_u/2 & 0 & 0 \\
0 & M_2 & g_2 v_d/2 & -g_2 v_u/2 & 0 & 0 \\
-g_1 v_d/2 & g_2 v_d/2 & 0 & -\mu_{\text{eff}} & -\mu_{\text{eff}} v_u/s & g_{Z'} z [H_d] v_d \\
g_1 v_u/2 & -g_2 v_u/2 & -\mu_{\text{eff}} & 0 & -\mu_{\text{eff}} v_d/s & g_{Z'} z [H_u] v_u \\
0 & 0 & -\mu_{\text{eff}} v_u/s & -\mu_{\text{eff}} v_d/s & 0 & g_{Z'} z [S] s \\
0 & 0 & g_{Z'} z [H_d] v_d & g_{Z'} z [H_u] n_u & g_{Z'} z [S] s & M_1 \\
\end{pmatrix}
$$

- **MSSM**: First $4 \times 4$ submatrix

$\rightarrow$ Easy to satisfy the relic density and direct detection constraints, since it has MSSM components which already do.

(Barger, Kao, Langacker, HL [hep-ph/0408120]) (Barger et al. [2007])
Sneutrino LSP dark matter candidate

- Pure left-handed sneutrino ($\tilde{\nu}_L$):

\[ \sigma_{SI}^{n} \sim G_F^2 \mu_{n-DM}^2 \sim 0.1 \text{pb} \gg 10^{-7} \text{pb} \quad (\text{CDMS/XENON}) \]

$Z$ mediated channels for sneutrino LSP has too large direct detection cross-section when it makes the right relic density.

(Falk, Olive, Srednicki [1994])
• Predominantly right-handed sneutrino ($\tilde{\nu}_R$):
  $N^c$: necessary for the neutrino mass ($LH_u N^c$).

$Z'$ mediated interaction can be suppressed by its mass and coupling.

(HL, Matchev, Nasri [hep-ph/0702223])
Predictions of relic density and direct detection cross-section

Yellow bands: right relic density \( (\Omega_{\tilde{\nu}_R} h^2 \sim 0.1) \) in the \( \tilde{Z}' \) mediation region \( (M_{\tilde{\nu}_R} \sim 45 \text{ GeV}) \) and \( Z' \) mediation region \( (M_{\tilde{\nu}_R} \sim M_{Z'}/2) \).

→ Sneutrino LSP is a viable thermal dark matter candidate in the \( U(1)' \)-extended MSSM.
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Supersymmetry + $U(1)'$ gauge symmetry without $R$-parity

Now, we consider the $R$-parity violating scenario.
Goal

Construct a stand-alone $R_p$ violating TeV scale SUSY model without

1. $\mu$-problem: $U(1)'$

2. proton decay problem

3. dark matter problem (non-LSP dark matter)

"$R$-parity violating $U(1)'$ model" as an alternative to the usual "$R$-parity conserving model".
Proton stability among the MSSM fields

Free parameters of the MSSM fields charges

Consider the MSSM Yukawa, effective $\mu$-term, $[SU(2)_L]^2 - U(1)'$ anomaly condition.

\[
\begin{align*}
H_u QU^c : & \quad z[H_u] + z[Q] + z[U^c] = 0 \\
H_d Q D^c : & \quad z[H_d] + z[Q] + z[D^c] = 0 \\
H_d L E^c : & \quad z[H_d] + z[L] + z[E^c] = 0 \\
S H_u H_d : & \quad z[S] + z[H_u] + z[H_d] = 0 \\
A_{221}' : & \quad 3(3z[Q] + z[L]) + (z[H_u] + z[H_d]) + \delta = 0 \\
\end{align*}
\]

with $\delta \equiv A_{221}'[SU(2)_L\text{ exotics}] = 0$ (assume no $SU(2)_L\text{ exotics}$).

8 unknown $U(1)'$ charges ($Q, U^c, D^c, L, E^c, H_u, H_d, S$) - 5 conditions

= 3 free parameters.
Proton and dark matter without $R$-parity

General solution of the MSSM fields

\[
\begin{pmatrix}
    z[Q] \\
    z[U^c] \\
    z[D^c] \\
    z[L] \\
    z[E^c] \\
    z[H_d] \\
    z[H_u] \\
    z[S]
\end{pmatrix} = \alpha \begin{pmatrix}
    1 \\
    -4 \\
    2 \\
    -3 \\
    6 \\
    -3 \\
    3 \\
    0
\end{pmatrix} + \beta \begin{pmatrix}
    1 \\
    -1 \\
    -1 \\
    -3 \\
    3 \\
    0 \\
    0 \\
    0
\end{pmatrix} + \gamma \begin{pmatrix}
    1 \\
    8 \\
    -1 \\
    0 \\
    0 \\
    0 \\
    -9 \\
    9
\end{pmatrix}
\]

1st vector $\propto$ hypercharge ($y$), 2nd vector $\propto B - L$.

\[
\alpha = -\frac{z[H_d]}{3} \quad \beta = \frac{z[H_d] - z[L]}{3} \quad \gamma = \frac{z[S]}{9}
\]
**Lepton number violating terms**

Since we already have

\[ y_E H_d L E^c, \ y_D H_d Q D^c, \ h S H_u H_d \]

allowing the \( \mathcal{L} \) violating terms means

\[ \lambda L L E^c, \ \lambda' L Q D^c, \ h' S H_u L \ \longleftrightarrow \ z[H_d] = z[L]. \]

Renormalizable \( \mathcal{L} \) violating couplings \((\lambda, \lambda', \mu')\) are either all allowed or all forbidden by the \( U(1)' \).
Proton and dark matter without $R$-parity

**LV-BV separation**

From MSSM Yukawa and $[SU(2)_L]^2 - U(1)'$ anomaly,

$$z[U^c D^c D^c] - z[LLE^c] + \frac{2}{3}(z[H_u H_d]) = 0$$

- BV term  
- LV term  
- original $\mu$-term

- $z[H_u H_d] \neq 0$ (\(\mu\)-problem solution).

- Either $z[U^c D^c D^c]$ or $z[LLE^c]$ should be non-zero (forbidden).

**LV-BV separation:** The LV terms ($\lambda LLE^c, \lambda' LQ D^c$) and the BV term ($\lambda'' U^c D^c D^c$) cannot coexist.

$$\lambda_{LV} \cdot \lambda_{BV} = 0$$

→ Proton does not decay through the MSSM dimension 4 operators.
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Also the dimension 5 LV and BV operators ($QQQL$, $U^cU^cD^cE^c$) are automatically forbidden.

$$z[QQQL] = -\frac{1}{3}z[H_uH_d] \neq 0$$
$$z[U^cU^cD^cE^c] = -\frac{5}{3}z[H_uH_d] \neq 0$$

Proton is sufficiently (up to dimension 5 level) stable among the MSSM fields in the $R$-parity violating $U(1)'$-extended MSSM.
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**Exotic colors**

$[SU(3)_C]^2 - U(1)'$ anomaly free condition:

$$3(2z[Q] + z[U^c] + z[D^c]) + A_{331'}[\text{exotic colors}] = 0$$

$$= -3(z[H_u] + z[H_d]) \neq 0 \quad \text{(µ-problem solution)}$$

due to the MSSM Yukawas.

$$\rightarrow A_{331'}[\text{exotic colors}] \neq 0$$

**Solving the µ-problem requires colored exotics.** (Well-known)

For definiteness, we assume three $SU(3)_C$ triplet ($K_i$) and antitriplet ($K_i^c$), which are $SU(2)_L$ singlets.

$$W_{\text{exotic colors}} = \eta_{ij}S K_i K_j^c$$
Right-handed neutrinos ($N^c$)

Observed neutrino mass ($m_\nu \lesssim 0.1$ eV) needs an explanation.

1. Majorana neutrino: with see-saw mechanism
   (Minkowski [1977]) (Yanagida [1979]) (Mohapatra, Senjanovic [1980])
   (Gell-Mann, Ramond, Slansky [1980])

   $$ W = y_N H_u L N^c + m N^c N^c $$

2. Dirac neutrino: natural suppression possible in $U(1)'$ model
   (Langacker [1998])

   $$ W = y_N \left( \frac{S}{M} \right)^a H_u L N^c $$
3. Lepton number violation: in the LV case

(Hall, Suzuki [1984]) (Grossman, Haber [1998])

\[ W = \mu' H_u L + \lambda L L E^c + \lambda' L Q D^c \]

The BV (\(\lambda'' U^c D^c D^c\)) case can have neutrino mass only through Dirac neutrino. (It does not allow \(N^c N^c, L L E^c, L Q D^c, H_u L\).)
General solution of the MSSM fields including $N^c$

We allow the (possibly high-dimensional) Dirac neutrino mass term in both LV and BV cases.

$$W = y_N \left( \frac{S}{M} \right)^a H_u L N^c$$

It gives $z[H_u] + z[L] + z[N^c] + az[S] = 0$ and

$$\begin{pmatrix}
    z[Q] \\
    z[U^c] \\
    z[D^c] \\
    z[L] \\
    z[N^c] \\
    z[E^c] \\
    z[H_d] \\
    z[H_u] \\
    z[S]
\end{pmatrix} = \alpha \begin{pmatrix} 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 2 \\ -3 \\ 0 \\ 6 \\ -3 \\ 3 \\ 3 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} 1 \\ -1 \\ -3 \\ 0 \\ 3 \\ 0 \\ 0 \\ 0 \\ 9 \end{pmatrix}.$$
Protecting proton from exotic particles

Proton is stable when MSSM fields are considered.

Is it still stable with exotic particles?

We will address this with the remnant discrete symmetry of the $U(1)'$. 
Proton and dark matter without $R$-parity

Brief review of residual discrete symmetry of $U(1)'$
Proton and dark matter without $R$-parity

Conditions to have $U(1)' \rightarrow Z_N$

A $Z_N$ emerges from $U(1)'$ if their charges satisfy (after normalization to integers):

- $z[F_i] = q[F_i] + n_i N$
- $z[S] = N$

($z[F_i]$: $U(1)'$ charge, $q[F_i]$: $Z_N$ charge) for each field $F_i$.

$q[S] = 0$: to keep the discrete symmetry unbroken after the $U(1)'$ symmetry is spontaneously broken by a Higgs singlet $S$.

(ex) In terms of discrete symmetry, $H_u H_d$ and $S H_u H_d$ are not distinguishable (their total discrete charge is same) by the $Z_N$. 
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Discrete symmetry compatible with MSSM sector

Most general \( Z_N \) of the MSSM sector (Ibanez, Ross [1992]) is

\[
Z_N : \quad g_N = B_N^b L_N^\ell
\]

with family-universal cyclic symmetries (\( \Phi_i \rightarrow e^{2\pi i \frac{q_i}{N}} \Phi_i \))

\[
B_N = e^{2\pi i \frac{q_B}{N}}, \quad L_N = e^{2\pi i \frac{q_L}{N}}
\]

and total discrete charge of \( Z_N \) is \( q = bq_B + \ell q_L \mod N \).

<table>
<thead>
<tr>
<th>( B_N )</th>
<th>( Q )</th>
<th>( U^c )</th>
<th>( D^c )</th>
<th>( L )</th>
<th>( E^c )</th>
<th>( N^c )</th>
<th>( H_u )</th>
<th>( H_d )</th>
<th>meaning of ( q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0 )</td>
<td>(-1)</td>
<td>( 1 )</td>
<td>(-1)</td>
<td>( 2 )</td>
<td>( 0 )</td>
<td>( 1 )</td>
<td>(-1)</td>
<td>(-B + y/3)</td>
<td></td>
</tr>
<tr>
<td>( L_N )</td>
<td>( 0 )</td>
<td>( 0 )</td>
<td>( 0 )</td>
<td>(-1)</td>
<td>( 1 )</td>
<td>( 1 )</td>
<td>( 0 )</td>
<td>( 0 )</td>
<td>(-L)</td>
</tr>
</tbody>
</table>
A discrete charge can be rewritten in terms of $B$ and $L$.

$$q = -(bB + \ell L) + b(y/3) \mod N$$

with a conserved quantity of $-(bB + \ell L) \mod N$.

(ex) Matter parity ($R_2 = B_2L_2^{-1}$):

$$q = -(B - L) + (y/3) \mod 2$$
Why 2 free parameters?

- 8 unknown discrete charges \((Q, U^c, D^c, L, E^c, N^c, H_u, H_d)\)
- 5 superpotential terms \((H_u Q U^c, H_d Q D^c, H_d L E^c, H_u L N^c, H_u H_d)\)
- 1 hypercharge shift invariance \((q[F_i] \rightarrow q[F_i] + \alpha y[F_i] \mod N)\)

= 2 free parameters
Family non-universal charges?

- Family non-universal discrete charges \((q[F_i])\)?
  : No, at least in quark sector.
  Mixing of quarks not allowed in contradiction to the CKM matrix.

- Family non-universal \(U(1)’\) charges \((z[F_i])\)?
  : Possible.
  It can still have family universal \(Z_N\), if the condition 
  \(z[F_i] = q[F_i] + n_iN\) is kept \((z[F_i] \text{ is family-dependent if } n_i \text{ is})\).
FCNC from family non-universal $U(1)'$ charges

Family non-universal charges may cause FCNC by $Z'$ at tree level. $U(1)'$ coupling matrix in mass eigenstate ($d_L = V_{dL} d^\text{int}_L$):

$$Q_{d_L} \equiv V_{dL} Q^\text{int}_{dL} V^\dagger_{dL} = V_{dL} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 + \delta \end{pmatrix} V^\dagger_{dL}$$

$Q_{d_L}$ has off-diagonal terms with phases originated from $V_{dL}$. (And similarly for $u$-type quark and/or right-handed coupling.)

The usual CKM matrix is given by $V_{CKM} = V_{uL} V^\dagger_{dL}$.
Flavor changing $Z'$ solution to $B$ anomalies

(Barger, Chiang, Langacker, HL [hep-ph/0310073], [hep-ph/0406126])

$B \rightarrow \phi K_S$

$B \rightarrow \pi K$

FCNC $Z'$ can explain the anomalies in both $B \rightarrow \phi K_S$ and $B \rightarrow \pi K$.

($B \rightarrow \phi K_S$ discrepancy disappeared by now, but the $B \rightarrow \pi K$ anomaly still remains a puzzle.)
Residual discrete symmetry of the RPV $U(1)'$ model

: Proton stability including TeV scale exotics

HL, Luhn, Matchev [arXiv:0712.3505]
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Discrete symmetries in presence of exotics

- The discrete symmetries may be changed with additional particles.
- The MSSM discrete symmetries still hold among the MSSM fields.

For a physics process which has only MSSM fields in its effective operators (such as proton decay), we can still discuss with $Z_{N}^{\text{MSSM}}$.

\[
\begin{align*}
\text{operator}[p\text{-decay}] & = \left( \frac{1}{M} \right)^m \begin{bmatrix} F_1 & F_2 & F_3 & F_4 & F_5 & \cdots \end{bmatrix} \\
\text{MSSM fields only}
\end{align*}
\]
Naturally suppressed LV and BV couplings

Experimental upper bounds:

\[ \lambda, \lambda' \lesssim 10^{-5} \]
\[ \lambda'' \lesssim 10^{-7} \]

In the \( U(1)' \) model, you can have the naturally suppressed \( \mathcal{L} \) and \( \mathcal{B} \) violating couplings from high-dimensional operators.

\[ \lambda = \hat{\lambda} \left( \frac{\langle S \rangle}{M} \right)^n \]

It does not affect discrete symmetry argument since \( q[S] = 0 \).
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\[
W_{\text{LV}} = \hat{\lambda} \left( \frac{S}{M} \right)^n LLE^c + \hat{\lambda}' \left( \frac{S}{M} \right)^n LQD^c + \hat{h}' \left( \frac{S}{M} \right)^n SLH_u
\]

\[
W_{\text{BV}} = \hat{\lambda}'' \left( \frac{S}{M} \right)^m U^c D^c D^c
\]

with \( \lambda_{\text{eff}} = \hat{\lambda} \left( \frac{\langle S \rangle}{M} \right)^n \), etc.

Generalized LV-BV separation:

\[
z [S^m U^c D^c D^c] - z [S^n LLE^c] - \left( \frac{2}{3} + (m - n) \right) z [S] = 0
\]

(The LV-BV separation still holds independent of \( n \) and \( m \).)
General $U(1)'$ charges in the LV case

Use another condition

$$S^n L L E^c : n z[S] + 2 z[L] + z[E^c] = 0$$

to reduce a parameter in the general $U(1)'$ charges.

$$
\begin{pmatrix}
z[Q] \\
z[U^c] \\
z[D^c] \\
z[L] \\
z[N^c] \\
z[E^c] \\
z[H_d] \\
z[H_u] \\
z[S]
\end{pmatrix}
= \alpha' \begin{pmatrix} 1 \\ -4 \\ 2 \\ -3 \\ 0 \\ 6 \\ -3 \\ 3 \\ 0 \end{pmatrix} + \beta' \begin{pmatrix} 0 \\ 3(1 + n) + 1 \\ -3n - 1 \\ 1 \\ 3(1 - a + n) \\ -3n - 2 \\ 3n + 1 \\ -3(1 + n) - 1 \\ 3 \end{pmatrix}
$$

→ **It is a $Z_3$ symmetry.**  ($N = z[S]$ after normalization to integers)
Discrete symmetry of the LV case

- First column ($\propto y$) is irrelevant $\rightarrow$ Take $\alpha' = 0$ and $\beta' = 1$.

- $q[F_i] = z[F_i] - n_i N \rightarrow q[F_i] = z[F_i] \mod 3$.

\[
\begin{pmatrix}
q[Q] \\
q[U^c] \\
q[D^c] \\
q[L] \\
q[N^c] \\
q[E^c] \\
q[H_d] \\
q[H_u] \\
q[S]
\end{pmatrix} = \begin{pmatrix}
0 \\
3(1+n) + 1 \\
-3n - 1 \\
1 \\
3(1 - a + n) \\
-3n - 2 \\
3n + 1 \\
-3(1 + n) - 1 \\
3
\end{pmatrix} \mod 3 = \begin{pmatrix}
0 \\
-1 \\
1 \\
-1 \\
0 \\
-1 \\
1 \\
0
\end{pmatrix} \mod 3
\]

Compare with charge table. $\rightarrow$ **LV model has $B_3$ (baryon triality).**

<table>
<thead>
<tr>
<th></th>
<th>$Q$</th>
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<td>0</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>$-B + y/3$</td>
</tr>
</tbody>
</table>
Selection rule of $B_3$

The discrete charge of $B_3$ for arbitrary operator is $(-B + y/3) \mod 3$.

$$\Delta B = 3 \times \text{integer}$$

for any process. (Castano, Martin [1994])

It dictates that baryon number can be violated by only $3 \times \text{integer}$ under the $B_3$.

- Proton decay ($\Delta B = 1$): Forbidden
- Neutron-antineutron oscillation ($\Delta B = 2$): Forbidden
Ensuring proton stability in the LV model ($B_3$)

1. Solve the $\mu$-problem with $U(1)'$ gauge symmetry.

2. Require $\mathcal{L}$ violating terms such as $\lambda' L Q D^c$. [$B_3$ is invoked]

3. Then proton is absolutely stable!
General $U(1)'$ charges for the BV case

Use another condition

$$S^m U^c D^c D^c : mz[S] + z[U^c] + 2z[D^c] = 0$$

to reduce a parameter in the general $U(1)'$ charges.

$$\begin{pmatrix}
  z[Q] \\
  z[U^c] \\
  z[D^c] \\
  z[L] \\
  z[N^c] \\
  z[E^c] \\
  z[H_d] \\
  z[H_u] \\
  z[S]
\end{pmatrix} = \alpha' \begin{pmatrix} 1 \\ -4 \\ 2 \\ -3 \\ 0 \\ 6 \\ -3 \\ 3 \\ 0 \end{pmatrix} + \beta' \begin{pmatrix} 0 \\ 3(2 + m) \\ -3(1 + m) \\ 1 \\ 3(2 - a + m) - 1 \\ -3(1 + m) - 1 \\ 3(1 + m) \\ -3(2 + m) \\ 3 \end{pmatrix}$$

→ It is a $Z_3$ symmetry. ($N = z[S]$ after normalization to integers)
Proton and dark matter without $R$-parity

Discrete symmetry of the BV case

- First column ($\propto y$) is irrelevant $\rightarrow$ Take $\alpha' = 0$ and $\beta' = 1$.

- $q[F_i] = z[F_i] - n_i N \rightarrow q[F_i] = z[F_i] \mod 3$.

$$
\begin{pmatrix}
q[Q] \\
q[U^c] \\
q[D^c] \\
q[L] \\
q[N^c] \\
q[E^c] \\
q[H_d] \\
q[H_u] \\
q[S]
\end{pmatrix} = \begin{pmatrix}
0 \\
3(2 + m) \\
-3(1 + m) \\
1 \\
3(2 - a + m) - 1 \\
-3(1 + m) - 1 \\
3(1 + m) \\
-3(2 + m) \\
3
\end{pmatrix} \mod 3 = -\begin{pmatrix}
0 \\
0 \\
0 \\
-1 \\
1 \\
1 \\
0 \\
0 \\
0
\end{pmatrix} \mod 3
$$

Compare with charge table. $\rightarrow$ **BV model has** $L_3$ (**lepton triality**).

<table>
<thead>
<tr>
<th></th>
<th>$Q$</th>
<th>$U^c$</th>
<th>$D^c$</th>
<th>$L$</th>
<th>$E^c$</th>
<th>$N^c$</th>
<th>$H_u$</th>
<th>$H_d$</th>
<th>meaning of $q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_3$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$-\mathcal{L}$</td>
</tr>
</tbody>
</table>
Selection rule of $L_3$

The discrete charge of $L_3$ for arbitrary operator is $-\mathcal{L} \mod 3$.

$$\Delta \mathcal{L} = 3 \times \text{integer}$$

for any process.

It dictates that $\mathcal{L}$ can be violated by only $3 \times \text{integer}$ under the $L_3$.

- $0\nu\beta\beta$ decay ($\Delta \mathcal{L} = 2$): Forbidden

Proton still may decay if the decay products has 3, 6, · · · leptons.

Discrete symmetry argument is not enough. $\rightarrow$ Need to consider the $U(1)'$ symmetry and exotic fields (model-dependent) to ensure proton stability.
Proton and dark matter without $R$-parity

Flowchart to check proton stability

Start

Does $LLE'$ exist effectively? No

You have $B_3$!

Yes

Does $U'D'D'$ exist effectively? No

You have $L_3$!

Yes

Is $\varepsilon[S] = \pm 3$? No

Identify your $Z_N$ symmetry!

Yes

You have $M_1$!

Yes

Compare with Table VI. Do you get an empty box? No

Do you have $(N')^3$ or $S(N')^3$? No

Do you have $U'D'D'$? No

See Table VI. Do you get $\otimes$? No

Do you have $N'N'N'$? No

Do you have any $\otimes$ term? No

Determine all BV/LV operators. Can you construct $p$-decay diagrams with at most $M_{\pi}^{-1}$ suppression? No

Are you sure? Yes

No

Yes

Yes

Yes

Yes

Yes

Yes

Yes

Too bad, the proton decays rapidly in your model!
Proton and dark matter without $R$-parity

START

Does $L E^c$ exist effectively?

Yes

You have $B_3$!

No

Does $U^c D^c D^c$ exist effectively?

Yes

You have $L_3$!

No

Compare with Table VI. Do you get an empty box?

No

Do you have $(N^c)^3$ or $S(N^c)^3$?

No

Yes
Ensuring proton stability in the BV model ($L_3$)

1. Solve the $\mu$-problem with $U(1)'$ gauge symmetry.

2. Require $\mathcal{B}$ violating term $\lambda'' U^c D^c D^c$. \([L_3 \text{ is invoked}]\)

3. Forbid $N^c N^c N^c$ and $S N^c N^c N^c$ by the $U(1)'$ charges$^a$.

4. Then proton is sufficiently (up to dimension 5) stable!

$^a$It holds in our choice of colored exotics ($K_i$, $K_i^c$) which have integer hypercharges (under normalization of $y[Q] = 1$).
Proton and dark matter without $R$-parity

Examples of anomaly-free $U(1)'$ charge assignments with stable proton
Free to be scaled by any normalization and shifted by hypercharge.

<table>
<thead>
<tr>
<th></th>
<th>LV ($B_3$)</th>
<th>BV ($L_3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>$z[Q]$</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>$z[U^c]$</td>
<td>8</td>
<td>24</td>
</tr>
<tr>
<td>$z[D^c]$</td>
<td>-1</td>
<td>-3</td>
</tr>
<tr>
<td>$z[L]$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$z[E^c]$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$z[N^c]$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$z[H_d]$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$z[S]$</td>
<td>9</td>
<td>27</td>
</tr>
<tr>
<td>$z[K_1]$</td>
<td>-5</td>
<td>-13</td>
</tr>
<tr>
<td>$z[K_2]$</td>
<td>-2</td>
<td>-4</td>
</tr>
<tr>
<td>$z[K_3]$</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$z[K_1^c]$</td>
<td>-4</td>
<td>-14</td>
</tr>
<tr>
<td>$z[K_2^c]$</td>
<td>-7</td>
<td>-23</td>
</tr>
<tr>
<td>$z[K_3^c]$</td>
<td>-10</td>
<td>-29</td>
</tr>
</tbody>
</table>

$y[K_i] = \{ \frac{1}{3}, -\frac{2}{3}, -\frac{2}{3} \}$
Recap of the goal
Construct a stand-alone $R_p$ violating TeV scale SUSY model without

1. $\mu$-problem: $U(1)'$

2. proton decay problem: $U(1)'$

3. dark matter problem (non-LSP dark matter)

A dark matter candidate without introducing an independent symmetry?
LUP dark matter
(in the $R$-parity conserving UMSSM)
Hur, HL, Nasri [arXiv:0710.2653]
SM-singlet (hidden sector) fields

SM-singlet exotics (hidden sector fields): often required for anomaly cancellations with $U(1)'$.

- $[\text{gravity}]^2 - U(1)'$: $\sum_i z[F_i] = \cdots + z[X] = 0$
- $[U(1)']^3$: $\sum_i z[F_i]^3 = \cdots + z[X]^3 = 0$

We consider Majorana fields for simplicity.

$$W_{\text{hidden}} = \frac{\xi}{2} SXX$$

These hidden sector fields ($X$) are neutral and massive particles.

$\rightarrow$ Potentially dark matter candidate if they are stable.
How to stabilize hidden sector field?

Introduce “$U$-parity”

$$U_p[\text{MSSM}] = \text{even}, \quad U_p[X] = \text{odd}$$

- Lightest $U$-parity Particle (LUP): Lightest $X \rightarrow$ stable
  
  either fermion ($\psi_X$) or scalar ($\phi_X$) component

It can be invoked as a residual discrete symmetry of the $U(1)'$.

$$Z^{hid}_N : \quad g^{hid}_2 = U_2 \quad (U\text{-parity})$$

$$z[F_i] = q[F_i] + 2n_i$$

<table>
<thead>
<tr>
<th></th>
<th>$Q$</th>
<th>$U^c$</th>
<th>$D^c$</th>
<th>$L$</th>
<th>$E^c$</th>
<th>$N^c$</th>
<th>$H_u$</th>
<th>$H_d$</th>
<th>$X$</th>
<th>meaning of $q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_2$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$-U$ ($X$ number)</td>
</tr>
</tbody>
</table>

(Other exotics: assumed to be heavier than the lightest $X$.)
Lightest $U$-parity Particle (LUP)

• It is a neutral, massive, and stable particle.

• It can be either a fermion or a scalar.

• It is neither the RH neutrino nor RH sneutrino ($H_u L N^c$).

• It naturally arises when an extra $U(1)$ gauge symmetry is present.
Annihilation channels for the LUP dark matter

For $\psi_X$ (fermionic) LUP,

1. $\psi_X\psi_X \rightarrow f \bar{f} \ (Z' \text{ mediated } s\text{-channel})$

2. $\psi_X\psi_X \rightarrow \bar{f}f^* \ (S \text{ mediated } s\text{-channel}, Z' \text{ mediated } s\text{-channel})$

3. $\psi_X\psi_X \rightarrow SS, Z'Z' \ (S \text{ mediated } s\text{-channel}, \psi_X \text{ mediated } t\text{-ch})$

4. $\psi_X\psi_X \rightarrow SZ' \ (Z' \text{ mediated } s\text{-channel}, \psi_X \text{ mediated } t\text{-channel})$

5. $\psi_X\psi_X \rightarrow \tilde{S}\tilde{S} \ (Z' \text{ mediated } s\text{-channel}, \phi_X \text{ mediated } t\text{-channel})$

6. $\psi_X\psi_X \rightarrow \tilde{Z}'\tilde{Z}' \ (\phi_X \text{ mediated } t\text{-channel})$

7. $\psi_X\psi_X \rightarrow \tilde{S}\tilde{Z}' \ (S \text{ mediated } s\text{-channel}, \phi_X \text{ mediated } t\text{-channel})$

and also similarly for $\phi_X$ (scalar) LUP.
LUP (+ LSP) dark matter can satisfy both the relic density and direct detection constraints.
Multiple dark matters scenario with $R$-parity and $U$-parity

LUP was first introduced in a $R$-parity conserving $U(1)'$-extended MSSM.

- $R$-parity: for proton stability (at renormalizable level)
  $\rightarrow$ LSP dark matter (SM charged particle: MSSM sector)

- $U$-parity: as a remnant of $U(1)'$
  $\rightarrow$ LUP dark matter (SM uncharged particle: hidden sector)

For each sector, discrete symmetries came from different origins.
Proton and dark matter without $R$-parity

Residual discrete symmetry extended to hidden sector
: LUP dark matter in the RPV-UMSSM

HL [arXiv:0802.0506]
Two discrete symmetries

$\mathbb{Z}_N$ is isomorphic (structure-preserving mapping in both directions) to $\mathbb{Z}_{N_1} \times \mathbb{Z}_{N_2}$, if $N_1$ and $N_2$ are coprime (their GCD = 1) and $N = N_1N_2$.

$$\mathbb{Z}_N = \mathbb{Z}_{N_1} \times \mathbb{Z}_{N_2}$$

(ex: $\mathbb{Z}_6 = \mathbb{Z}_2 \times \mathbb{Z}_3$).

What does it mean?

- No need of two gauge origins for $\mathbb{Z}_{N_1}, \mathbb{Z}_{N_2}$ (if $N_1, N_2$ coprime).

  $$U(1)' \rightarrow \mathbb{Z}_{N_1}, \quad U(1)'' \rightarrow \mathbb{Z}_{N_2}$$

- Only one $U(1)$ which has $\mathbb{Z}_N$ as a residual discrete symmetry.

  $$U(1)' \rightarrow \mathbb{Z}_N = \mathbb{Z}_{N_1} \times \mathbb{Z}_{N_2}$$
Discrete symmetries over the MSSM and the hidden sectors

Consider \( Z_{N}^{tot} = Z_{N_1}^{obs} \times Z_{N_2}^{hid} \) (where \( N_1 \) and \( N_2 \) are coprime)
as the most general residual discrete symmetry from a common \( U(1)' \) gauge symmetry.

\[
Z_{N}^{tot} : \quad g_{N}^{tot} = \quad B_{N_1}^{b} L_{N_1}^{\ell} \times U_{N_2}^{u} \\
= \quad B_{N}^{bN_2} L_{N}^{\ell N_2} U_{N}^{uN_1}
\]
Proton and dark matter without $R$-parity

Simplest example: $U(1)' \rightarrow Z_6 (= B_3 \times U_2)$

The residual discrete symmetry of the $U(1)'$ is therefore

$$Z_6^{\text{tot}} : g_6^{\text{tot}} = B_6^2 U_6^3$$

and its total discrete charge is given by $q = 2q_B + 3q_U \mod 6$.

$$q[Q] = 0 \quad q[U^c] = -2 \quad q[D^c] = 2$$
$$q[L] = -2 \quad q[E^c] = -2 \quad q[N^c] = 0$$
$$q[H_u] = 2 \quad q[H_d] = -2 \quad q[X] = -3$$

(Other exotic fields: assumed to be heavier than proton and the LUP → not stable due to the discrete symmetry.)
A unified picture of the stabilities in the observable and hidden sectors

\[ U(1)' \rightarrow Z_{N_1}^{obs} \times Z_{N_2}^{hid} \]

A single \( U(1)' \) gauge symmetry provides stabilities for proton (MSSM sector) and dark matter (hidden sector).
Proton and dark matter without $R$-parity

Light gravitino problem of the GMSB

In the gauge mediated SUSY breaking (GMSB) scenario, gravitino is the LSP.

$$
\left( m_{3/2} \sim \frac{\langle F \rangle}{M_{Pl}} \right) \ll \left( m_{\text{soft}} \sim \frac{\alpha_a}{4\pi} \frac{\langle F \rangle}{M_{\text{mess}}} \right)
$$

The gravitino relic density (assuming $R$-parity) is approximately given by (Pagels, Primack [1982])

$$
\Omega_{3/2} h^2 \sim \frac{m_{3/2}}{1 \text{ keV}}.
$$

Dark matter relic density constrains $m_{3/2} \sim \mathcal{O}(\text{keV})$

$\rightarrow$ warm dark matter, which cannot explain the matter power spectrum. (Viel et al. [2005])
Cure of light gravitino problem with LUP and $R$-parity violation

When the LUP is the only (or dominant) dark matter, there is no conflict with matter power spectrum.

- lighter gravitino LSP ($m_{3/2} \ll 1$ keV): maybe still long-lived (small coupling and mass) as a subdominant dark matter

- heavier gravitino LSP ($m_{3/2} \gg 1$ keV): decays through the $R$-parity violating couplings

The next-to-lightest superparticle (NLSP) will decay into the SM particles through the $R$-parity violating processes before BBN.

→ **LUP in RPV model can be an appealing solution to the light gravitino problem of the GMSB.** (Need numerical study).
Proton and dark matter without $R$-parity

Future studies

1. Extension of the hidden sector fields to the Dirac particles ($Z_{N}^{hid}$ with $N \geq 2$ is possible), and explicit model buildings including $L_3$ etc.

2. Collider signals (RPV signals, LUP signals).

3. Indirect detection signals of the LUP dark matter.

4. Quantitative study of gravitino problem solution with RPV and LUP.
Summary
### $R$-parity conserving MSSM vs. $R$-parity violating UMSSM

<table>
<thead>
<tr>
<th></th>
<th>$R_p$</th>
<th>$U(1)' \rightarrow B_3 \times U_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RPV signals</td>
<td>impossible</td>
<td>possible</td>
</tr>
<tr>
<td>$\mu$-problem</td>
<td>not addressed</td>
<td>solvable ($U(1)'$)</td>
</tr>
<tr>
<td>proton</td>
<td>unstable w/ dim 5 op. ($R_p$)</td>
<td>stable ($B_3$)</td>
</tr>
<tr>
<td>dark matter</td>
<td>stable LSP ($R_p$)</td>
<td>stable LUP ($U_p$)</td>
</tr>
<tr>
<td>light $\tilde{G}$ problem</td>
<td>not addressed</td>
<td>solvable</td>
</tr>
</tbody>
</table>

Proton and dark matter without $R$-parity

Fermilab 2008

Hye-Sung Lee
**Proton and dark matter without \( R \)-parity**

\( R \)-parity conserving MSSM vs. \( R \)-parity violating UMSSM

<table>
<thead>
<tr>
<th></th>
<th>( R_p )</th>
<th>( U(1)' \to B_3 \times U_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>RPV signals</td>
<td>impossible</td>
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<tr>
<td>( \mu )-problem</td>
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<tr>
<td>proton</td>
<td>unstable w/ dim 5 op. (( R_p ))</td>
<td>stable (( B_3 ))</td>
</tr>
<tr>
<td>dark matter</td>
<td>stable LSP (( R_p ))</td>
<td>stable LUP (( U_p ))</td>
</tr>
<tr>
<td>light ( \tilde{G} ) problem</td>
<td>not addressed</td>
<td>solvable</td>
</tr>
</tbody>
</table>

**Conclusion:** TeV scale \( U(1)' \) is an attractive alternative to \( R \)-parity.