The assault on one-loop QCD

R. Keith Ellis
Fermilab

Fermilab, October 2008

http://mcfm.fnal.gov/

MCFM overview

Monte Carlo for FeMtobarn processes. At LHC few of the cross sections are expressed in fb, so MCFM. Parton level cross sections predicted to NLO in $\alpha_{S.}$ Currently released version 5.2, July 2007

Features-Less sensitivity to unphysical μ_R and $\mu_{F,}$ better normalization for rates, fully differential distributions.
Shortcomings- low parton multiplicity (no showering), no hadronization, hard to model detector effects.

$p\bar{p} \to W^\pm/Z$	$p\bar{p} \to W^+ + W^-$
$p\bar{p} \to W^\pm + Z$	$p\bar{p} \to Z + Z$
$p\bar{p} \to W^\pm + \gamma$	$p\bar{p} \to W^\pm/Z + H$
$p\bar{p} \to W^\pm + g^\star\ (\to b\bar{b})$	$p\bar{p} \to Zb\bar{b}$
$p\bar{p} \to W^\pm/Z + 1$ jet	$p\bar{p} \to W^\pm/Z + 2$ jets
$p\bar{p}(gg) \to H$	$p\bar{p}(gg) \to H + 1$ jet
$p\bar{p}(VV) \to H + 2$ jets	$p\bar{p} \to t + X$
$pp \to t + W$	

Work by John Campbell and Keith Ellis with appearances by guest celebrities, Fabio Maltoni, Francesco Tramontano, Scott Willenbrock & Giulia Zanderighi.

Why NLO?

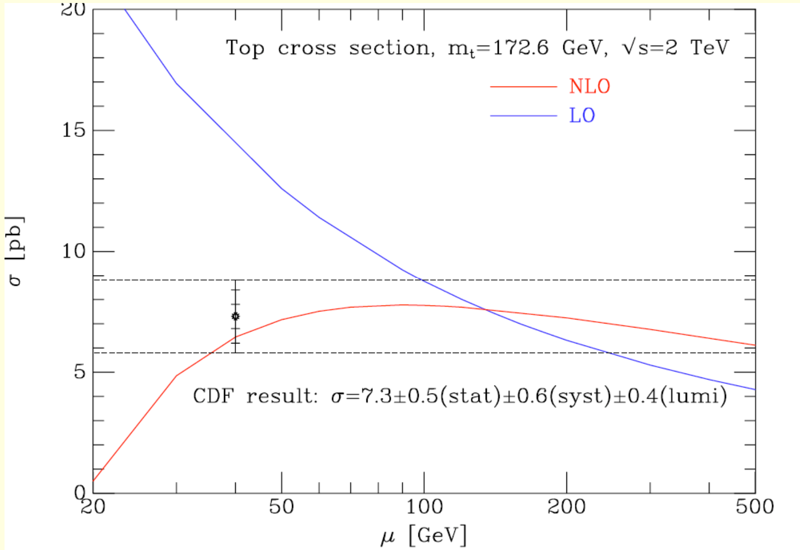
- Less sensitivity to unphysical input scales (eg. renormalization and factorization scales).
- First real prediction of normalization of observables occurs at NLO.
- It is a necessary prerequisite for other techniques, matching with resummed calculations, (MC@NLO, POWHEG, etc).
- More physics (a) parton merging to give structure in jets, (b) initial state radiation, (c) More species of incoming partons enter at NLO.

Improved scale dependence

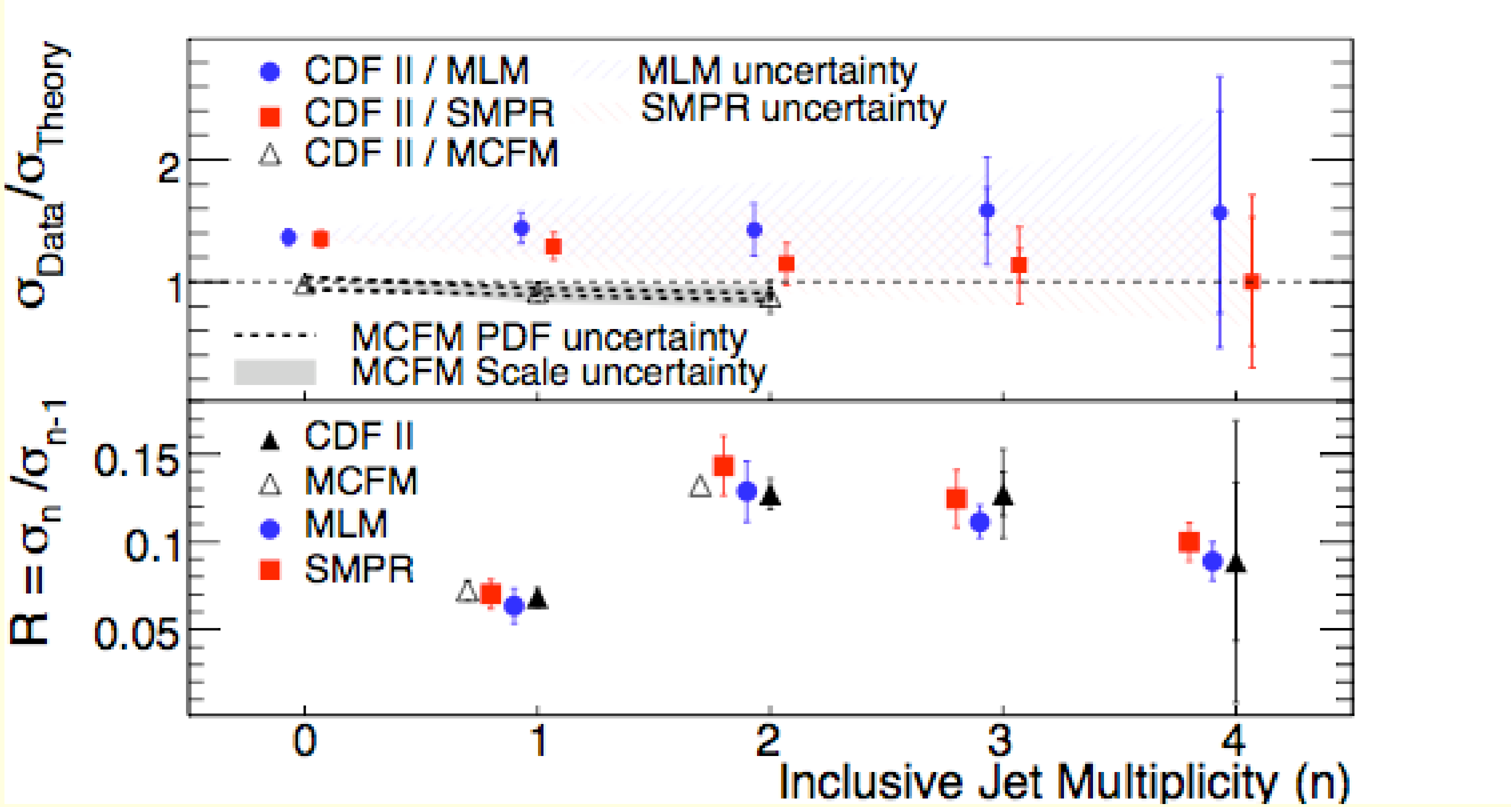
- Variations of renormalization scale are themselves NLO effects. So without NLO calculation one has no idea about the choice of renormalization (or factorization) scale.
- Example: Top cross section at the Tevatron.
- Performing the calculation at NLO reduces the dependence on unphysical scales.
- μ is the renormalization and factorization scale.

$$\alpha_S(\mu_0^2) = \frac{1}{b \ln \frac{\mu_0^2}{\Lambda^2}}$$

$$\begin{aligned} \alpha_S(\mu_1^2) &= \frac{1}{b \ln \frac{\mu_1^2}{\Lambda^2}} \equiv \frac{1}{b(\ln \frac{\mu_0^2}{\Lambda^2} + \ln \frac{\mu_1^2}{\mu_0^2})} \\ &\approx \alpha_S(\mu_0^2) - b \ln \frac{\mu_1^2}{\mu_0^2} \alpha_S^2(\mu_0^2) \end{aligned}$$



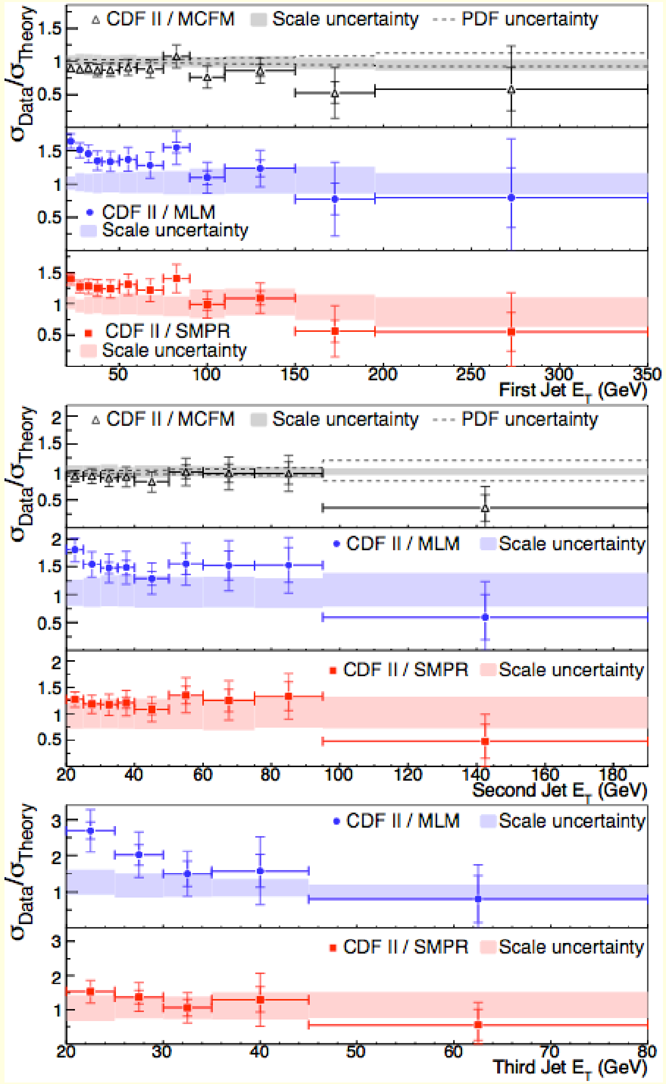
W+n jet rates from CDF



Both uncertainty on rates and deviation of Data/Theory from 1 are smaller than other calculations. “Berends” ratio agrees well for all calculations, but unfortunately only available for n≤2 from MCFM.

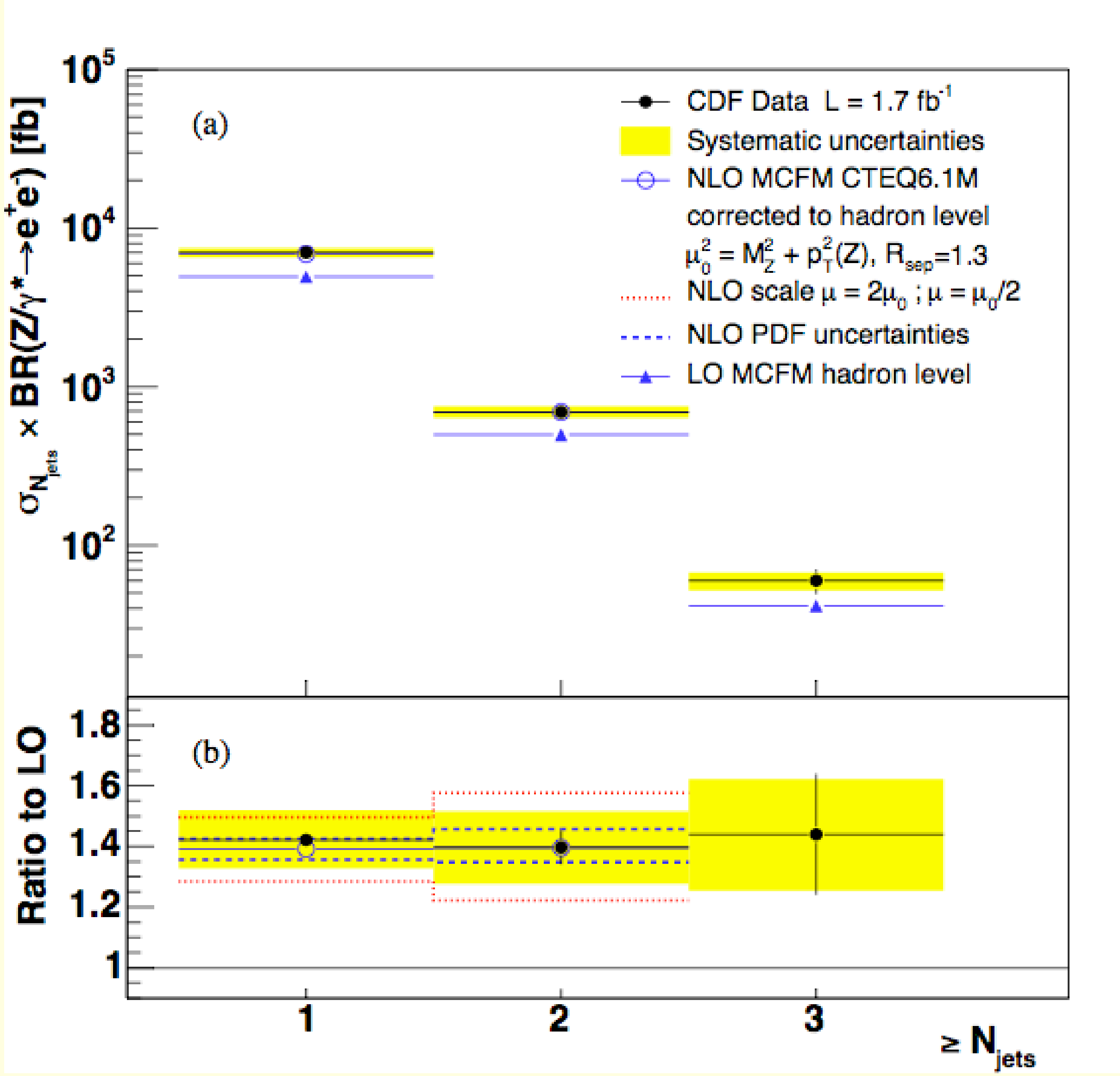
PRD77 011108R

CDF results for W+jets



Ratio of data over theory (MCFM) for first and second jet appears to agree well. MCFM results are not available at NLO for third jet.

Z + n jets rate agrees well with NLO QCD from MCFM



Recent additions to MCFM

- WW+1jet (Campbell, RKE, Zanderighi, arXiv:0710.1832)
- H+2jet (Campbell, RKE, Zanderighi, hep-ph/0608194)

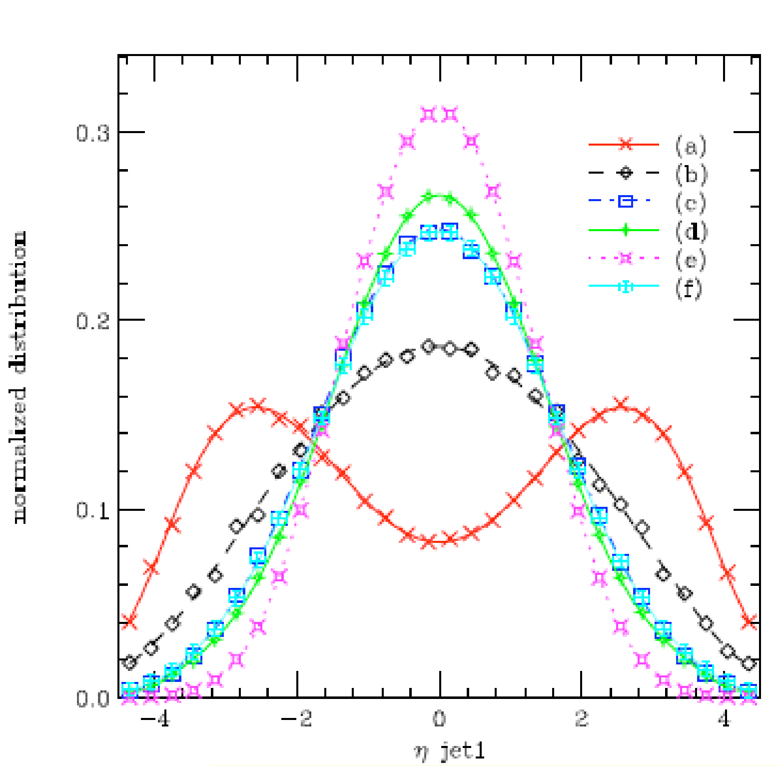
Unfortunately neither of these processes are yet included in the publically released code.

Campbell et al, arXiv:0710.1832 [hep-ph]

Dittamaier et al, arXiv:0710.1577 [hep-ph]

WW+1jet

WW+1 jet and impact on
Higgs->WW + 1 jet search



Rates with cuts I+II

Process	σ_{LO} (fb)	σ_{NLO} (fb)
(a) $H \to WW$ (WBF)	10.6	10.6
(b) $H \to WW$ (gluon fusion)	8.6	18.0
(c) WW+jet	11.7	20.2
(d) $W + t$	7.8	7.6
(e) $t\bar{t}$	12.7	-
(f) ZZ+jet	0.44	-

Standard Cuts I : $P_{t,j1} > 30$ GeV, $|\eta_{j1}| < 4.5$

$P_{t,miss} > 30$ GeV , $P_{t,\ell_1} > 20$ GeV , $P_{t,\ell_2} > 10$ GeV , $|\eta_{\ell_1(\ell_2)}| < 2.5$.

Cuts II: $|\eta_{j_1}| > 1.8$ $|\eta_{j_2}| > 2.5$ $\phi_{\ell_1,\ell_2} < 1.2$ $m_{\ell_1,\ell_2} < 75$ GeV

Campbell, Ellis, Zanderighi

Higgs+2 jets at NLO

- Calculation performed using an effective Lagrangian, valid in the large m_t limit.

$$\mathcal{L}_{\rm eff} = \frac{1}{4} A(1+\Delta) H G^{a}_{\mu\nu} G^{a\,\mu\nu}$$

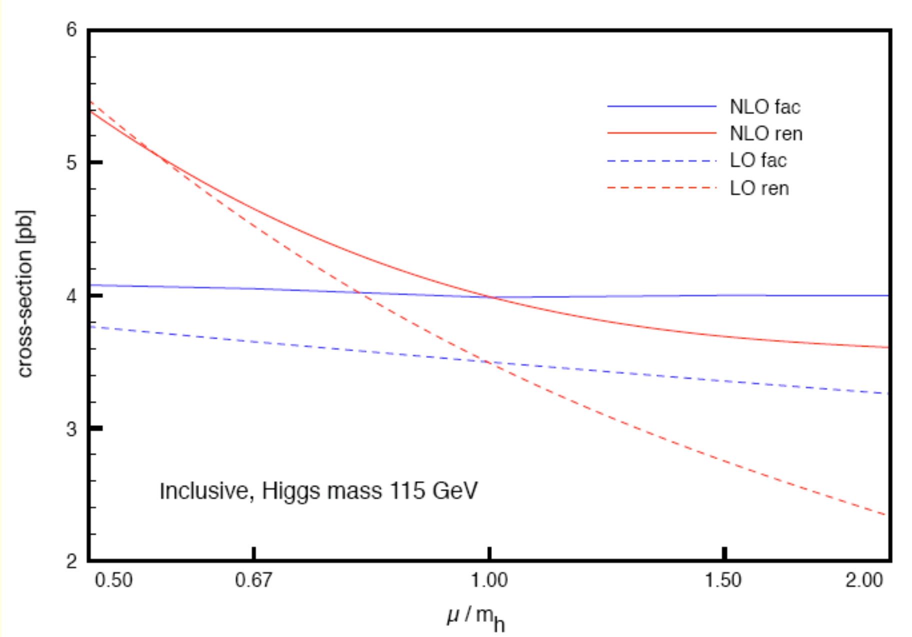
Three basic processes at lowest order.

$$\begin{array}{ll} A) & 0 \to H q\bar{q}q'\bar{q}' g \,, \\ B) & 0 \to H q\bar{q}ggg \,, \\ C) & 0 \to H ggggg \,. \end{array}$$

Higgs + 2 jet continued

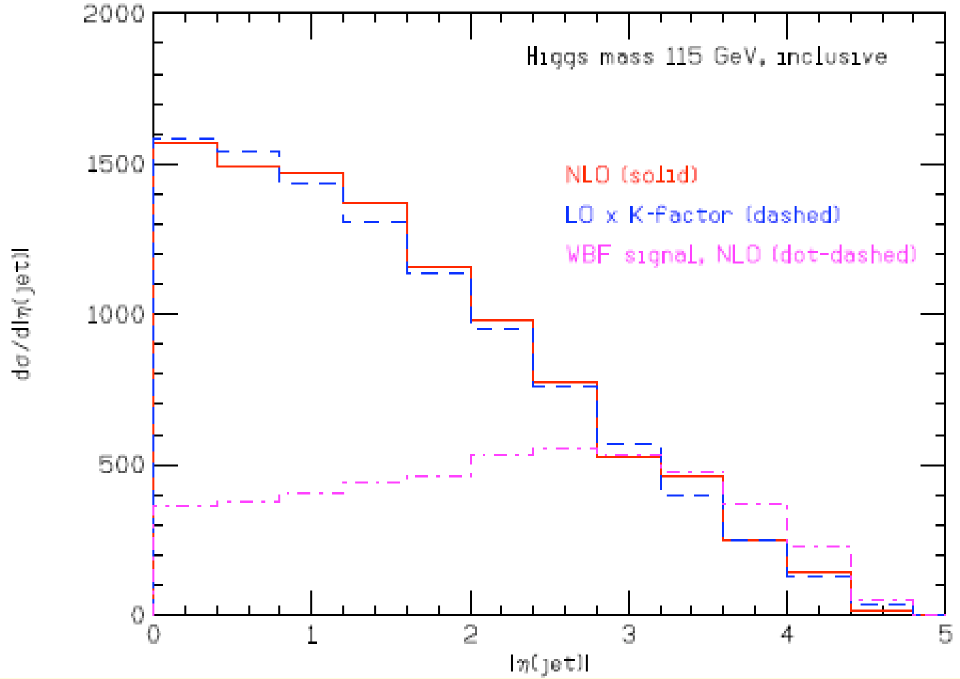
- NLO corrections are quite mild, increasing LO cross section by 15%
- NLO cross section contains a considerable residual scale uncertainty.

Higgs mass	115 GeV	160 GeV
σ_{LO} [pb]	3.50	2.19
σ_{NLO} [pb]	4.03	2.76
σ_{WBF} [pb]	1.77	1.32



Higgs + 2 jets rapidity distribution versus WBF

- Shape of NLO result, similar to LO in rapidity.
- WBF shape is quite different at NLO.



An experimenter's wishlist

Run II Monte Carlo Workshop

Single Boson	Diboson	Triboson	Heavy Flavour
$W+\leq 5j$	$WW+\leq 5j$	$WWW+\leq 3j$	$t\bar{t}+\leq 3j$
$W+b\bar{b}\leq 3j$	$W+b\bar{b}+\leq 3j$	$WWW+b\bar{b}+\leq 3j$	$t\bar{t}+\gamma+\leq 2j$
$W+c\bar{c}\leq 3j$	$W+c\bar{c}+\leq 3j$	$WWW+\gamma\gamma+\leq 3j$	$t\bar{t}+W+\leq 2j$
$Z+\leq 5j$	$ZZ+\leq 5j$	$Z\gamma\gamma+\leq 3j$	$t\bar{t}+Z+\leq 2j$
$Z+b\bar{b}+\leq 3j$	$Z+b\bar{b}+\leq 3j$	$ZZZ+\leq 3j$	$t\bar{t}+H+\leq 2j$
$Z+c\bar{c}+\leq 3j$	$ZZ+c\bar{c}+\leq 3j$	$WZZ+\leq 3j$	$t\bar{b}\leq 2j$
$\gamma+\leq 5j$	$\gamma\gamma+\leq 5j$	$ZZZ+\leq 3j$	$b\bar{b}+\leq 3j$
$\gamma+b\bar{b}\leq 3j$	$\gamma\gamma+b\bar{b}\leq 3j$		single top
$\gamma+c\bar{c}\leq 3j$	$\gamma\gamma+c\bar{c}\leq 3j$		
	$WZ+\leq 5j$		
	$WZ+b\bar{b}\leq 3j$		
	$WZ+c\bar{c}\leq 3j$		
	$W\gamma+\leq 3j$		
	$Z\gamma+\leq 3j$		

Extension to higher leg processes

- MCFM does not include W/Z+3 jets,W/Z+4 jets at NLO.
- We know the tree graphs, we know the subtraction procedure, with enough effort we can write an efficient phase space generator.
- The bottleneck is the calculation of multi-leg, one-loop diagrams.
- Straightforward numerical integration of one-loop diagrams is complicated, by the presence of soft, collinear and UV divergences
- Analytic calculation may be too painstaking.
- Our preferred method is a semi-numerical approach . Scalar integrals are calculated analytically and their coefficients calculated numerically.

Components of a NLO calculation

- Tree graphs (both for lowest order process and real radiation)
- One-loop correction to the Born level process
- Subtraction terms to remove singularities from real radiation graphs.
- Phase space generator.

General Method for NLO parton integrator

- We want to compute a jet cross section σ to NLO, namely

$$\sigma = \sigma^{LO} + \sigma^{NLO} \ .$$

- Born approximation involves m partons in the final state.

$$\sigma^{LO} = \int_m d\sigma^B \ ,$$

- At NLO we have the real cross section $d\sigma^R$ with $m+1$ partons in the final-state and the one-loop correction $d\sigma^V$ to the process with m partons in the final state:

$$\sigma^{NLO} \equiv \int d\sigma^{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V \ .$$

- The two integrals are separately divergent in (four dimensions), although their sum is finite.

Finiteness

- The general idea of the subtraction method for writing a general-purpose Monte Carlo program is to use the identity

$$d\sigma^{NLO} = \left[d\sigma^R - d\sigma^A\right] + d\sigma^A + d\sigma^V \quad ,$$

where $d\sigma^A$ is a proper approximation of $d\sigma^R$ such as to have the same singular behaviour point-by-point as $d\sigma^R$ itself. Thus, $d\sigma^A$ acts as a local counterterm for $d\sigma^R$ and, introducing the phase space integration,

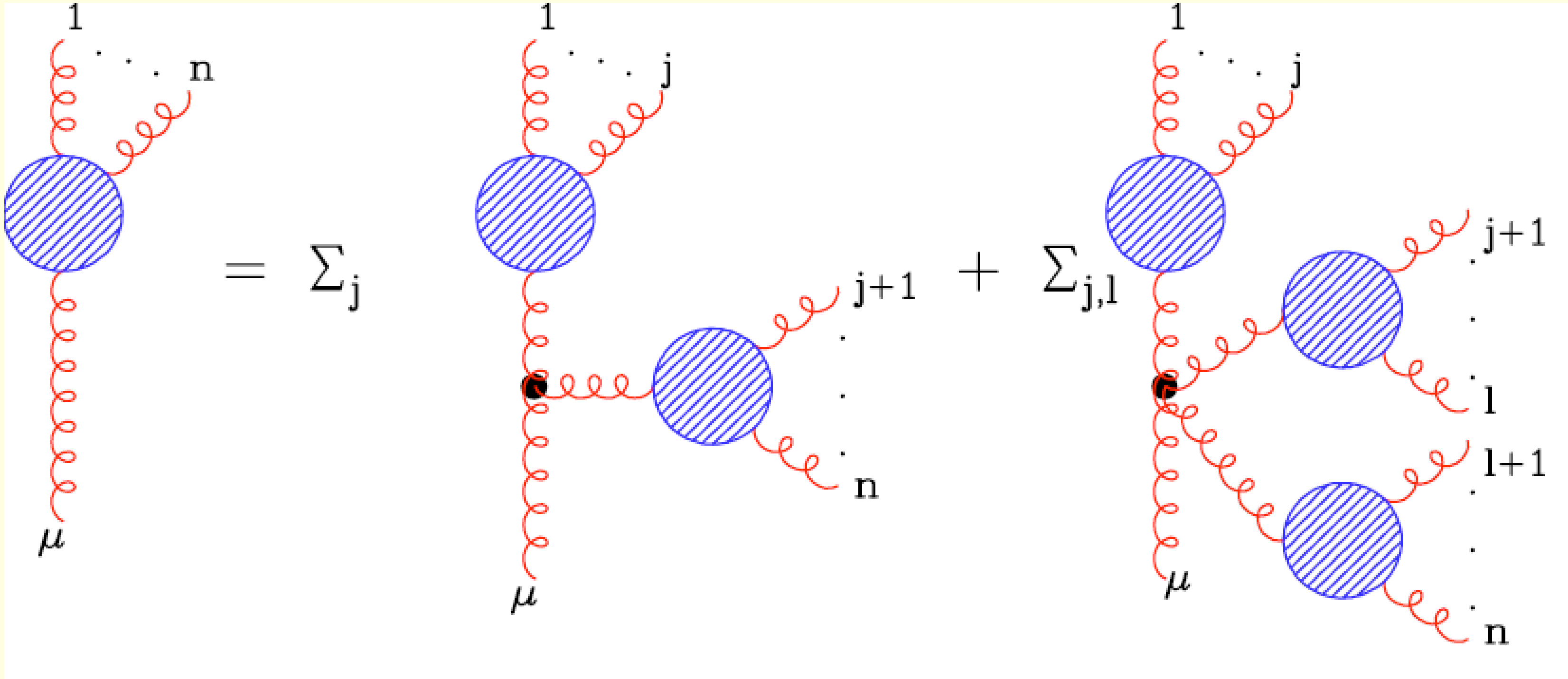
$$\sigma^{NLO} = \int_{m+1} \left[d\sigma^R - d\sigma^A\right] + \int_{m+1} d\sigma^A + \int_m d\sigma^V \quad ,$$

- one can safely perform the limit $\epsilon \to 0$ under the integral sign in the first term on the right-hand side. The first term can be integrated numerically in four dimensions.

The major outstanding problem is the calculation of one loop amplitudes

Berends-Giele recursion (for Born+real)

Building blocks are non-gauge invariant color-ordered off-shell currents. Off-shell currents with n legs are related to off-shell currents with fewer legs (shown here for the pure gluon case).



Despite the fact that it is constructing the complete set of Feynman diagrams, BG recursion is a very economical scheme.

Comparison of methods for tree graphs

Final State	BG		BCF		CSW	
	CO	CD	CO	CD	CO	CD
$2g$	0.24	0.28	0.28	0.33	0.31	0.26
$3g$	0.45	0.48	0.42	0.51	0.57	0.55
$4g$	1.20	1.04	0.84	1.32	1.63	1.75
$5g$	3.78	2.69	2.59	7.26	5.95	5.96
$6g$	14.2	7.19	11.9	59.1	27.8	30.6
$7g$	58.5	23.7	73.6	646	146	195
$8g$	276	82.1	597	8690	919	1890
$9g$	1450	270	5900	127000	6310	29700
$10g$	7960	864	64000	-	48900	-

Color dressed Berends-Giele approach avoids factorial growth.
The numerical calculation of tree graphs can be considered a solved problem.

Duhr, Hoeche and Maltoni hep-ph/060705

The calculation of one loop amplitudes

- The classical paradigm for the calculation of one-loop diagrams was established in 1979.
- Complete calculation of one-loop scalar integrals
- Reduction of tensors one-loop integrals to scalars.

Nuclear Physics B153 (1979) 365–401

SCALAR ONE-LOOP INTEGRALS

G. 't HOOFT and M. VELTMAN
Institute for Theoretical Physics, University of Utrecht, Netherlands*

Received 16 January 1979

Nuclear Physics B160 (1979) 151–207

ONE-LOOP CORRECTIONS FOR e^+e^- ANNIHILATION INTO $\mu^+\mu^-$ IN THE WEINBERG MODEL

G. PASSARINO* and M. VELTMAN
Institute for Theoretical Physics, University of Utrecht, Utrecht, The Netherlands

Received 22 March 1979

The ingredients are almost the same,
but neither technique will be adequate for present-day purposes.

Techniques for one loop diagrams

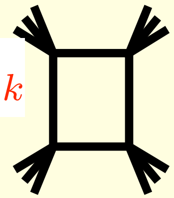
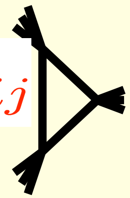
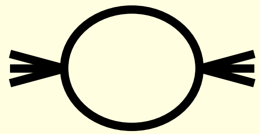
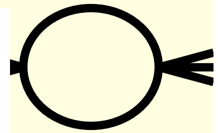
- QCDLoop project: allows one to evaluate numerically an arbitrary one-loop scalar integral,
  (Ellis, Zanderighi, arXiv:0712.1851)
- Unitarity techniques for one-loop amplitudes

Ossola, Papadopoulos, Pittau and

Ellis, Giele, Kunszt arXiv:0708.2398,	4-dimensional method
Giele, Kunszt, Melnikov arXiv:0801.2237,	d-dimensional method
Giele and Zanderighi, arXiv:0805.2152,	Application of d-dimensional method (gluons)
Ellis, Giele, Kunszt, Melnikov, arXiv:0806.3467,	Application with massive fermions (t tbar gg..)
Ellis,Giele,Kunszt,Melnikov,Zanderighi, arXiv:0810.2762	Amplitudes for W+3jets processes.

Basis set of scalar integrals

Any one-loop amplitude can be written as a linear sum of boxes, triangles, bubbles and tadpoles

$\mathcal{A}_N(\{p_i\}) = \sum d_{ijk}$  $+ \sum c_{ij}$  $+ \sum b_i$  $+ \sum_i a_i$ 

In addition, in the context of NLO calculations, scalar higher point functions, can always be expressed as sums of box integrals. Passarino, Veltman - Melrose ('65)

- Scalar hexagon can be written as a sum of six pentagons.
- For the purposes of NLO calculations, the scalar pentagon can be written as a sum of five boxes.
- In addition to the 'tH-V integrals we need integrals containing infrared and collinear divergences.

Scalar one-loop integrals

- ‘t Hooft and Veltman’s integrals contain internal masses; however in QCD many lines are (approximately) massless. The consequent soft and collinear divergences are regulated by dimensional regularization.
- So we need general expressions for boxes, triangles, bubbles and tadpoles, including the cases with one or more vanishing internal masses.

Scalar triangle integrals

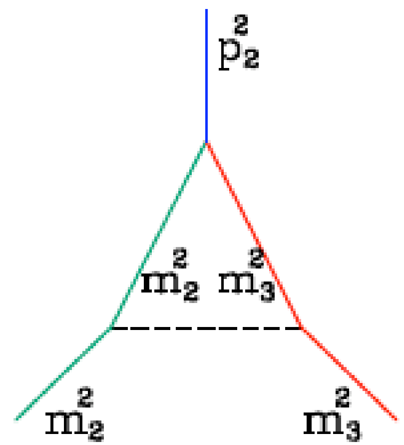
$$I_3^D(p_1^2,p_2^2,p_3^2;m_1^2,m_2^2,m_3^2) = \frac{\mu^{4-D}}{i\pi^{\frac{D}{2}} r_\Gamma}$$
$$\times \int d^D l \, \frac{1}{(l^2-m_1^2+i\varepsilon)((l+q_1)^2-m_2^2+i\varepsilon)((l+q_2)^2-m_3^2+i\varepsilon)}$$

$$I_3^D(p_1^2,p_2^2,p_3^2;m_1^2,m_2^2,m_3^2) = -\frac{\mu^{2\epsilon}\Gamma(1+\epsilon)}{r_\Gamma} \prod_{i=1}^{3} \int_0^1 da_k \, \frac{\delta(1-\sum_k a_k)}{\left[\sum_{i,j} a_i a_j Y_{ij} - i\varepsilon\right]^{1+\epsilon}}$$

Y is the modified Cayley matrix $Y_{ij} \equiv \frac{1}{2}\left[m_i^2 + m_j^2 - (q_{i-1}-q_{j-1})^2\right]$

$$Y_{i+1\,i+1} = Y_{i+1\,i+2} = Y_{i+1\,i} = 0\,, \qquad \text{soft singularity}$$

$$Y_{i\,i} = Y_{i+1\,i+1} = Y_{i\,i+1} = 0\,, \qquad \text{collinear singularity}$$



$$Y_{\text{soft}} = \begin{pmatrix} \dots & 0 & \dots & \dots \\ 0 & 0 & 0 & \dots \\ \dots & 0 & \dots & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}, \quad Y_{\text{collinear}} = \begin{pmatrix} \dots & \dots & \dots & \dots \\ \dots & 0 & 0 & \dots \\ \dots & 0 & 0 & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}$$

Basis set of divergent integrals

By classsifying the integral in terms of the number of zero internal masses, and the number of distinct Cayley matrices we can create a basis set of divergent integrals

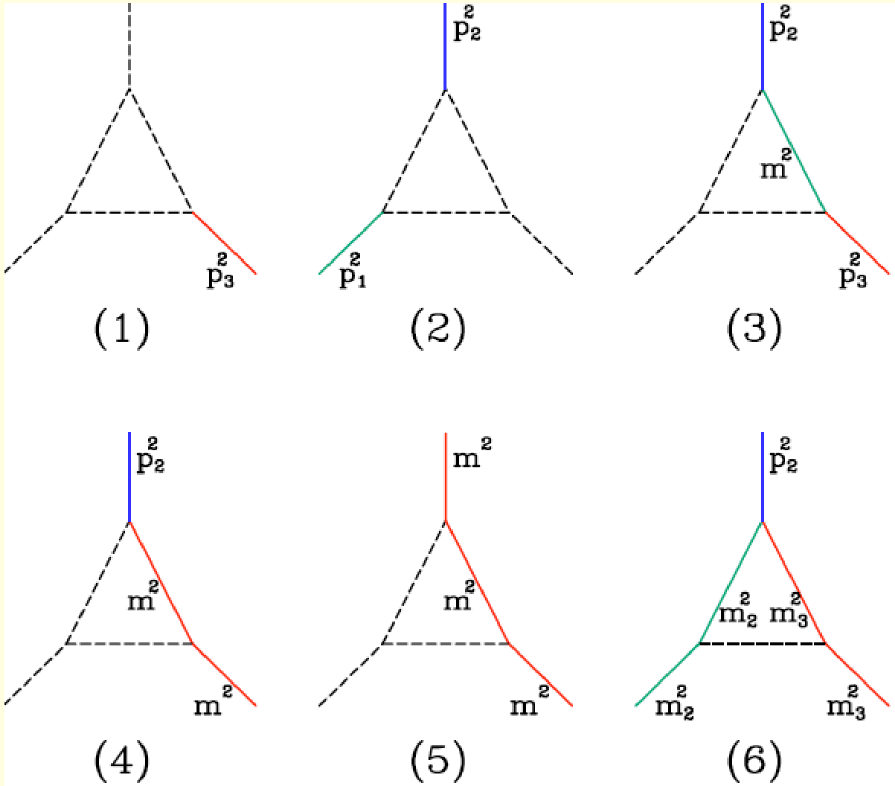
The basis set of divergent triangles contains 6 integrals

$$\begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & 0 \\ \times & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ \times & \times & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}$$

(1) (2) (3)

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & \times \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}$$

(4) (5) (6)



Similarly, the modified Cayley matrices for 16 divergent box integrals

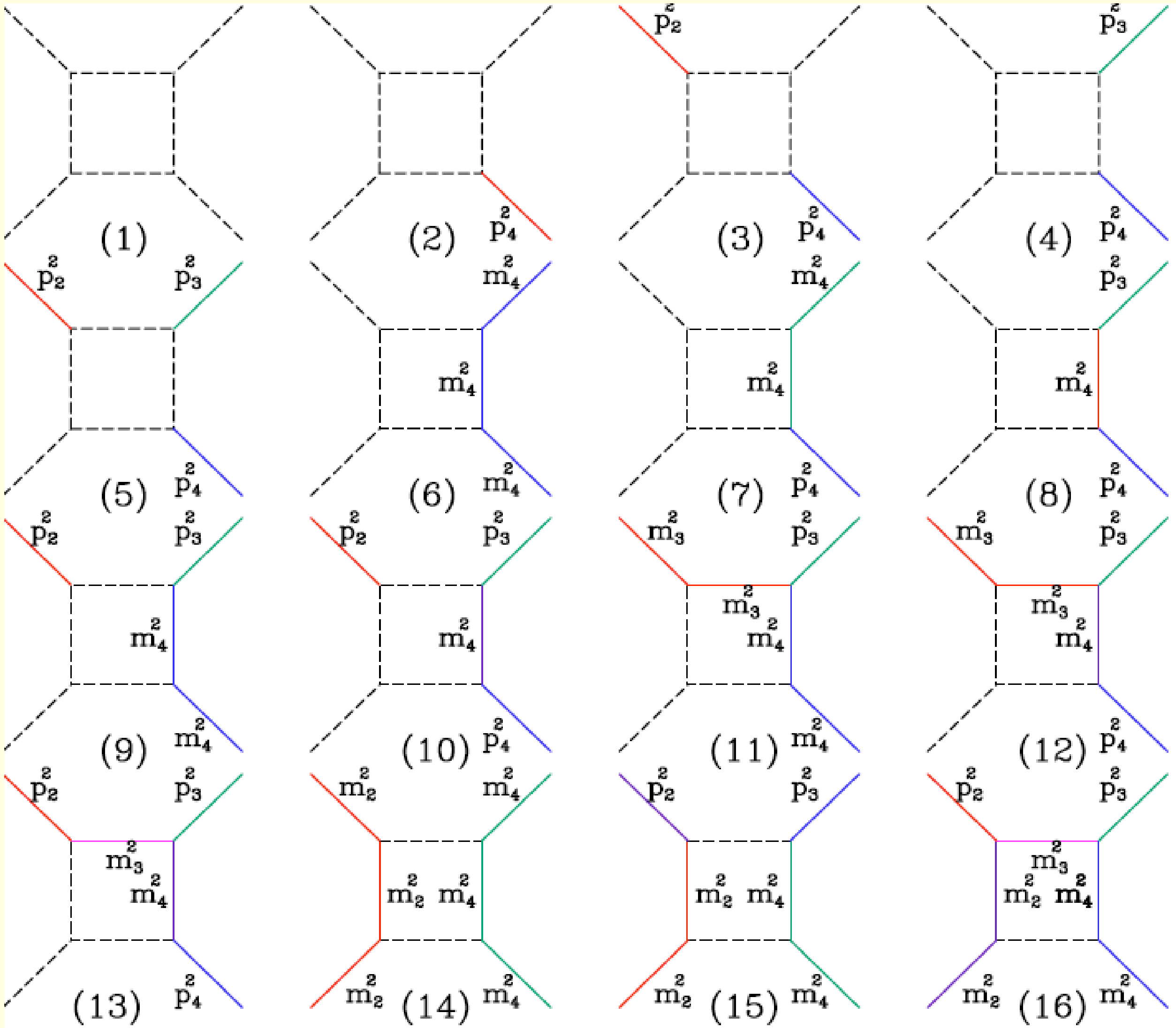
$$\underset{(1)}{\begin{pmatrix} 0 & 0 & \times & 0 \\ 0 & 0 & 0 & \times \\ \times & 0 & 0 & 0 \\ 0 & \times & 0 & 0 \end{pmatrix}} \quad \underset{(2)}{\begin{pmatrix} 0 & 0 & \times & \times \\ 0 & 0 & 0 & \times \\ \times & 0 & 0 & 0 \\ \times & \times & 0 & 0 \end{pmatrix}} \quad \underset{(3)}{\begin{pmatrix} 0 & 0 & \times & \times \\ 0 & 0 & \times & \times \\ \times & \times & 0 & 0 \\ \times & \times & 0 & 0 \end{pmatrix}} \quad \underset{(4)}{\begin{pmatrix} 0 & 0 & \times & \times \\ 0 & 0 & 0 & \times \\ \times & 0 & 0 & \times \\ \times & \times & \times & 0 \end{pmatrix}}$$

$$\underset{(5)}{\begin{pmatrix} 0 & 0 & \times & \times \\ 0 & 0 & \times & \times \\ \times & \times & 0 & \times \\ \times & \times & \times & 0 \end{pmatrix}} \quad \underset{(6)}{\begin{pmatrix} 0 & 0 & \times & 0 \\ 0 & 0 & 0 & \times \\ \times & 0 & 0 & 0 \\ 0 & \times & 0 & \times \end{pmatrix}} \quad \underset{(7)}{\begin{pmatrix} 0 & 0 & \times & \times \\ 0 & 0 & 0 & \times \\ \times & 0 & 0 & 0 \\ \times & \times & 0 & \times \end{pmatrix}} \quad \underset{(8)}{\begin{pmatrix} 0 & 0 & \times & \times \\ 0 & 0 & 0 & \times \\ \times & 0 & 0 & \times \\ \times & \times & \times & \times \end{pmatrix}}$$

$$\underset{(9)}{\begin{pmatrix} 0 & 0 & \times & 0 \\ 0 & 0 & \times & \times \\ \times & \times & 0 & \times \\ 0 & \times & \times & \times \end{pmatrix}} \quad \underset{(10)}{\begin{pmatrix} 0 & 0 & \times & \times \\ 0 & 0 & \times & \times \\ \times & \times & 0 & \times \\ \times & \times & \times & \times \end{pmatrix}} \quad \underset{(11)}{\begin{pmatrix} 0 & 0 & \times & 0 \\ 0 & 0 & 0 & \times \\ \times & 0 & \times & \times \\ 0 & \times & \times & \times \end{pmatrix}} \quad \underset{(12)}{\begin{pmatrix} 0 & 0 & \times & \times \\ 0 & 0 & 0 & \times \\ \times & 0 & \times & \times \\ \times & \times & \times & \times \end{pmatrix}}$$

$$\underset{(13)}{\begin{pmatrix} 0 & 0 & \times & \times \\ 0 & 0 & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \end{pmatrix}} \quad \underset{(14)}{\begin{pmatrix} 0 & 0 & \times & 0 \\ 0 & \times & 0 & \times \\ \times & 0 & 0 & 0 \\ 0 & \times & 0 & \times \end{pmatrix}} \quad \underset{(15)}{\begin{pmatrix} 0 & 0 & \times & 0 \\ 0 & \times & \times & \times \\ \times & \times & 0 & \times \\ 0 & \times & \times & \times \end{pmatrix}} \quad \underset{(16)}{\begin{pmatrix} 0 & 0 & \times & 0 \\ 0 & \times & \times & \times \\ \times & \times & \times & \times \\ 0 & \times & \times & \times \end{pmatrix}}$$

The basis set of box integrals contains 16 integrals.



QCDLoop.fnal.gov

QCDLoop web page giving access to hyper-linked PDF web-pages which give the results for the basis integrals, together with references, special cases etc.

11 of the 16 divergent box integrals were known in the literature. The rest are new.

QCDloop: A repository for one-loop scalar integrals

This is a repository of one-loop scalar Feynman integrals, evaluated close to four dimensions. For integrals with all massive internal lines the integrals are all known, both analytically and numerically. This website therefore concentrates on integrals with some internal masses vanishing; in general, these integrals contain infra-red and collinear singularities which are here regulated dimensionally. The integrals are described in a PDF file for every known integral. The browser must be set to use hypertex-aware tool, such as Acrobat reader, and for best viewing, should open the pdf files in the browser. For general notation for the loop integrals click here

- Box integrals definitions and generalities
  - Basis set of divergent box integrals
  - Index of all box integrals currently in the repository
- Triangle integrals
  - Divergent triangle integrals
  - Finite triangle integrals
- Bubble integrals
- Tadpole integral

The results in this web-site are also available in the paper arXiv:0712.1851v1 by R.K. Ellis and G. Zanderighi
The corresponding fortran 77 code which calculates an arbitrary one-loop scalar integral, finite or divergent can be downloaded,
QCDLoop-1.3.tar.gz (version 1.3, date 2008-Feb-21). If you encounter any problems with the code, please notify the authors.

Other associated tools for one-loop diagrams:-
Looptools
the FF package by G.J. van Oldenborgh

R. Keith Ellis
Last modified: Thu Feb 21 06:39:34 CST 2008

Fortran code

Fortran code is available. It calculates finite integrals using the ff library, and calculates divergent integrals using the QCDLoop library.

```
=====================================================
 This is QCDLoop - version 1.3
 Authors: Keith Ellis and Giulia Zanderighi
 (ellis@fnal.gov, g.zanderighi1@physics.ox.ac.uk)
 For details see FERMILAB-PUB-07-633-T,OUTP-07/16P
 arXiv:0712.1851 [hep-ph]
=====================================================
=====================================================
 FF 2.0, a package to evaluate one-loop integrals
written by G. J. van Oldenborgh, NIKHEF-H, Amsterdam
=====================================================
for the algorithms used see preprint NIKHEF-H 89/17,
'New Algorithms for One-loop Integrals', by G.J. van
Oldenborgh and J.A.M. Vermaseren, published in
Zeitschrift fuer Physik C46(1990)425.
=====================================================
ffini: precx =   4.4408921E-16
ffini: precc =   4.4408921E-16
ffini: xalogm =   4.94065646E-324
ffini: xclogm =   4.94065646E-324

p1sq,p2sq,p3sq,p4sq,s12,s23,m1sq,m2sq,m3sq,m4sq,musq  1.5 -1.7  2.3  2.9  37.
-15.7  3.  5.  9.  1.3  1.1

ep,qlI4(p1sq,p2sq,p3sq,p4sq,s12,s23,m1sq,m2sq,m3sq,m4sq,musq,ep) -2 (0.,0.)
ep,qlI4(p1sq,p2sq,p3sq,p4sq,s12,s23,m1sq,m2sq,m3sq,m4sq,musq,ep) -1 (0.,0.)
ep,qlI4(p1sq,p2sq,p3sq,p4sq,s12,s23,m1sq,m2sq,m3sq,m4sq,musq,ep) 0
(0.00404258529,0.0176915481)

test of divergent boxes
ep,qlI4(0d0,0d0,p3sq,p4sq,s12,s23,0d0,0d0,0d0,m4sq,musq,ep) -2
(-0.00158982512,0.)
ep,qlI4(0d0,0d0,p3sq,p4sq,s12,s23,0d0,0d0,0d0,m4sq,musq,ep) -1
(0.0138506054,0.00499458292)
ep,qlI4(0d0,0d0,p3sq,p4sq,s12,s23,0d0,0d0,0d0,m4sq,musq,ep) 0
(-0.043279838,0.0270258376)
```

Numerical checks

We can perform a numerical check of the code, by using the relation between boxes, triangles and the six-dimensional box.

$$I_4^D = \frac{1}{2}\left(\sum_{i=1}^{4} c_i I_3^D[i] + (3-D)c_0 I_4^{D+2}\right)$$

$$c_i = \sum_{j=1}^{4}(Y^{-1})_{ij}\,, \quad c_0 = \sum_{i=1}^{4} c_i\,.$$

In D=6, the box integral is finite - no UV,IR or collinear divergences So we can check this relation numerically, (including in the physical region by setting the causal ε equal to a very small number.

$$I_4^D(p_1^2,p_2^2,p_3^2,p_4^2;s_{12},s_{23};m_1^2,m_2^2,m_3^2,m_4^2) = \frac{\mu^{2\epsilon}\Gamma(2+\epsilon)}{r_\Gamma}\prod_{i=1}^{4}\int_0^1 da_k\,\frac{\delta(1-\sum_k a_k)}{\left[\sum_{i,j} a_i a_j Y_{ij} - i\varepsilon\right]^{2+\epsilon}}$$

Divergent Box Integral 10: $I_4^{\{D=4-2\epsilon\}}(0,p_2^2,p_3^2,p_4^2;s_{12},s_{23};0,0,0,m^2)$

Page contributed by R.K. Ellis

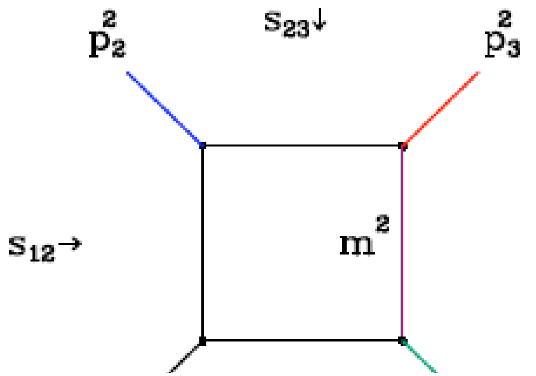
The result for this box (see figure) is

$$I_4^{\{D=4-2\epsilon\}}(0,p_2^2,p_3^2,p_4^2;s_{12},s_{23};0,0,0,m^2) = \frac{1}{(s_{12}s_{23}-m^2s_{12}-p_2^2p_4^2+m^2p_2^2)}$$
$$\times \left[\frac{1}{\epsilon}\ln\left(\frac{(m^2-p_4^2)p_2^2}{(m^2-s_{23})s_{12}}\right)+\mathrm{Li}_2\left(1+\frac{(m^2-p_3^2)(m^2-s_{23})}{p_2^2m^2}\right)-\mathrm{Li}_2\left(1+\frac{(m^2-p_3^2)(m^2-p_4^2)}{s_{12}m^2}\right)\right.$$
$$+\ 2\,\mathrm{Li}_2\left(1-\frac{m^2-s_{23}}{m^2-p_4^2}\right)-2\,\mathrm{Li}_2\left(1-\frac{p_2^2}{s_{12}}\right)+2\,\mathrm{Li}_2\left(1-\frac{p_2^2(m^2-p_4^2)}{s_{12}(m^2-s_{23})}\right)$$
$$\left.+\ 2\ln\left(\frac{\mu m}{m^2-s_{23}}\right)\ln\left(\frac{(m^2-p_4^2)p_2^2}{(m^2-s_{23})s_{12}}\right)\right]+\mathcal{O}(\epsilon)$$

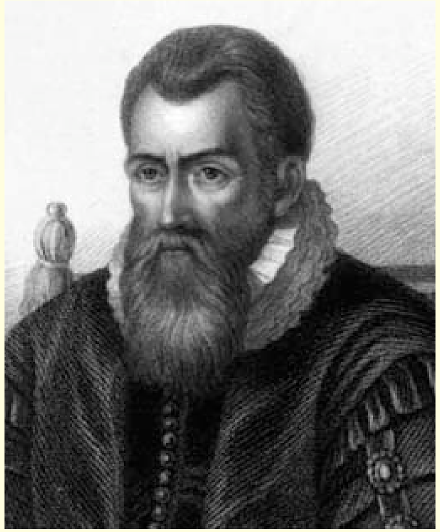
See the file on notation.

References

[1] R. K. Ellis and G. Zanderighi, "Scalar one-loop integrals for QCD," arXiv:0712.1851 [hep-ph]



Scottish functions.



- We have expressed the one-loop integrals entirely in Scottish functions, logarithms (Napier) and dilogarithms (Spence).
- Shown are John Napier, 1550-1617, laird of Merchiston, inventor of the Napierian logarithm and William Spence of Greenock (1777-1815) author of "Essay on logarithmic transcendents"

Algebraic reduction, subtraction terms

- Ossola, Papadopoulos and Pittau showed that there is a systematic way of calculating the subtraction terms at the integrand level.
- We can re-express the rational function in an expansion over 4,3,2, and 1 propagator terms.
- The residues of these pole terms contain the l-independent master integral coefficients plus a finite number of spurious terms, which vanish after integration.

$$\mathcal{A}_N(p_1, p_2, \ldots, p_N | l) = \frac{\mathcal{N}(p_1, p_2, \ldots, p_N; l)}{d_1 d_2 \cdots d_N}$$

$$= \sum_{1 \le i_1 < i_2 < i_3 < i_4 \le N} \frac{\bar{d}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{1 \le i_1 < i_2 < i_3 \le N} \frac{\bar{c}_{i_1 i_2 i_3}(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{1 \le i_1 < i_2 \le N} \frac{\bar{b}_{i_1 i_2}(l)}{d_{i_1} d_{i_2}} + \sum_{1 \le i_1 \le N} \frac{\bar{a}_{i_1}(l)}{d_{i_1}}$$

Decomposing in terms of boxes, triangles,…

$$\begin{aligned}\mathcal{A}_N(p_1, p_2, \ldots, p_N) &= \sum_{1\leq i_1<i_2<i_3<i_4\leq N} d_{i_1 i_2 i_3 i_4}(p_1, p_2, \ldots, p_N) I_{i_1 i_2 i_3 i_4} \\ &+ \sum_{1\leq i_1<i_2<i_3\leq N} c_{i_1 i_2 i_3}(p_1, p_2, \ldots, p_N) I_{i_1 i_2 i_3} \\ &+ \sum_{1\leq i_1<i_2\leq N} b_{i_1 i_2}(p_1, p_2, \ldots, p_N) I_{i_1 i_2} \\ &+ \sum_{1\leq i_1\leq N} a_{i_1}(p_1, p_2, \ldots, p_N) I_{i_1} \ ,\end{aligned}$$

$$I_{i_1 \cdots i_M} = \int [d\,l] \, \frac{1}{d_{i_1} \cdots d_{i_M}}$$

- Without the integral sign, the identification of the coefficients is straightforward.
- Determine the coefficients of a multipole rational function.

van Neerven-Vermaseren basis

Example: solve for the box coefficients by setting $d_i = d_j = d_k = d_l = 0$

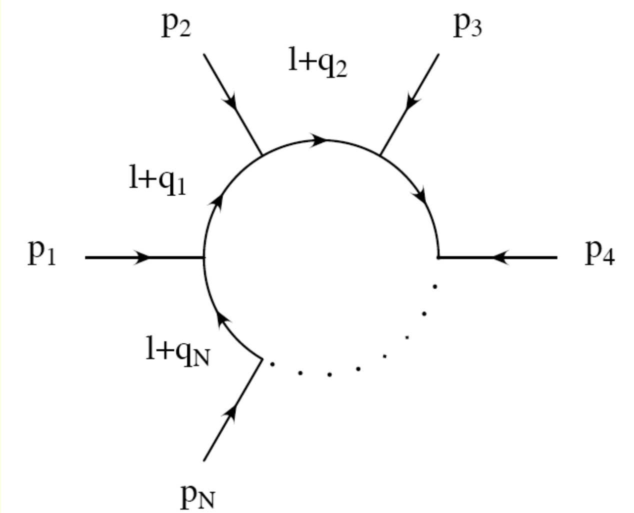
We find two complex solutions $l_\pm^\mu = V_4^\mu \pm i\sqrt{V_4^2 - m_l^2} \times n_1^\mu$

$$V_4^\mu = -\frac{1}{2}(q_i^2 - m_i^2 + m_l^2)\, v_1^\mu - \frac{1}{2}(q_j^2 - q_i^2 - m_j^2 + m_i^2)\, v_2^\mu - \frac{1}{2}(q_k^2 - q_j^2 - m_k^2 + m_j^2)\, v_3^\mu$$

$$v_1^\mu = \frac{\delta^{\mu p_2 p_3}_{p_1 p_2 p_3}}{\Delta(p_1,p_2,p_3)};\ v_2^\mu = \frac{\delta^{p_1 \mu p_3}_{p_1 p_2 p_3}}{\Delta(p_1,p_2,p_3)};\ v_3^\mu = \frac{\delta^{p_1 p_2 \mu}_{p_1 p_2 p_3}}{\Delta(p_1,p_2,p_3)};\ n_1^\mu = \frac{\varepsilon^{\mu p_1 p_2 p_3}}{\sqrt{\Delta(p_1,p_2,p_3)}}$$

$$v_i \cdot p_j = \delta_{ij};\ n_1 \cdot v_i = 0$$

p is external momentum,
q is the denominator
offset momentum



Systematic algebraic reduction at the integrand level

Box residues 2, triangle residues 7, bubble residues 9 structures:

18 l-independent parameters, vanishing (spurious) integrals

Parameterization of the loop momentum

The loop momenta can be decomposed in terms of a set of basis vectors, determined by the external vectors in the problem.

we use: **dual momenta v_i** $p_i v_j = \delta_{ij}$

and orthogonal **unit vectors n_i**

Decomposition of the loop momentum

$$l^\mu = V_R^\mu + \sum_{i=1}^{D_P} \frac{1}{2}(d_i - d_{i-1})\, v_i^\mu + \sum_{i=1}^{D_T} \alpha_i\, n_i^\mu \ ,$$

$$V_R^\mu = -\frac{1}{2} \sum_{i=1}^{D_P} \left((q_i^2 - m_i^2) - (q_{i-1}^2 - m_{i-1}^2) \right) v_i^\mu$$

Solving the unitarity conditions

Contributions with four cut propagators $d_i = d_j = d_k = d_l = 0$ two solutions

$$l^\mu = V_4^\mu + \alpha_1\, n_1^\mu$$

$$l_\pm^\mu = V_4^\mu \pm i\,\sqrt{V_4^2 - m_l^2} \times n_1^\mu$$

Complex valued loop momenta

Triangle, infinite # of solutions (on a circle)

Bubble, infinite # of solutions (on a “sphere”)

The coefficients are obtained by solving algebraic equations

The residue is taken at special loop momentum defined by the unitarity conditions.

$$\overline{d}_{ijkl}(l) = \mathsf{Res}_{ijkl}\Big(\mathcal{A}_N(l)\Big)$$

$d_i=d_j=d_k=d_l=0$ two solutions

$d_i=d_j=d_k=0$

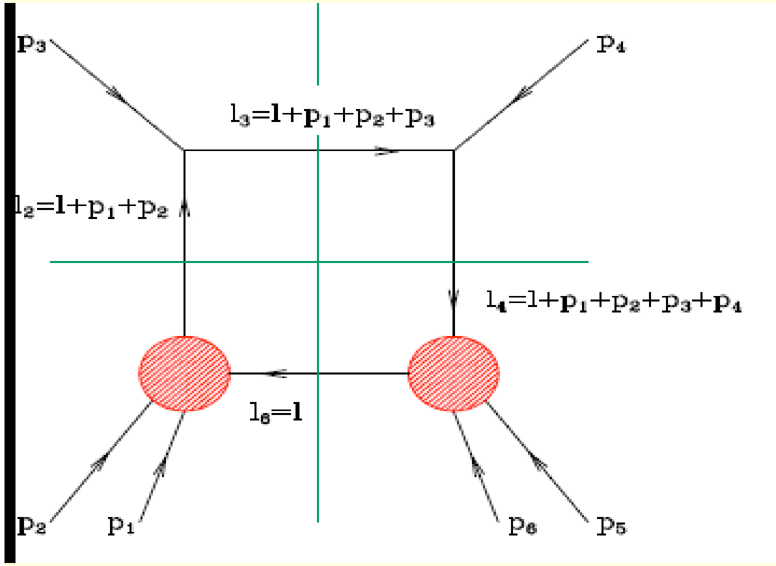
infinite solutions

$d_i=d_j=0$

The residue of the amplitudes factorize to the product of tree amplitudes

The box residue

$\times \mathcal{M}_4^{(0)}(l_4^{\pm}; p_5, p_6; -l_6^{\pm})$ = $\bar{d}_{ijkl}(l) = d_{ijkl} + \tilde{d}_{ijkl}\, l \cdot n_1$



The residues of the poles = sum over factorized tree amplitudes

Numerical Implementation

Check the singular parts:

Compare CPU time with those of the traditional method in case of 6g, 5g ,… amplitudes

EGZ: 9s per ordered amplitude on 2.8GHz Pentium processor
EGK: 0.01s per ordered amplitude on 2.8GHz Pentium processor

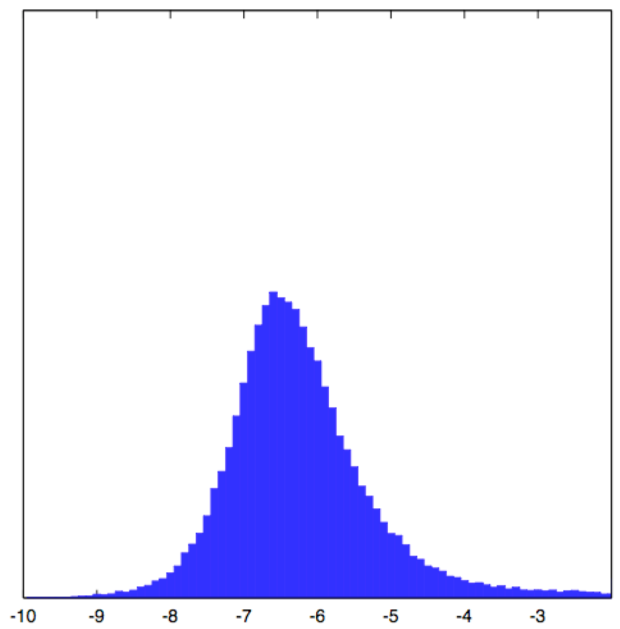
	ev.time	# of cuts
4 gluon:	0.0009s	6
5 gluon:	0.0035s	20
6 gluon :	0.0107s	44

Computer time: scales with _ n^4 (# of cuts) not as n!

Ellis, Giele, Kunszt

Result for six gluon amplitude

- Results shown here for the cut-constructible part
- The relative error for the finite part of the 6-gluon amplitude compared to the analytic result, for the (+ + - - - -) helicity choice. The horizontal axis is the log of the relative error, the vertical axis is the number of events in arbitrary linear units.
- For most events the error is less than 10^{-6}, although there is a tail extending to higher error.



Giele, Kunszt,Melnikov, arXiv 0801.2237

Numerical Unitarity Method in D-dimension for gluon amplitudes

Two sources of D-dependence

i) spin-polarization states live in D_s .

ii) loop momentum component live in D. (D_s>D)

$$\mathcal{A}_{(D,D_s)}(\{p_i\},\{J_i\}) = \int \frac{d^D l}{i(\pi)^{D/2}} \frac{\mathcal{N}^{(D_s)}(\{p_i\},\{J_i\};l)}{d_1 d_2 \cdots d_N}.$$

$$\sum_{i=1}^{D_s-2} e_\mu^{(i)}(l) e_\nu^{(i)}(l) = -g_{\mu\nu}^{(D_s)} + \frac{l_\mu b_\nu + b_\mu l_\nu}{l \cdot b},$$

$$l^2 = \bar{l}^2 - \tilde{l}^2 = l_1^2 - l_2^2 - l_3^2 - l_4^2 - \sum_{i=5}^{D} l_i^2$$

Two key features

Dependence on Ds is linear

$$\mathcal{N}^{(D_s)}(l) = \mathcal{N}_0(l) + (D_s - 4)\mathcal{N}_1(l)$$

- **Choose two integer values $D_s = D_1$ and $D_s = D_2$ to reconstruct the full D_s dependence.**
- **Suitable for numerical implementation.**
- **$D_s=4-2\varepsilon$ ‘t Hooft Veltman scheme, $D_s=4$ FDHS**

The loop momentum effectively has only 4+1 component

$$N(l) = N(l_4\,,\ \mu^2) \qquad \mu^2 = -l_5^2 - \ldots - l_D^2$$

maximum 5 constraints: we need to consider also pentagon cuts.

Reduction in D-dimensions

The parametrization of the N-particle amplitude

$$\frac{\mathcal{N}^{(D_s)}(l)}{d_1 d_2 \cdots d_N} = \sum_{[i_1|i_5]} \frac{\bar{e}^{(D_s)}_{i_1 i_2 i_3 i_4 i_5}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4} d_{i_5}} + \sum_{[i_1|i_4]} \frac{\bar{d}^{(D_s)}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}}$$

$$+ \sum_{[i_1|i_3]} \frac{\bar{c}^{(D_s)}_{i_1 i_2 i_3}(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{[i_1|i_2]} \frac{\bar{b}^{(D_s)}_{i_1 i_2}(l)}{d_{i_1} d_{i_2}} + \sum_{[i_1|i_1]} \frac{\bar{a}^{(D_s)}_{i_1}(l)}{d_{i_1}}$$

Parametrization of the residues

Pentuple residue:

$$\bar{e}_{ijklm}(l) = e^{(D_s,(0))}_{ijklm}$$

Box residue:

$$\bar{d}^{\mathrm{FDH}}_{ijkn}(l) = d^{(0)}_{ijkn} + d^{(1)}_{ijkn} s_1 + (d^{(2)}_{ijkn} + d^{(3)}_{ijkn} s_1) s_e^2 + d^{(4)}_{ijkn} s_e^4$$

$$s_1 = n_1 \cdot l; \; s_\epsilon^2 = -\sum_{i=5}^{D} (n_i \cdot l)^2$$

Three extra structures for triple, three for double and zero for single cuts, only even powers of

Four new master integrals

Four of the s_e^2 dependent master integrals are not spurious (ie they give a contribution)

$$\int \frac{\mathrm{d}^D l}{(i\pi)^{D/2}} \frac{s_e^2}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} = -\frac{D-4}{2} I^{D+2}_{i_1 i_2 i_3 i_4}, \quad \ldots.$$

We obtain the full D-dependence of the amplitude

$$\mathcal{A}_{(D)} = \sum_{[i_1|i_5]} e^{(0)}_{i_1 i_2 i_3 i_4 i_5} \; I^{(D)}_{i_1 i_2 i_3 i_4 i_5}$$

+ similar terms for triangle, bubble and tadpole contributions.

As ε -> 0 the new master integrals can be decomposed in the old basis and generate ε dependent bubble coefficients !

One-loop amplitudes up to terms of order ε

One loop amplitudes as sum of cut-constructible and rational parts:

$$\mathcal{A}_N = \mathcal{A}_N^{CC} + R_N.$$

The cut constructible part is as before (EGK):

The rational part is new (GKM):

Giele, Kunszt,Melnikov, arXiv 0801.2237

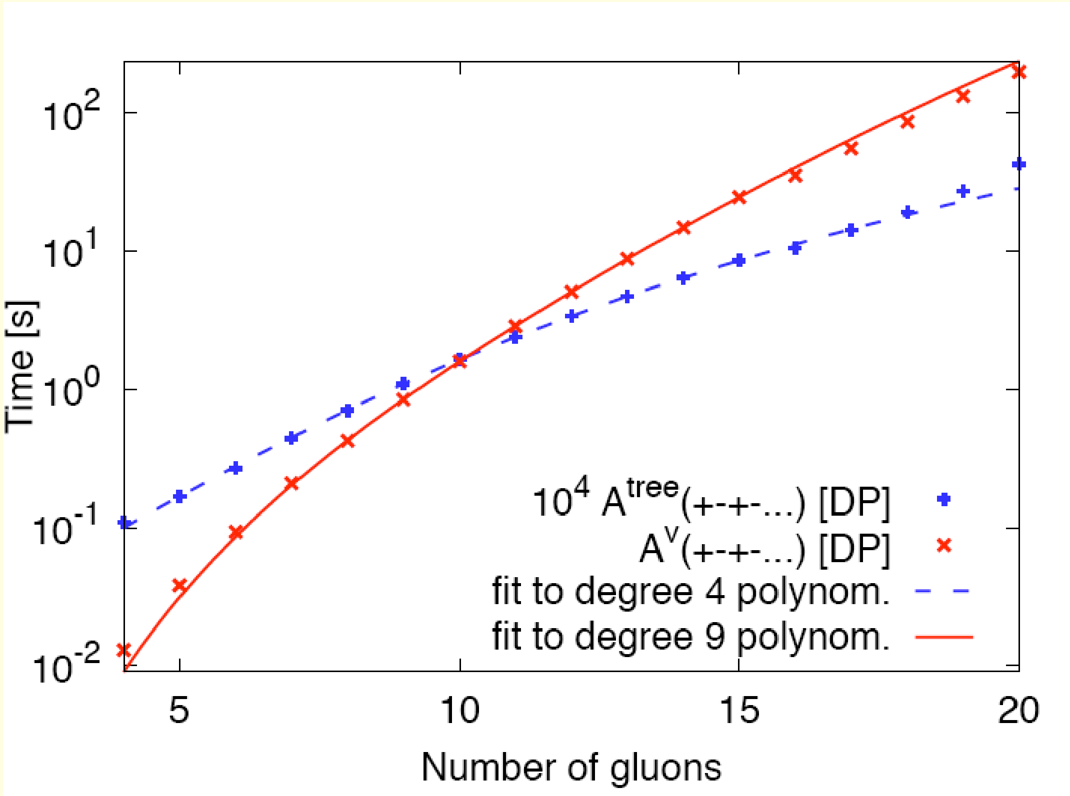
Results for full amplitude

- Keep dimensions of virtual unobserved particles integer and perform calculations in more than one dimension.
- Arrive at non-integer values D=4-2ε by linear interpolation.
- Results for six-gluon amplitudes agree with original Feynman diagram calculation of RKE, Giele, Zanderighi.

$\lambda_1, \lambda_2, \ldots, \lambda_6$	$\Delta^{\rm cut}$	$\Delta^{\rm rat}$	Δ
- - + + + +	-19.481065+78.147162 i	28.508591-74.507275 i	9.027526+3.639887 i
- + - + + +	-241.10930+27.176200 i	250.27357-25.695269 i	9.164272+1.480930 i
- + + - + +	5.4801516-12.433657 i	0.19703574+0.25452928 i	5.677187-12.179127 i
- - - + + +	15.478408-2.7380153 i	2.2486654+1.0766607 i	17.727073-1.661354 i
- - + - + +	-339.15056-328.58047 i	348.65907+336.44983 i	9.508509+7.869351 i
- + - + - +	31.947346+507.44665 i	-17.430910-510.42171 i	14.516436-2.975062 i

Scaling property of tree and loop amplitudes

Results for one-loop 20 gluon amplitude!



Giele and Zanderighi arXiv:0805.2152

W+3 jet production

We need to calculate two amplitudes

$$0 \to \bar{u} + d + g + g + g + W^+,$$
$$0 \to \bar{u} + d + \bar{Q} + Q + g + W^+,$$

Process 1) 1203 + 104 Feynman diagrams
Process 2) 258 + 18 Feynman diagrams

New ! Ellis,Giele,Kunszt,Melnikov,Zanderighi, ArXiv:0810.2762

Color decomposition for W+multi-gluon amplitude

Tree amplitude for qbar q g g ….g W can be written as

$$\mathcal{A}_n^{\text{tree}}(1_{\bar{q}}, 2_q, 3_g, \ldots, n_g) \;=\; g^{n-2} \sum_{\sigma \in S_{n-2}} (T^{a_{\sigma(3)}} \ldots T^{a_{\sigma(n)}})_{i_2}^{\;\bar{\imath}_1} \; A_n^{\text{tree}}(1_{\bar{q}}, 2_q; \sigma(3)_g, \ldots, \sigma(n)_g)$$

At one loop the amplitude is decomposed into primitive amplitudes

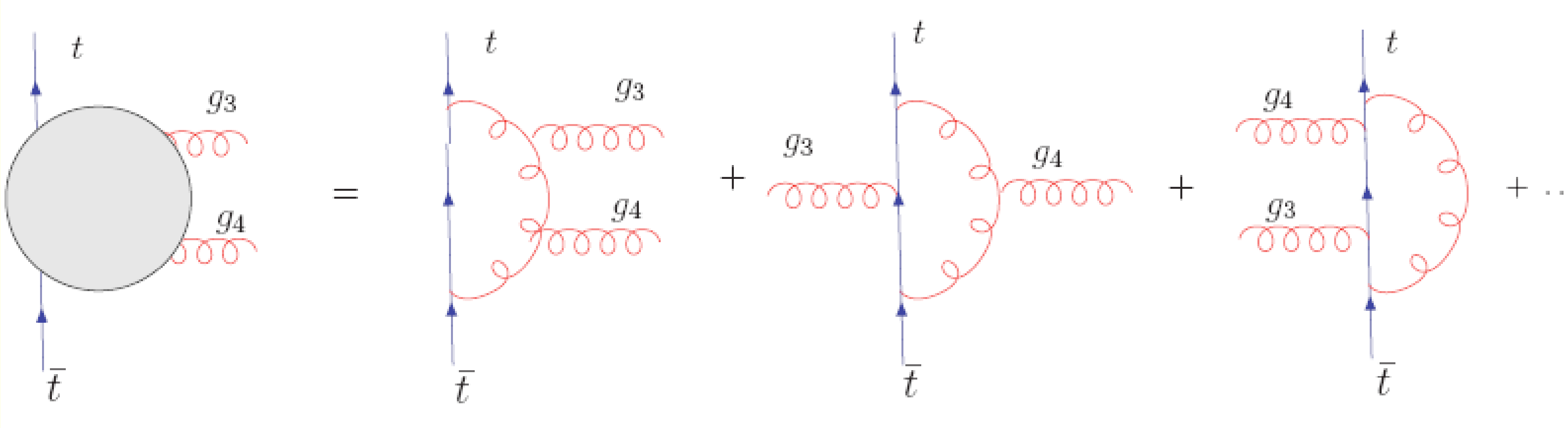
$$\begin{aligned}
\mathcal{A}_n^{\text{1-loop}}(1_{\bar{q}}, 2_q, 3_g, \ldots, n_g) = g^n \Bigg[& \sum_{p=2}^{n} \sum_{\sigma \in S_{n-2}} (T^{x_2} T^{a_{\sigma_3}} \cdots T^{a_{\sigma_p}} T^{x_1})_{i_2}^{\;\bar{\imath}_1} (F^{a_{\sigma_{p+1}}} \cdots F^{a_{\sigma_n}})_{x_1 x_2} \\
& \times (-1)^n A_n^L(1_{\bar{q}}, \sigma(p)_g, \ldots, \sigma(3)_g, 2_q, \sigma(n)_g, \ldots, \sigma(p+1)_g) \qquad (4.4) \\
& + \frac{n_f}{N_c} \sum_{j=1}^{n-1} \sum_{\sigma \in S_{n-2}/S_{n;j}} \mathrm{Gr}_{n;j}^{(\bar{q}q)}(\sigma_3 \ldots, \sigma_n) A_{n;j}^{[1/2]}(1_{\bar{q}}, 2_q; \sigma(3)_g, \ldots, \sigma(n)_g) \Bigg],
\end{aligned}$$

Full information about the (gauge invariant) primitives specifies 1-loop amplitude. Only one highest level N-point function per primitive.

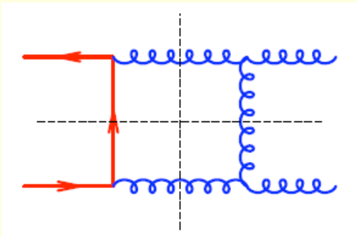
Primitive amplitudes

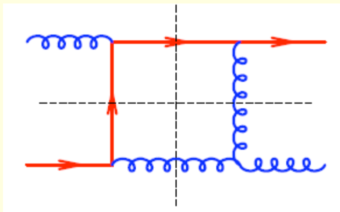
Bern, Dixon, Kosower (1994)

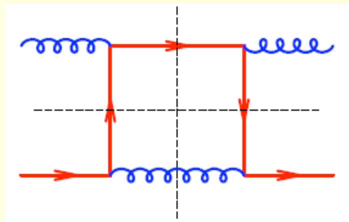
Special role: the flavor of the cut lines are uniquely defined



Three distinct quadruple cuts → three gauge invariant primitive amplitudes







qbar q Qbar Q g W amplitude

Color decomposition at tree graph level

$$\mathcal{B}^{\text{1-loop}}(1_{\bar{q}}, 2_q, 3_{\bar{Q}}, 4_Q, 5_g) = g^5 \Bigg[N_c (T^{a_5})_{i_4}^{\;\bar{\imath}_1} \delta_{i_2}^{\;\bar{\imath}_3} B_{7;1} + (T^{a_5})_{i_2}^{\;\bar{\imath}_1} \delta_{i_4}^{\;\bar{\imath}_3} B_{7;2}$$

$$+ N_c (T^{a_5})_{i_2}^{\;\bar{\imath}_3} \delta_{i_4}^{\;\bar{\imath}_1} B_{7;3} + (T^{a_5})_{i_4}^{\;\bar{\imath}_3} \delta_{i_2}^{\;\bar{\imath}_1} B_{7;4} \Bigg].$$

Similar at one-loop level

$$\mathcal{B}^{\text{tree}}(1_{\bar{q}}, 2_q, 3_{\bar{Q}}, 4_Q, 5_g) = g^3 \Bigg[(T^{a_5})_{i_4}^{\;\bar{\imath}_1} \delta_{i_2}^{\;\bar{\imath}_3} B_{7;1}^{\text{tree}} + \frac{1}{N_c} (T^{a_5})_{i_2}^{\;\bar{\imath}_1} \delta_{i_4}^{\;\bar{\imath}_3} B_{7;2}^{\text{tree}}$$

$$+ (T^{a_5})_{i_2}^{\;\bar{\imath}_3} \delta_{i_4}^{\;\bar{\imath}_1} B_{7;3}^{\text{tree}} + \frac{1}{N_c} (T^{a_5})_{i_4}^{\;\bar{\imath}_3} \delta_{i_2}^{\;\bar{\imath}_1} B_{7;4}^{\text{tree}} \Bigg],$$

Numerical results for primitive amplitudes

Helicity	$1/\epsilon^2$	$1/\epsilon$	ϵ^0
$A^{\text{tree}}(1_{\bar{q}}^+\, 2_q^-\, 3_g^+\, 4_g^+\, 5_g^+\, 6_{\bar{l}}^+\, 7_l^-)$			$-0.006873 + i\,0.011728$
$r_L^{[1]}(1_{\bar{q}}^+\, 2_q^-\, 3_g^+\, 4_g^+\, 5_g^+\, 6_{\bar{l}}^+\, 7_l^-)$	-4.00000	$-10.439578 - i\,9.424778$	$5.993700 - i\,19.646278$
$A^{\text{tree}}(1_{\bar{q}}^+\, 2_q^-\, 3_g^+\, 4_g^+\, 5_g^-\, 6_{\bar{l}}^+\, 7_l^-)$			$0.010248 - i\,0.007726$
$r_L^{[1]}(1_{\bar{q}}^+\, 2_q^-\, 3_g^+\, 4_g^+\, 5_g^-\, 6_{\bar{l}}^+\, 7_l^-)$	-4.00000	$-10.439578 - i\,9.424778$	$-14.377555 - i\,37.219716$
$A^{\text{tree}}(1_{\bar{q}}^+\, 2_q^-\, 3_g^-\, 4_g^+\, 5_g^+\, 6_{\bar{l}}^+\, 7_l^-)$			$0.495774 - i\,1.274796$
$r_L^{[1]}(1_{\bar{q}}^+\, 2_q^-\, 3_g^-\, 4_g^+\, 5_g^+\, 6_{\bar{l}}^+\, 7_l^-)$	-4.00000	$-10.439578 - i\,9.424778$	$-1.039489 - i\,30.210418$
$A^{\text{tree}}(1_{\bar{q}}^+\, 2_q^-\, 3_g^-\, 4_g^+\, 5_g^-\, 6_{\bar{l}}^+\, 7_l^-)$			$-0.294256 - i\,0.223277$
$r_L^{[1]}(1_{\bar{q}}^+\, 2_q^-\, 3_g^-\, 4_g^+\, 5_g^-\, 6_{\bar{l}}^+\, 7_l^-)$	-4.00000	$-10.439578 - i\,9.424778$	$-1.444709 - i\,26.101951$

Results for all primitives in our paper,

$$r_L^{[j]}(1,2,3,4,5,6,7) = \frac{1}{c_\Gamma}\frac{A_L^{[j]}(1,2,3,4,5,6,7)}{A^{\text{tree}}(1,2,3,4,5,6,7)}, \quad c_\Gamma = \frac{\Gamma(1+\epsilon)\Gamma(1-\epsilon)^2}{(4\pi)^{2-\epsilon}\Gamma(1-2\epsilon)},$$

Timing and accuracy

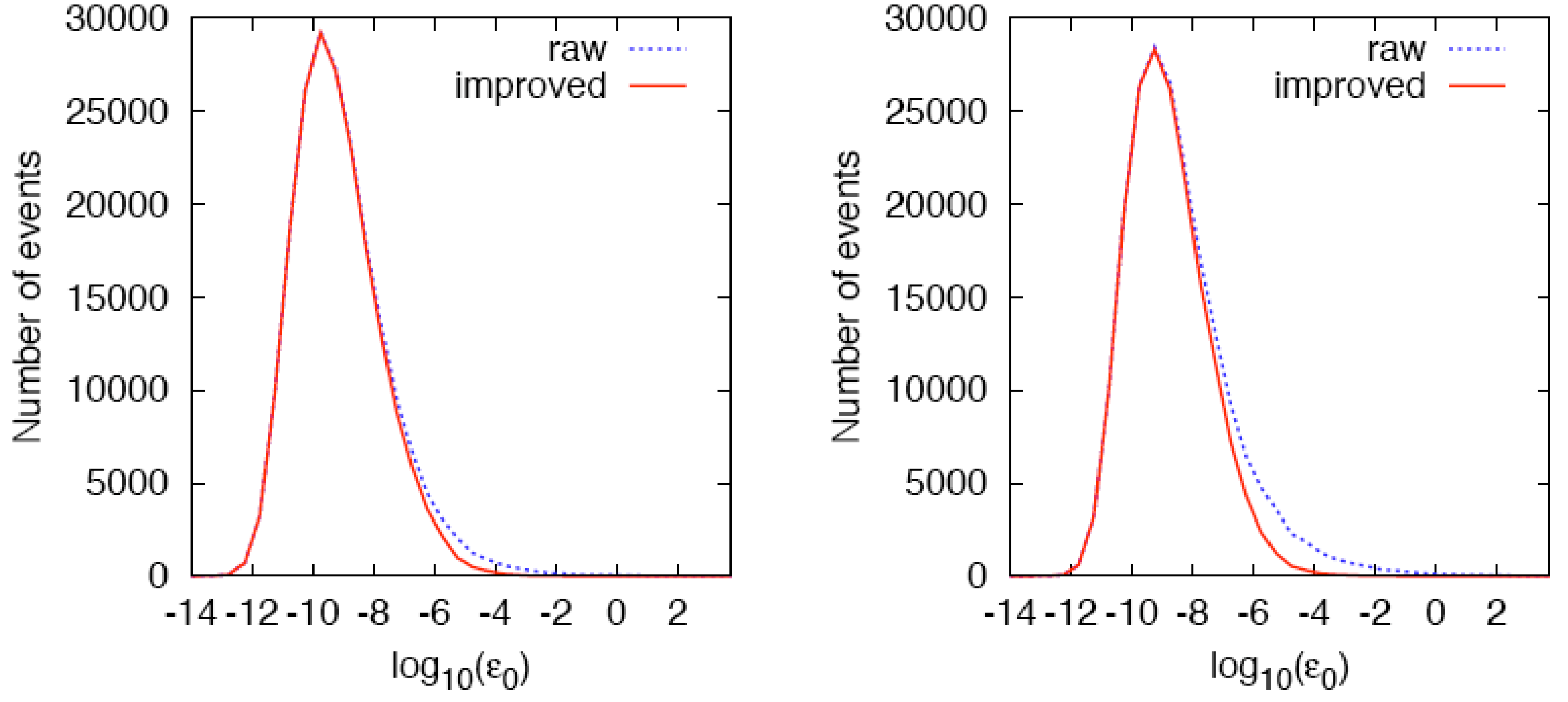


Figure 2: Accuracy for $A_5^L(1_{\bar{q}}^+, 2_q^-, 3_g^-, 4_g^+, 5_g^-)$ (left panel) and for $A_5^L(1_{\bar{q}}^+, 3_g^-, 4_g^+, 5_g^-, 2_q^-)$ (right

45-50 ms for most leading color, 160 for most subleading, 2.33GHz Pentium processor

Rocket science!

Eruca sativa =Rocket=roquette=arugula=rucola

Recursive unitarity calculation of one-loop amplitudes

On a more general side, the current version of Rocket computes one-loop amplitudes for processes $0 \to\ n$ gluons, $0 \to \bar{q}q + n$ gluons, $0 \to \bar{q}qW + n$ gluons and $0 \to \bar{q}q\bar{Q}QW + 1$ gluon. It is straightforward to extend the program to include similar processes with the Z boson and processes with massive quarks $0 \to \bar{t}t + n$ gluons. This list is a testimony to the power of the method and indicates that the development of automated programs for one-loop calculations may finally be within reach.

But it still must be tested in battle conditions, ie a real physical process

Summary

- MCFM appears to describe untagged W/Z+1 jet and W/Z+2 jet data well.
- Even at the Tevatron the known results on multi-leg processes are inadequate.
- Calculation of one-loop scalar integrals is now complete: QCDLoop
- There is much theoretical effort on the calculation of one-loop multi-leg diagrams.
- D-dimensional unitarity techniques can calculate full one-loop amplitudes (ie both cut-constructible and rational parts).
- But so far, practical calculations have used either a) analytic results, b) PV reduction, or c) Giele-Glover style reduction.
- Semi-numerical d-dimensional unitarity-based methods have obtained results not attainable with other methods (eg 20 gluon amplitudes)
- Full results one-loop available for W+5 partons (and more)
- As yet the unitarity methods have been little tested in full calculations.

Numerical evaluation of the primitive amplitudes for ttgg and ttggg

INPUT

i) Born primitive amplitudes are calculated using BG recursion

tadpole cuts: six-, seven-leg tree amplitudes

ii) we have to calculate renormalized one loop primitive amplitudes

Z_2, Z_m factors + mass counter term diagrams (restores gauge invariance)

iii) test: correct soft collinear limits, + traditional calculation

iv) Master integral input from QCDloop

Numerical D-dimensional unitarity algorithm for massive fermions

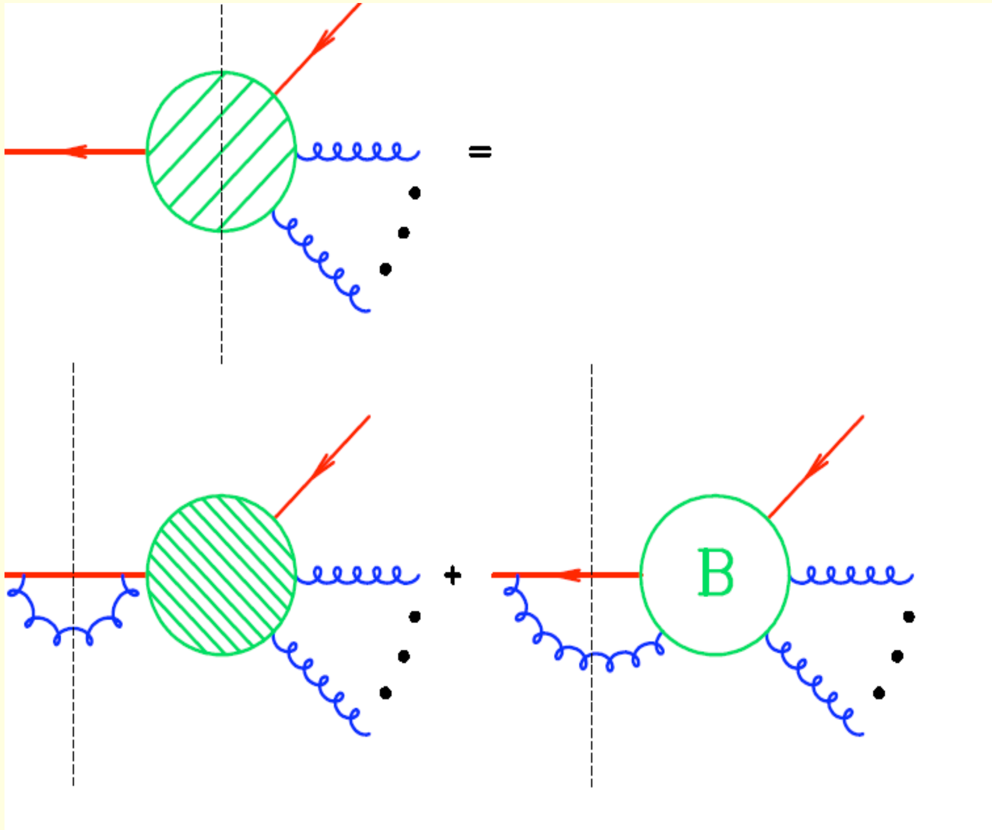
Application to ggtt and gggtt

- We have to choose even values for D_s

$$\mathcal{A}^{\mathsf{FDH}} = 2\mathcal{A}_{(D,D_s=6)} - \mathcal{A}_{(D,D_s=8)}$$

- Pentagon, box, triangle ,bubble and tadpole cuts
- The treatment of bubble and tadpole cuts is more subtle: difficulty with self-energy insertions on external lines.
- Particles of different flavors: more sophisticated bookkeeping
- Master integrals from with masses from QCDLoop (Ellis,Zanderighi)

Self-energy on external massive fermion leg



The external self-energy is doubly counted
Tree amplitude on the right hand side is not well defined.

Self energy contribution in unitarity algorithm

In a Feynman diagram calculation:

i) one particle reducible self-energy corrections on external legs are discarded

ii) Their effects are included by renormalization constants (Z_2 ,Z_m)

Follow the same path:

i) Discard the term in the tree amplitude generating one particle reducible diagrams

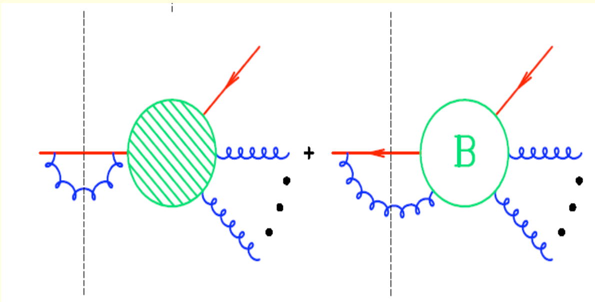
BG recursion relations can accommodate it by truncating the recursive steps

ii) It is taken into account by adding later wave function renormalization

The remaining part of the amplitude (B) is not gauge invariant.

iii) The gauges used to calculate Z_2 and B must be the same, (eg Feynman gauge)

It mildly violates "unitarity ": sum over non-physical states



Results for the tree and loop primitive amplitudes

Computer time from ggtt to gggtt scales the same way as in case of gluons

Fortran77 code

Amplitude	tree	c^{cut}	c
$+_{\bar{t}}, +_t, +_3, +_4, +_5$	-0.000533-0.000137 i	9.584144+6.530925 i	51.8809+6.543042 i
$+_{\bar{t}}, -_t, +_3, -_4, +_5$	-0.004540 + 0.018665 i	19.65913-11.77003 i	23.00306-9.699584 i
$+_{\bar{t}}, +_t, -_3, +_4, -_5$	-0.004726+ 0.014201 i	33.15950-1.832717 i	33.71943 -3.142751 i
$+_{\bar{t}}, -_t, -_3, +_4, +_5$	0.045786 + 0.010661 i	22.84043-6.540697 i	23.03114-7.313041 i
$+_{\bar{t}}, +_3, +_t, +_4, +_5$	0.000182 + 0.001369 i	6.517366-1.277070 i	19.37656+7.563101 i
$+_{\bar{t}}, +_3, -_t, -_4, +_5$	0.0467366-0.006020 i	19.440997-7.639466 i	20.93024-9.936409 i
$+_{\bar{t}}, -_3, +_t, +_4, -_5$	0.019275 -0.0732138 i	15.31910 -3.9278496 i	15.176306-4.102803 i
$+_{\bar{t}}, -_3, -_t, +_4, +_5$	-0.018203-0.111312 i	24.13158+1.431596 i	24.70002+1.018096 i
$+_{\bar{t}}, +_3, +_4, +_t, +_5$	0.00060-0.001377 i	13.13854+6.157043 i	10.13113+13.83997 i
$+_{\bar{t}}, +_3, -_4, -_t, +_5$	-0.047199-0.021516 i	23.90539 -2.168867 i	22.905695-4.284617 i
$+_{\bar{t}}, -_3, +_4, +_t, -_5$	-0.015110+0.063118 i	13.54258-7.800591 i	13.50273-8.018127 i
$+_{\bar{t}}, -_3, +_4, -_t, +_5$	-0.048800+ 0.112645 i	21.77602+ 2.078051 i	22.52784+1.424481 i
$+_{\bar{t}}, +_3, +_4, +_5, +_t$	-0.000252+0.000144 i	-10.35085+45.26276 i	-98.81384+52.81712 i
$+_{\bar{t}}, +_3, -_4, +_5, -_t$	0.0050023+0.008871 i	23.944473+2.862220 i	20.92683-0.968026 i
$+_{\bar{t}}, -_3, +_4, -_5, +_t$	0.000561-0.004105 i	-2.987822-42.00048 i	-3.834451-43.67103 i
$+_{\bar{t}}, -_3, +_4, +_5, -_t$	0.021216-0.011994 i	19.72995-2.120128 i	20.94996-1.684734 i