Top quark production at Tevatron and LHC

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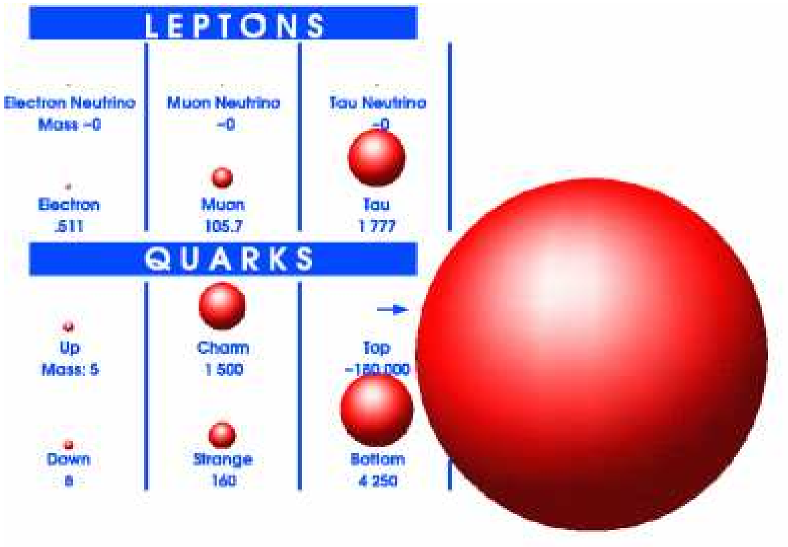
– Theory Seminar, FNAL, May 05, 2009 –

Plan

- Some new results on the heaviest elementary particle



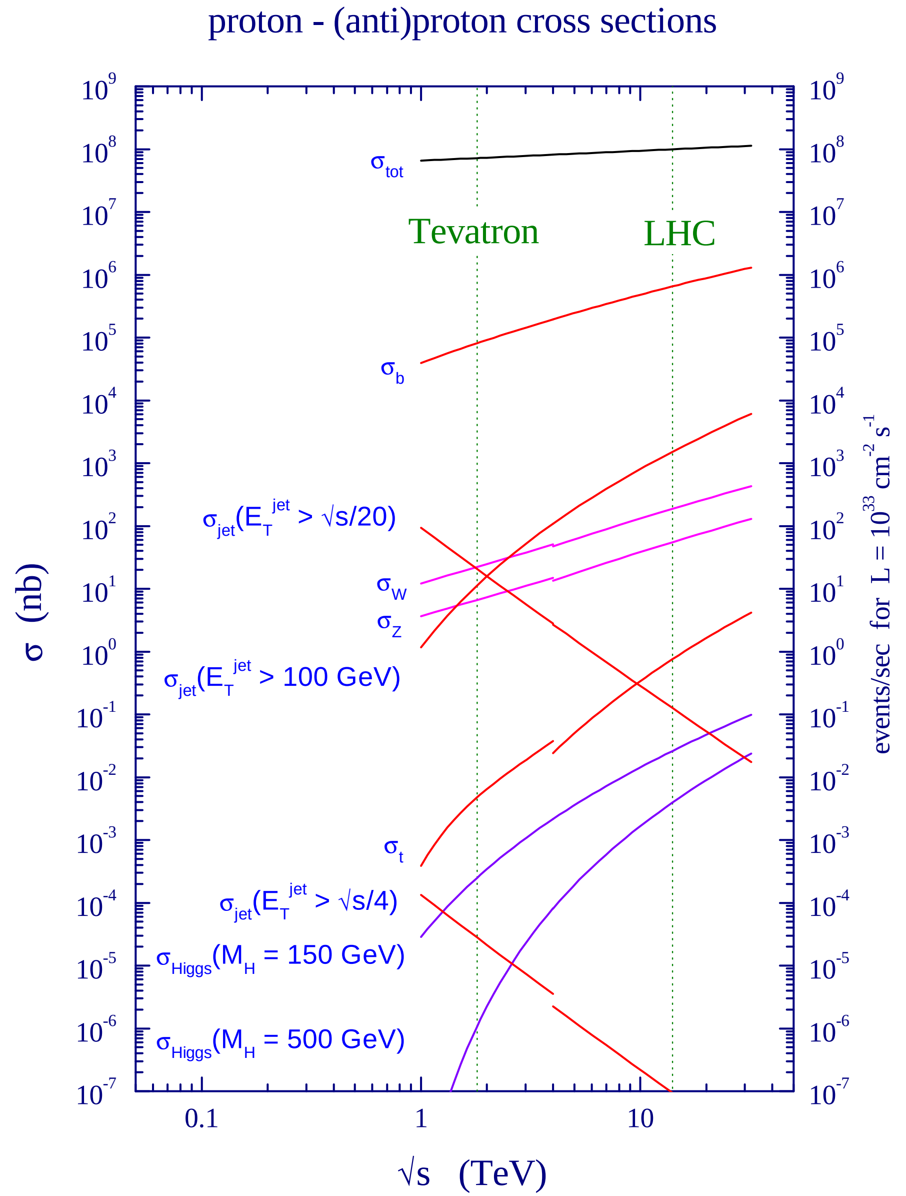
- Report based on recent work done in collaboration with
  - P. Uwer on arXiv:0804.1476 and on arXiv:0807.2794
  - Y. Kiyo, J.H. Kühn, M. Steinhauser and P. Uwer on arXiv:0812.0919
  - U. Langenfeld on arXiv:0901.0802
  - U. Langenfeld and P. Uwer to appear

Proton colliders

- Tevatron: energy frontier at $\sqrt{S} = 1.96$TeV
  top quark discovery
- LHC: in comissioning phase
  Higgs boson search at highest energies: $\sqrt{S} = 14$TeV

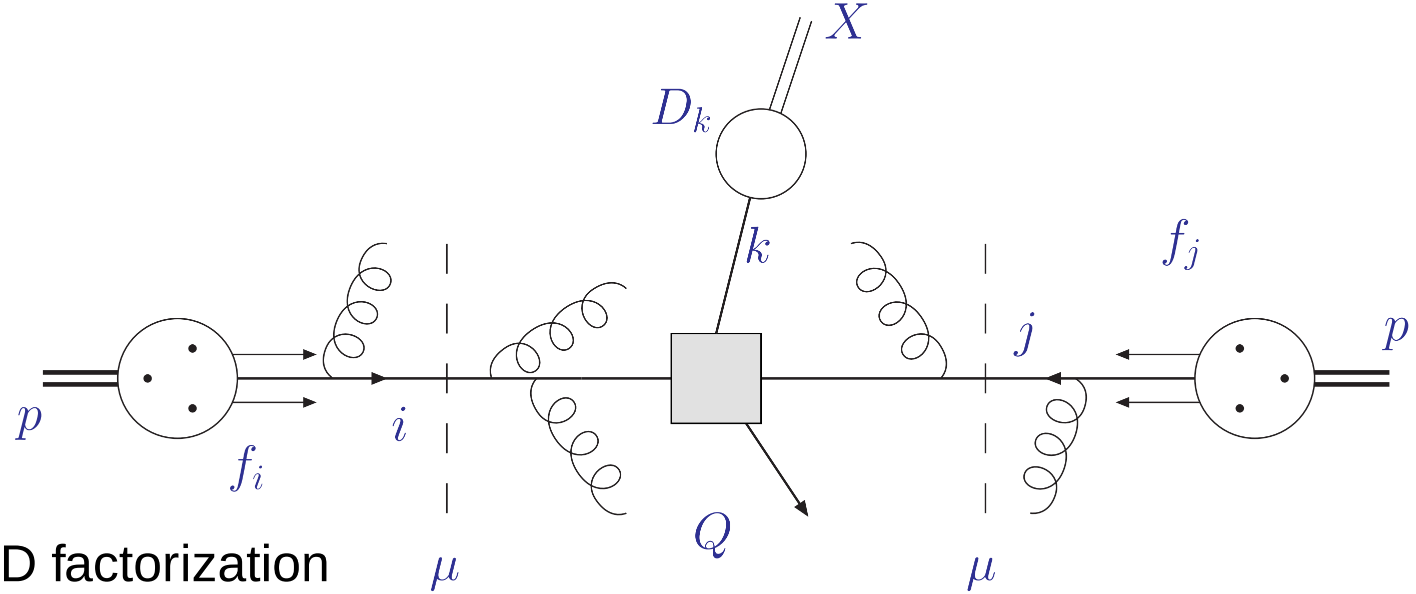
Top quarks at proton colliders

- Top quark discovery at Tevatron
  - a lot of integrated luminosity for analyses
  - $t\bar{t}$-pairs and single top
- LHC will accumulate very high statistics for $t\bar{t}$-pairs
  - low luminosity run: $8 \cdot 10^6$ events/year (high luminosity run: 10 times more)
  - mass measurement $\Delta m_t = \mathcal{O}(1)$GeV (constraints on Standard Model Higgs mass m_h)
  - top-spin correlations
  - search for anomalous couplings
- Tops make up large part of background in Higgs or new physics searches



Perturbative QCD at colliders

- Hard hadron-hadron scattering
  - constituent partons from each incoming hadron interact at short



- QCD factorization
  - separate sensitivity to dynamics from different scales

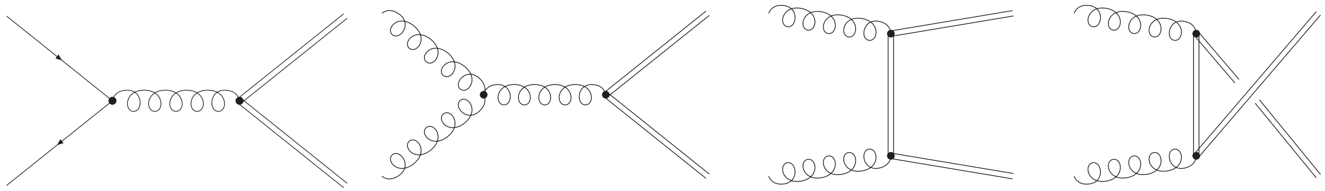
$$\sigma_{pp \to X} = \sum_{ijk} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij \to k}\left(\alpha_s(\mu^2), Q^2, \mu^2\right) \otimes D_{k \to X}(\mu^2)$$

  - factorization scale μ, subprocess cross section $\hat{\sigma}_{ij \to k}$ for parton types i, j and hadronic final state X

Top quark production

- Leading order Feynman diagrams

$$\mathrm{q} + \bar{\mathrm{q}} \longrightarrow \mathrm{Q} + \bar{\mathrm{Q}}$$
$$\mathrm{g} + \mathrm{g} \longrightarrow \mathrm{Q} + \bar{\mathrm{Q}}$$



- NLO in QCD Nason, Dawson, Ellis ‘88; Beenakker, Smith, van Neerven ‘89; Mangano, Nason, Ridolfi ‘92; Bernreuther, Brandenburg, Si, Uwer ‘04; Mitov, Czakon ‘08; ...
  - accurate to $\mathcal{O}(15\%)$ at LHC
- Much activity towards higher orders in QCD
  - one-loop squared terms (NLO $\times$ NLO) Anastasiou, Mert Aybat ‘08; Kniehl, Merebashvili, Körner, Rogal ‘08
  - analytic two-loop fermionic corrections for $\mathrm{q}\bar{\mathrm{q}} \to \mathrm{t}\bar{\mathrm{t}}$ Bonciani, Ferroglia, Gehrmann, Maitre, Studerus ‘08
  - numerical result for two-loop virtual $\mathrm{q}\bar{\mathrm{q}} \to \mathrm{t}\bar{\mathrm{t}}$ Czakon ‘08

Strategy

- First steps towards higher orders in QCD: explore limits

Strategy

- First steps towards higher orders in QCD: explore limits

Small mass limit

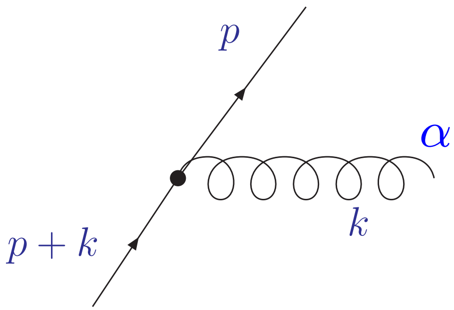
- Study of massive QCD amplitudes in limit $m \to 0$
  - look at soft and collinear limits
  - exploit relation of massive to massless amplitudes
- Two-loop virtual corrections to $\mathrm{q\bar{q}} \to \mathrm{t\bar{t}}$ and $\mathrm{gg} \to \mathrm{t\bar{t}}$ in small-mass limit $m^2 \ll s, t, u$ S.M., Czakon, Mitov ‘07

Threshold resummation

- Partonic threshold $s \simeq 4m^2$
  - Sudakov-type logarithms $\ln \beta$ with velocity of heavy quark $\beta = \sqrt{1 - 4m^2/s}$
  - long resummation Kidonakis, Sterman ‘97; Bonciani, Catani, Mangano, Nason ‘98; Kidonakis, Laenen, S.M., Vogt ‘01; . . .

Scattering amplitudes in small mass limit

- Soft/collinear regions of phase space
  - massless partons



$$\frac{1}{(p+k)^2} = \frac{1}{2p\cdot k} = \frac{1}{2E_q E_g (1-\cos\theta_{qg})}$$

$$\alpha_s \int d^4k \frac{1}{(p+k)^2} \longrightarrow \alpha_s \int dE_g\, d\theta_{qg}\, \frac{1}{2E_q E_g (1-\cos\theta_{qg})}$$

$$\longrightarrow \alpha_s \frac{1}{\epsilon^2} \times (\dots)$$ in dim. reg. $D = 4 - 2\epsilon$

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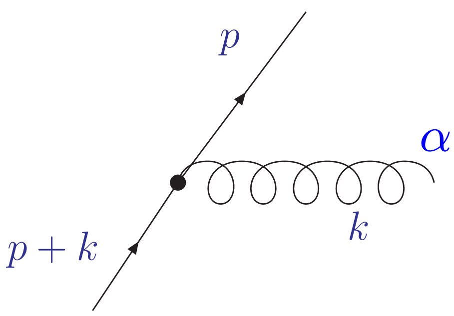

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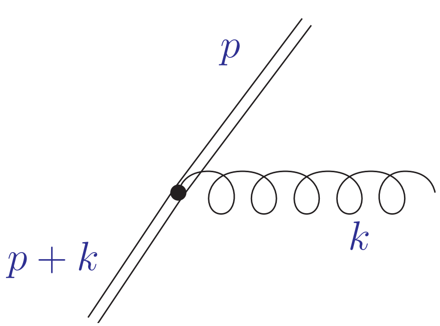
$$\longrightarrow \alpha_s \frac{1}{\epsilon^2} \times (\dots) \quad \text{in dim. reg.} \quad D = 4 - 2\epsilon$$

- Parton masses regulate collinear singularity

$$\frac{1}{(p+k)^2 - m_q^2} = \frac{1}{2p\cdot k} = \frac{1}{2E_qE_g(1-\beta\cos\theta_{qg})}$$

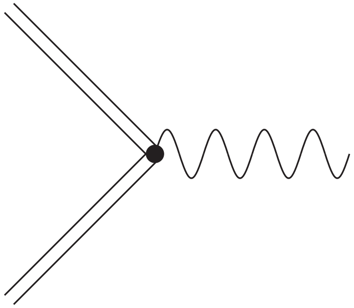


with $\beta = \left(1 - \frac{m_q^2}{E_q^2}\right)^{1/2} < 1$

$$\alpha_s \int d^4k \frac{1}{(p+k)^2 - m_q^2} \longrightarrow \alpha_s \frac{1}{\epsilon} \ln(m_q^2) \times (\dots)$$

Quark form factor

- Form factor for (massive) quarks (on-shell)



$$\Gamma_\mu(k_1, k_2) \; = \; ie_{\mathrm{q}}\, \bar{u}(k_1) \left(\gamma_\mu \; \mathcal{F}_1(Q^2, m^2, \alpha_s) + \frac{1}{2m}\, \sigma_{\mu\nu}\, q^\nu \; \mathcal{F}_2(Q^2, m^2, \alpha_s) \right) u(k_2)$$

- QCD corrections to vertex
  - gauge invariant quantity
  - infrared divergent (dimesional regularization $D = 4 - 2\epsilon$)

Massless form factor

- Form factor $\mathcal{F}(Q^2, \alpha_s)$ exponentiates
  Collins '80; Sen '81; Korchemsky '88; Magnea, Sterman '90; Contopanagos, Laenen, Sterman '97; Magnea '00

$$Q^2 \frac{\partial}{\partial Q^2} \ln \mathcal{F}\left(\frac{Q^2}{\mu^2}, \alpha_s, \epsilon\right) = \frac{1}{2} K(\alpha_s, \epsilon) + \frac{1}{2} G\left(\frac{Q^2}{\mu^2}, \alpha_s, \epsilon\right) .$$

  - all Q^2-dependence in finite function G
  - function K pure counterterm (series of poles in ϵ)
- Renormalization group equations for functions G and K

$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s, \epsilon) \frac{\partial}{\partial \alpha_s}\right) \{G, K\} = \{A(\alpha_s), -A(\alpha_s)\}$$

  - anomalous dimension A (e.g. from Wilson line with cusp)
- Solution for $\ln \mathcal{F}$ with D-dim. coupling $\bar{a}(1, a_s, \epsilon) = a_s \equiv \alpha_s/(4\pi)$
  Magnea, Sterman '90

$$2 \ln \mathcal{F}\left(\frac{Q^2}{\mu^2}, \alpha_s, \epsilon\right) = \int\limits_0^{Q^2/\mu^2} \frac{d\xi}{\xi} \left\{G(\bar{a}, \epsilon) + K(\alpha_s, \epsilon) - \int\limits_1^{\xi} \frac{d\lambda}{\lambda} A(\bar{a}(\lambda, \epsilon))\right\}$$

Result (massless)

- Expansion in terms of bare coupling α_s^b

$$\mathcal{F}\left(\alpha_s^b, Q^2\right) = 1 + \sum_{l=1} \left(\frac{\alpha_s^b}{4\pi}\right)^l \left(\frac{Q^2}{\mu^2}\right)^{-l\epsilon} \mathcal{F}_l$$

  - $\mathcal{F}_2$: Hamberg, van Neerven, Matsuura ‘88; Harlander ‘00; Gehrmann, Huber, Maitre ’05; S.M. Vermaseren, Vogt ’05
  - $\mathcal{F}_3$: S.M. Vermaseren, Vogt ’05

- Result up to three loops in terms of expansion coefficients of A and G

$$\begin{aligned}
\mathcal{F}_1 &= -\frac{1}{2}\frac{1}{\epsilon^2}A_1 - \frac{1}{2}\frac{1}{\epsilon}G_1 \\
\mathcal{F}_2 &= \frac{1}{8}\frac{1}{\epsilon^4}A_1^2 + \frac{1}{8}\frac{1}{\epsilon^3}A_1(2G_1 - \beta_0) + \frac{1}{8}\frac{1}{\epsilon^2}(G_1^2 - A_2 - 2\beta_0 G_1) - \frac{1}{4}\frac{1}{\epsilon}G_2 \\
\mathcal{F}_3 &= \ldots
\end{aligned}$$

Massive form factor

- Renormalization group equation factorizes into functions G and K
  - all Q^2-dependence again in finite function G
  - function K now dependent on infrared sector (parton mass m)

$$\mu^2 \frac{\partial}{\partial \mu^2} \ln \mathcal{F}\left(\frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}, \alpha_{\mathrm{s}}, \epsilon\right) = \frac{1}{2} K\left(\frac{m^2}{\mu^2}, \alpha_{\mathrm{s}}, \epsilon\right) + \frac{1}{2} G\left(\frac{Q^2}{\mu^2}, \alpha_{\mathrm{s}}, \epsilon\right)$$

- Solution for evolution equation Mitov, S.M. '06

$$2 \ln \mathcal{F}\left(\frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}, \alpha_s, \epsilon\right) =$$

$$\int\limits_0^{Q^2/\mu^2} \frac{d\xi}{\xi} \left\{ G(\bar{a}(\xi\mu^2, \epsilon)) + K(\bar{a}(\xi\mu^2 m^2/Q^2, \epsilon)) - \int\limits_{\xi m^2/Q^2}^{\xi} \frac{d\lambda}{\lambda} A(\bar{a}(\lambda\mu^2, \epsilon)) \right\}$$

  - Double logarithms $L = \ln(Q^2/m^2)$ from integral over A
  - A and G same functions as in massless calculations
  - K determined matching to fixed order results

Result (massive)

- Massive form factor with logarithms $L = \ln(Q^2/m^2)$
  - expansion in terms of coefficients A, G, K and constant terms C (all finite in m^2 and ϵ)
- Expansion up to two loops Bernreuther, Bonciani, Gehrmann, Heinesch, Leineweber, Mastrolia, Remiddi '04; Mitov, S.M. '06

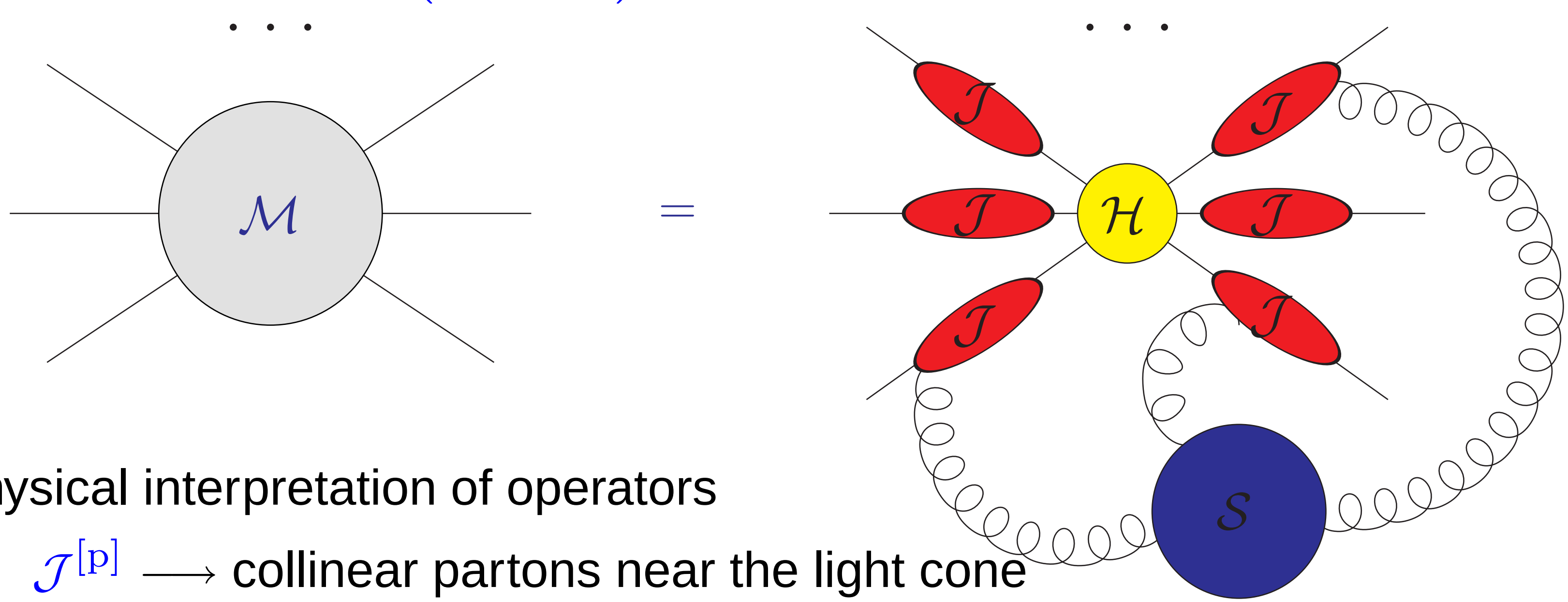
$$
\begin{aligned}
\mathcal{F}_1 &= \frac{1}{\epsilon}\left\{\frac{1}{2}A_1 L + \frac{1}{2}(G_1 + K_1)\right\} - \frac{1}{4}A_1 L^2 - \frac{1}{2}G_1 L + C_1 \\
\mathcal{F}_2 &= \frac{1}{\epsilon^2}\left\{\frac{1}{8}A_1^2 L^2 + \frac{1}{4}A_1(G_1 + K_1 - \beta_0)L + \frac{1}{8}(G_1 + K_1)(G_1 + K_1 - 2\beta_0)\right\} \\
&\quad + \frac{1}{\epsilon}\left\{-\frac{1}{8}A_1^2 L^3 - \frac{1}{8}A_1(3G_1 + K_1)L^2 + \frac{1}{4}(A_2 - G_1^2 - K_1 G_1 + 2A_1 C_1)L \right. \\
&\quad \left. + \frac{1}{4}(G_2 + K_2) + \frac{1}{2}C_1(G_1 + K_1)\right\} + \frac{7}{96}A_1^2 L^4 \\
&\quad + \frac{1}{24}A_1(7G_1 + K_1 + 2\beta_0)L^3 + \frac{1}{8}G_1(2G_1 + K_1 + 2\beta_0)L^2 \\
&\quad - \frac{1}{4}(A_2 + A_1 C_1)L^2 - \frac{1}{2}(G_2 + G_1 C_1)L + C_2
\end{aligned}
$$

Scattering amplitudes

Soft and collinear factorization

- Amplitude $\mathcal{M}$ factorizes into various functions $\mathcal{J}^{[\mathrm{p}]}$, $\mathcal{S}^{[\mathrm{p}]}$ and $\mathcal{H}^{[\mathrm{p}]}$

$$|\mathcal{M}_{\mathrm{p}}\rangle = \mathcal{J}^{[\mathrm{p}]}\left(Q^2, \alpha_s, \epsilon\right) \mathcal{S}^{[\mathrm{p}]}\left(\{k_i\}, \alpha_s, \epsilon\right) |\mathcal{H}_{\mathrm{p}}\rangle$$



- Physical interpretation of operators
  - $\mathcal{J}^{[\mathrm{p}]} \longrightarrow$ collinear partons near the light cone
  - $\mathcal{S}^{[\mathrm{p}]} \longrightarrow$ soft partons of long wave-length at large angle (matrix in color space)
  - $\mathcal{H}^{[\mathrm{p}]} \longrightarrow$ hard off-shell partons at short distances (vector in color space)

Jet function

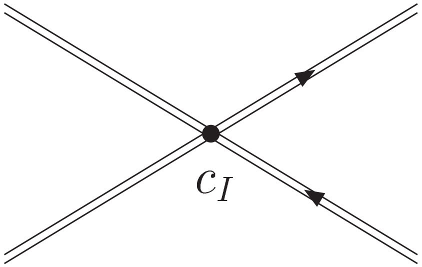
- Jet function defined by parton form factors
  - definition ensures exponentiation of QCD corrections

$$\mathcal{J}^{[i]}\left(\frac{Q^2}{\mu^2},\frac{m_i^2}{\mu^2},\alpha_s(\mu^2),\epsilon\right) \quad = \quad \left(\mathcal{F}^{[i]}\left(\frac{Q^2}{\mu^2},\frac{m_i^2}{\mu^2},\alpha_s,\epsilon\right)\right)^{\frac{1}{2}}, \qquad i = q, g$$

- QCD corrections to form factor $\mathcal{F}$ exponentiate (anomalous dimensions are universal)
  Collins '80; Sen '81; Korchemsky '88; Magnea, Sterman '90; Contopanagos, Laenen, Sterman '97; Magnea '00; S.M. Mitov '06
  - massless case: double and single poles in $\frac{1}{\epsilon}$
  - massive case: logarithms in parton masses $\ln(m)$ and poles in $\frac{1}{\epsilon}$

Soft function

- Sensitivity to color structure of scattering process
  - color mixing through soft gluon exchange
- Construction of $\mathcal{S}$ as composite operator
  Korchemsky, Korchemskaya '94; Contopanagos, Laenen, Sterman '97; Aybat, Dixon, Sterman '06; ...
  - coupling to Wilson-lines
    $\longrightarrow$ partons in eikonal approximation



- Renormalization group equations
  with soft anomalous dimension $\Gamma^{[\mathrm{p}]}$

$$\mu^2 \frac{d}{d\mu^2} \mathcal{S}^{[\mathrm{p}]}_{IJ} = -\left(\Gamma^{[\mathrm{p}]}\right)_{IK} \mathcal{S}^{[\mathrm{p}]}_{KJ}$$

  - $\Gamma^{[\mathrm{p}]}$ with smooth limit $m \to 0$
    - much progress recently on soft anomalous dimension
      massless: Becher, Neubert '09; Gardi, Magnea '09
      massive: Mitov, Sterman, Sung '09; Becher, Neubert '09
- Solution as path-ordered exponential (matrix in color space)

Massless amplitudes

- Singularity structure of massless amplitudes $|\mathcal{M}_\mathrm{p}\rangle$
  - process p for $2 \to n$ parton scattering
  - poles in $\frac{1}{\epsilon}$ in terms of universal anomalous dimensions Catani '98
  - soft and collinear divergences exhibit exponentiation to all orders
    Tejeda-Yeomans, Sterman '02

$$
\begin{aligned}
|\mathcal{M}_\mathrm{p}^{(0)}\rangle &= |\mathcal{H}_\mathrm{p}^{(0)}\rangle \\
|\mathcal{M}_\mathrm{p}^{(1)}\rangle &= \frac{1}{2} \sum_{i\in\,\{\text{all legs}\}} \mathcal{F}_1^{[i]}|\mathcal{H}_\mathrm{p}^{(0)}\rangle + \mathcal{S}_1^{[\mathrm{p}]}|\mathcal{H}_\mathrm{p}^{(0)}\rangle + |\mathcal{H}_\mathrm{p}^{(1)}\rangle \\
|\mathcal{M}_\mathrm{p}^{(2)}\rangle &= \frac{1}{2} \sum_{i\in\,\{\text{all legs}\}} \left(\mathcal{F}_2^{[i]} - \frac{1}{4}\left(\mathcal{F}_1^{[i]}\right)^2 + \frac{1}{2}\,\mathcal{F}_1^{[i]}\mathcal{S}_1^{[\mathrm{p}]}\right)|\mathcal{H}_\mathrm{p}^{(0)}\rangle \\
&\quad + \frac{1}{2} \sum_{i\in\,\{\text{all legs}\}} \mathcal{F}_1^{[i]}|\mathcal{H}_\mathrm{p}^{(1)}\rangle + \mathcal{S}_2^{[\mathrm{p}]}|\mathcal{H}_\mathrm{p}^{(0)}\rangle + \mathcal{S}_1^{[\mathrm{p}]}|\mathcal{H}_\mathrm{p}^{(1)}\rangle + |\mathcal{H}_\mathrm{p}^{(2)}\rangle
\end{aligned}
$$

- Checks with explicit calculations of NNLO QCD $2 \to 2$ amplitudes
  Anastasiou, Bern, v.d.Bij, De Freitas, Dixon, Garland, Gehrmann, Ghinculov, Glover, Koukoutsakis, S.M., Oleari, Remiddi, Schmidt, Tejeda-Yeomans, Uwer, Weinzierl, Wong '01-'04

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massless form factor

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Massive amplitudes

- Singularity structure of massive amplitudes $|\mathcal{M}_{\mathrm{p},\{m_i\}}\rangle$
  - process p for $2 \to n$ parton scattering
  - generalization of Catani's massless formula and one-loop massive results Catani, Dittmaier, Trocsanyi '00
  - amplitude factorizes in terms of three functions $\mathcal{F}$, $\mathcal{S}_{\mathrm{p}}$ and $|\mathcal{H}_{\mathrm{p}}\rangle$ ($\mathcal{S}_{\mathrm{p}}$ and $|\mathcal{H}_{\mathrm{p}}\rangle$ largely same as in massless case) Mitov S.M. '06

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|\mathcal{M}^{(0)}_{\mathrm{p},\{m_i\}}\rangle &= |\mathcal{H}^{(0)}_{\mathrm{p}}\rangle \, , \\
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|\mathcal{M}^{(2)}_{\mathrm{p},\{m_i\}}\rangle &= \frac{1}{2} \sum_{i \in \,\{\text{all legs}\}} \left(\mathcal{F}^{[i]}_2 - \frac{1}{4} \left(\mathcal{F}^{[i]}_1 \right)^2 + \frac{1}{2} \mathcal{F}^{[i]}_1 \mathcal{S}^{[\mathrm{p}]}_1 \right) |\mathcal{H}^{(0)}_{\mathrm{p}}\rangle \\
&\quad + \frac{1}{2} \sum_{i \in \,\{\text{all legs}\}} \mathcal{F}^{[i]}_1 |\mathcal{H}^{(1)}_{\mathrm{p}}\rangle + \mathcal{S}^{[\mathrm{p}]}_2 |\mathcal{H}^{(0)}_{\mathrm{p}}\rangle + \mathcal{S}^{[\mathrm{p}]}_1 |\mathcal{H}^{(1)}_{\mathrm{p}}\rangle + |\mathcal{H}^{(2)}_{\mathrm{p}}\rangle
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\end{aligned}
$$

massive form factor

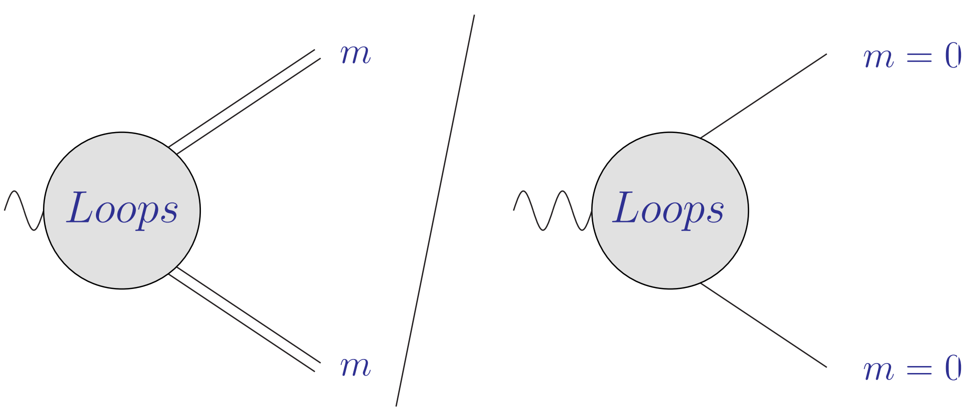
Upshot

- Simple multiplicative relation between massless and massive amplitudes to all orders Mitov S.M. '06
  - hierarchy of scales $m^2 \ll s, t, u$
  - relation correct up to power suppressed terms $\mathcal{O}(m)$

$$\mathcal{M}^{[\mathrm{p}],(m)}\left(\{k_i\}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) =$$

$$\prod_{i \in\ \{\text{all legs}\}} \left(Z_{[i]}^{(m|0)}\left(\frac{m^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right)\right)^{\frac{1}{2}} \times \mathcal{M}^{[\mathrm{p}],(m=0)}\left(\{k_i\}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right)$$

- factor $Z_{[i]}^{(m|0)}$ with $i = q, g$
  - determined by ratio of massless and massive form factor
  - however, no internal heavy loops



Heavy-quark hadro-production at two loops in QCD

- Amplitude for heavy-quark production in $q\bar{q}$-annihilation

$$|\mathcal{M}\rangle = 4\pi\alpha_s \left[|\mathcal{M}^{(0)}\rangle + \left(\frac{\alpha_s}{2\pi}\right)|\mathcal{M}^{(1)}\rangle + \left(\frac{\alpha_s}{2\pi}\right)^2 |\mathcal{M}^{(2)}\rangle + \mathcal{O}(\alpha_s^3) \right]$$

- Relate massive amplitude (interference with Born) up to power corrections in the mass $\mathcal{O}(m)$
  - use massless result for $q\bar{q} \to q'\bar{q}'$-scattering
    Anastasiou, Glover, Oleari, Tejeda-Yeomans '00

$$\mathrm{Re}\,\langle\mathcal{M}^{(0)}|\mathcal{M}^{(2)}\rangle^{(m)} =$$

$$\mathrm{Re}\,\langle\mathcal{M}^{(0)}|\mathcal{M}^{(2)}\rangle^{(m=0)} + Z^{(1)}\mathrm{Re}\,\langle\mathcal{M}^{(0)}|\mathcal{M}^{(1)}\rangle^{(m=0)} + Z^{(2)}\langle\mathcal{M}^{(0)}|\mathcal{M}^{(0)}\rangle^{(m=0)}$$

Heavy-quark hadro-production at two loops in QCD

- Amplitude for heavy-quark production in $q\bar{q}$-annihilation

$$|\mathcal{M}\rangle = 4\pi\alpha_s \left[|\mathcal{M}^{(0)}\rangle + \left(\frac{\alpha_s}{2\pi}\right) |\mathcal{M}^{(1)}\rangle + \left(\frac{\alpha_s}{2\pi}\right)^2 |\mathcal{M}^{(2)}\rangle + \mathcal{O}(\alpha_s^3) \right]$$

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- Independent check: direct calculation of massive Feynman diagrams
  - generate Feynman diagrams, reduce integrals to masters, construct Mellin-Barnes representation, expand in small mass, evaluate Mellin-Barnes by summing up series representations
  - software DIAGEN/IDSOLVER, MB, SUMMER, XSUMMER, PSLQ, ...

Results

- Full result for heavy-quark hadro-production at two loops in QCD in limit $m^2 \ll s, t, u$ S.M., Czakon, Mitov '07
  - number of colors N and light/heavy quarks n_l/n_h

$$2\mathrm{Re}\,\langle \mathcal{M}^{(0)} | \mathcal{M}^{(2)} \rangle = 2(N^2 - 1)\Bigg(N^2 A + B$$

$$+ \frac{1}{N^2} C + N n_l D_l + N n_h D_h + \frac{n_l}{N} E_l + \frac{n_h}{N} E_h + (n_l + n_h)^2 F \Bigg)$$

Results

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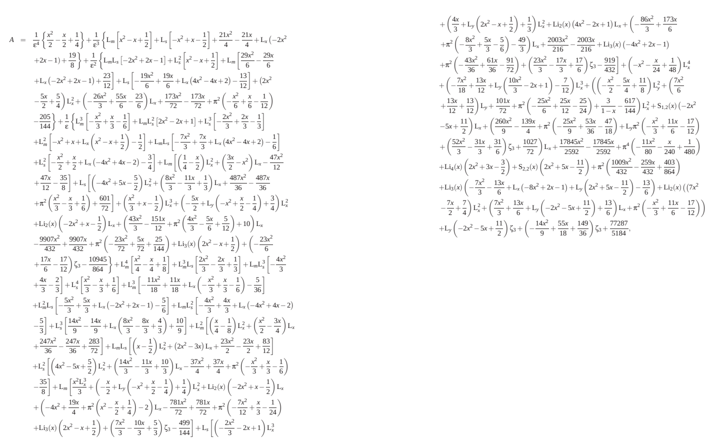
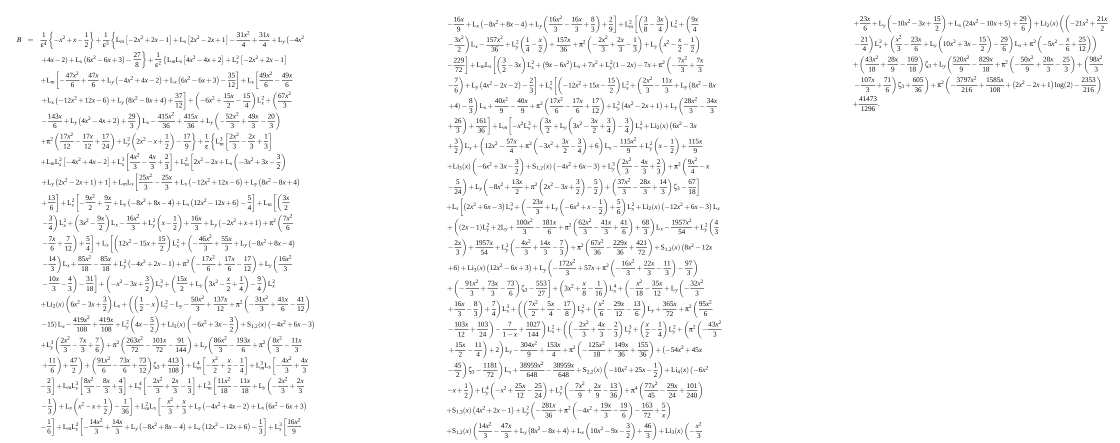


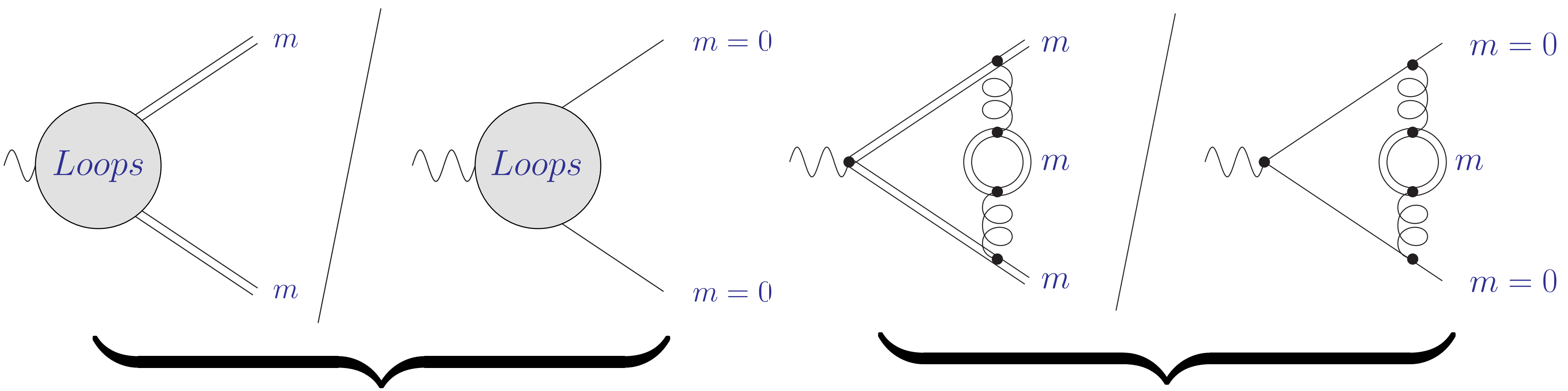
A



B

Remarks

- $Z^{(m|0)}_{[i]}$ relates theories with same total number of flavors n_f
  - require distinction according to number of internal heavy loops



- Sidecondition for $Z^{(m|0)}_{[q]}$-factor:
  no internal heavy loops

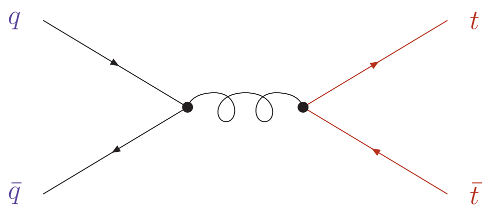
- Linear terms in n_h accounted for by separate ratio e.g. $Z^{(m|0)}_{[q]}\Big|_{n_h}$ (as defined above)
- Additional internal heavy loops in soft function $\mathcal{S}^{[\mathrm{p}]}$ (process dependent)

Phenomenological applications

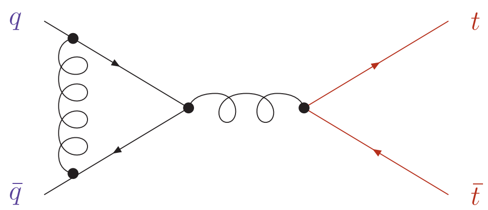
- Top production at LHC
  $\longrightarrow$ singularities and logarithms $\ln(m)$ in amplitudes to two-loop
  - $q\bar{q} \to Q\bar{Q}$ accomplished
  - $gg \to Q\bar{Q}$ accomplished
- QCD corrections to heavy (colored) BSM particles
  - squark and gluino production
- Subtraction schemes with massive partons for real emission contributions
  - extensions beyond one loop

Sudakov logarithms

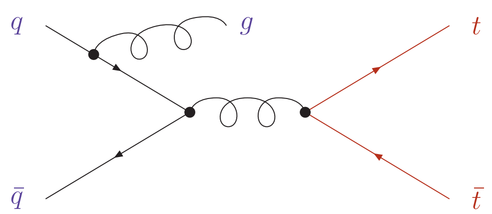
- Intuitive aspects of higher order corrections
  - lowest order, elastic



  - first order correction
    $virtual < 0$ (elastic)



  - first order correction
    $Brems > 0$ (inelastic)

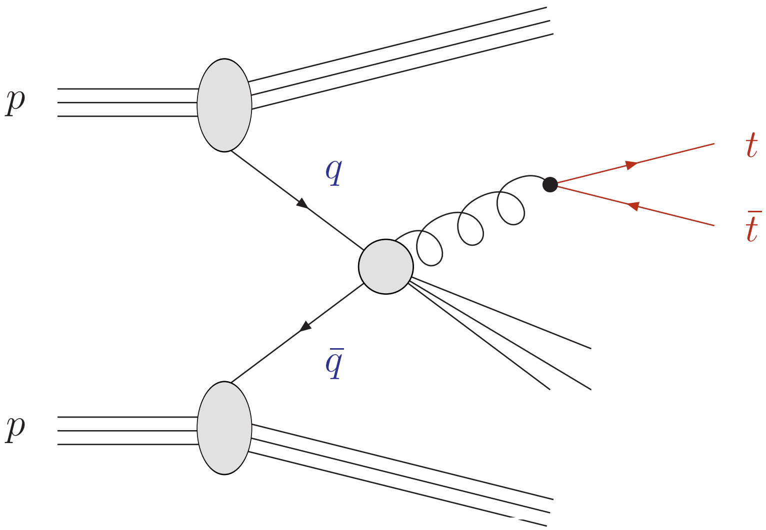


- at threshold for $t\bar{t}$-creation
  - strong Sudakov-supression inelastic tendency

$$\sigma \sim \exp\left[-\alpha_s \ln^2(1 - 4m_t^2/s)\right]$$

  - universal factor for parton splittings (leading log accuracy)
    modelling of MC parton showers

- Hadronic reaction $p\bar{p}$:
  - recall master equation $\sigma_{pp\to t\bar{t}} = \sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij\to t\bar{t}}$
  - initial partons: also Sudakov-supressed



- Parton cross section $\hat{\sigma}_{ij\to t\bar{t}}$
  - Sudakov-enhancement after mass factorization

$$\hat{\sigma}_{ij\to t\bar{t}} = \frac{\sigma_{pp\to t\bar{t}}}{f_i \otimes f_j} = \frac{\mathrm{e}^{-\alpha_s \ln^2(\dots)}}{\left(\mathrm{e}^{-\alpha_s \ln^2(\dots)}\right)^2} = \mathrm{e}^{+\alpha_s \ln^2(\dots)}$$

  - large double logarithms

Threshold resummation

- Threshold at $s \simeq 4m_t^2$
  - parton cross section exhibit Sudakov-type logarithms $\ln(\beta)$ with velocity of heavy quark $\beta = \sqrt{1 - 4m_t^2/s}$ at n$^{\mathrm{th}}$-order
- All order resummation of large logarithms $\alpha_s^n \ln^{2n}(\beta) \longleftrightarrow \alpha_s^n \ln^{2n}(N)$
  - resummation in Mellin space (renormalization group equation)
- Resummed cross section in Mellin space

$$\frac{\hat{\sigma}^N_{ij,\,I}(m^2)}{\hat{\sigma}^{(0),N}_{ij,\,I}(m^2)} = g^0_{ij,\,I}(m^2) \cdot \exp\left(G^{N+1}_{ij,\,I}(m^2)\right) + \mathcal{O}(N^{-1} \ln^n N)$$

  - exponent in singlet-octet color basis decomposition $I = 1, 8$

$$G^N_{q\bar{q}/gg,\,I} \quad = \quad G^N_{\mathrm{DY/Higgs}} + \delta_{I,8} G^N_{Q\bar{Q}}$$

- Renormalization group equations for functions $G^N_{\mathrm{DY/Higgs}}$ and $G^N_{Q\bar{Q}}$
  - well-known exponentiation from factorization in soft/collinear limit

The radiative factors

- Production of color singlet final state from parton-parton scattering described by $G^N_{\mathrm{DY/Higgs}}$

  $G^N_{\mathrm{DY/Higgs}} =$

$$\int\limits_0^1 dz\, \frac{z^{N-1}-1}{1-z} \int_{\mu_f^2}^{4m^2(1-z)^2} \frac{dq^2}{q^2}\, 2\, A_i(\alpha_s(q^2)) + D_i(\alpha_s(4m^2[1-z]^2))$$

  - well known anomalous dimensions A_i (collinear gluon emission) and D_i (process dependent gluon emission at large angles)

    Vogt '00; Catani, Grazzini, de Florian, Nason '03; S.M., Vermaseren, Vogt '05

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    Vogt '00; Catani, Grazzini, de Florian, Nason '03; S.M., Vermaseren, Vogt '05

- $G^N_{Q\bar{Q}}$ accounts for gluon emission from octet final state

  $$G^N_{Q\bar{Q}} \quad = \quad \int\limits_0^1 dz\, \frac{z^{N-1}-1}{1-z}\, D_{Q\bar{Q}}(\alpha_s([1-z]^2 4m^2))$$

  - anomalous dimension $D_{Q\bar{Q}}$ (cf. pole of form factor for massive quarks)

    $$D^{(1)}_{Q\bar{Q}} = -A^{(1)}_g\,, \qquad\qquad D^{(2)}_{Q\bar{Q}} = -A^{(2)}_g$$

    $D^{(2)}_{Q\bar{Q}}$ consistent with Mitov, Sterman, Sung '09; Becher, Neubert '09

Accuracy under control

- Control over logarithms $\ln(N)$ with $\lambda = \beta_0 \alpha_s \ln(N)$ to N^{k}LL accuracy

$$G^N_{ij\,I} = \ln N \cdot g^1_{ij}(\lambda) + g^2_{ij,\,I}(\lambda) + \alpha_s\, g^3_{ij,\,I}(\lambda) + \ldots$$

  - $g^1(\lambda)$: LL
    Laenen, Smith, v.Neerven ‘92; Berger, Contopanagos ‘95; Catani, Mangano, Nason, Trenatedue ‘96
  - $g^2(\lambda)$: NLL
    Bonciani, Catani, Mangano, Nason ‘98; Kidonakis, Laenen, S.M., Vogt ‘01
  - $g^3(\lambda)$: NNLL
    S.M., Uwer ‘08
- Resummed G^N predicts fixed orders in perturbation theory
  - generating functional for towers of large logarithms

New results

- NNLO cross section for heavy-quark hadro-production near threshold (all powers of $\ln \beta$ and Coulomb corrections) S.M., Uwer '08; Langenfeld, S.M., Uwer to appear
  - e.g. gg-fusion for $n_f = 5$ light flavors at $\mu = m_t$

$$\hat{\sigma}^{(1)}_{gg\to t\bar{t}} = \hat{\sigma}^{(0)}_{gg\to t\bar{t}} \left\{ 96 \ln^2 \beta - 9.5165 \ln \beta + 35.322 + 5.1698 \frac{1}{\beta} \right\}$$

$$\hat{\sigma}^{(2)}_{gg\to t\bar{t}} = \hat{\sigma}^{(0)}_{gg\to t\bar{t}} \left\{ 4608 \ln^4 \beta - 1894.9 \ln^3 \beta + \left(-912.35 + 496.30 \frac{1}{\beta} \right) \ln^2 \beta \right.$$
$$\left. + \left(3031.1 + 321.14 \frac{1}{\beta} \right) \ln \beta + 68.547 \frac{1}{\beta^2} - 196.93 \frac{1}{\beta} + C^{(2)}_{gg} \right\}$$

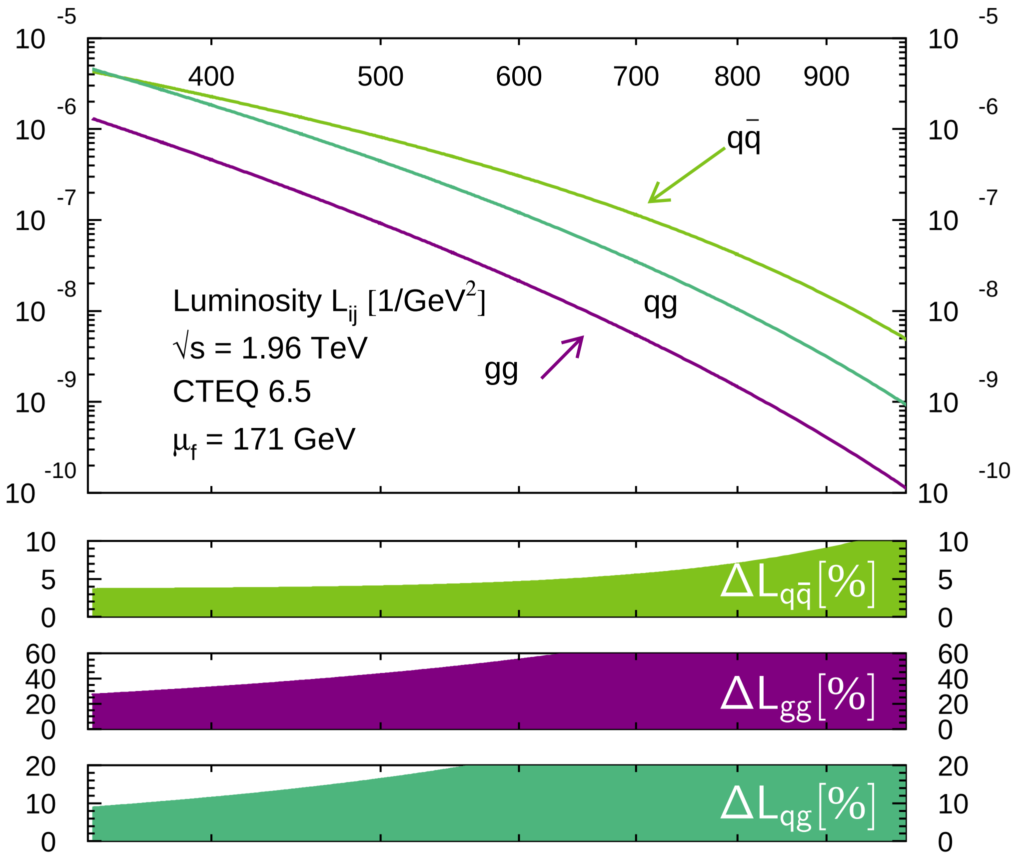
- Add all scale dependent terms
  - $\ln(\mu/m_t)$-terms exactly known from renormalization group methods

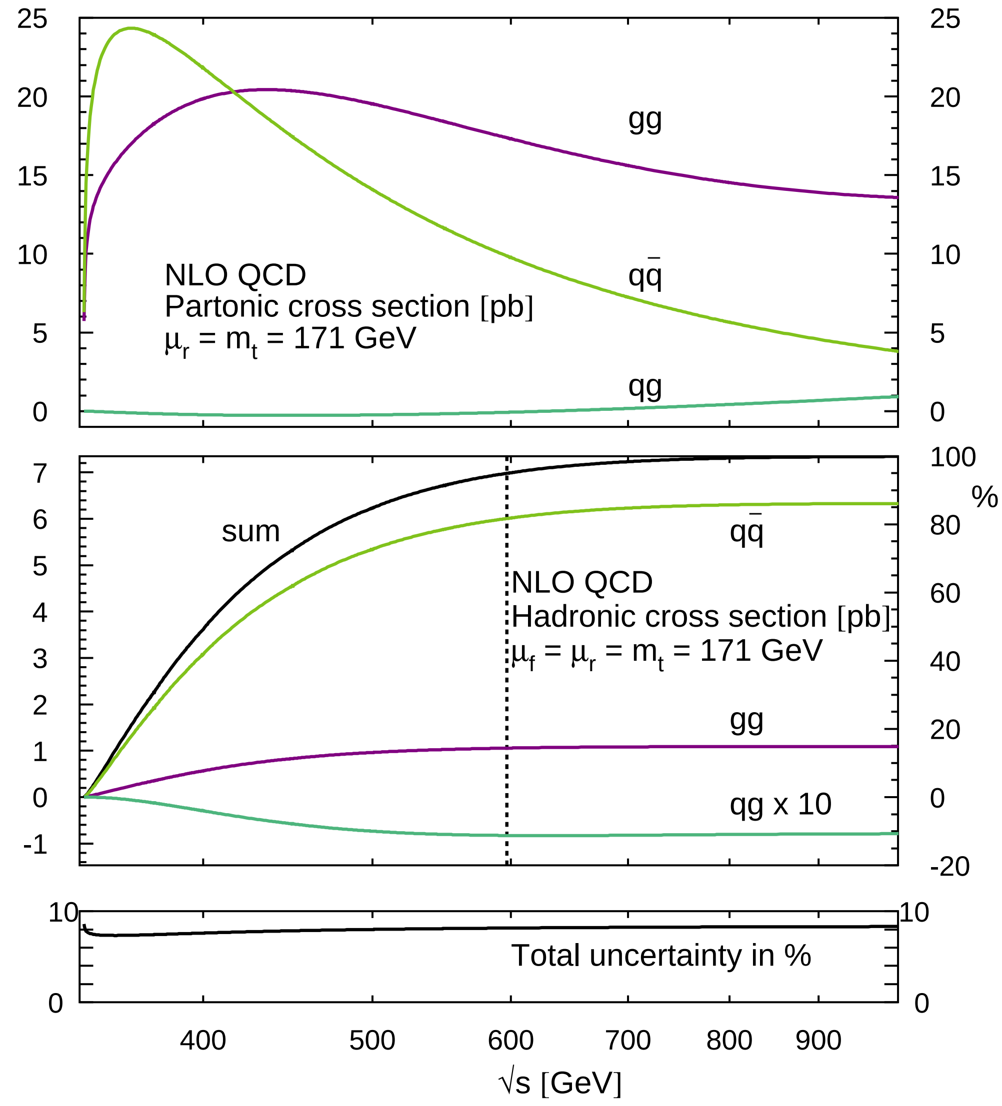
Upshot

- Best approximation to complete NNLO
- Similar results for new massive colored particles (4th generation quarks, squarks, gluinos, ...) S.M., Uwer '08; S.M., Langenfeld '08

Total cross section at Tevatron

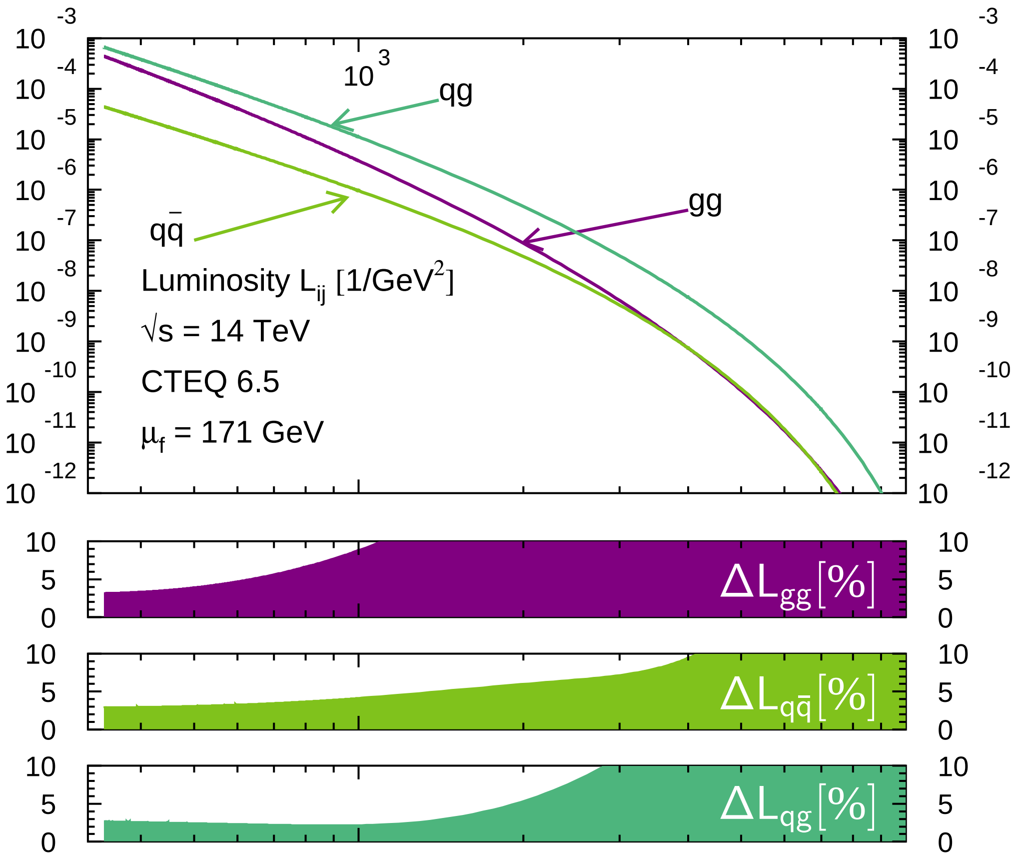
$$\sigma_{pp\to t\bar{t}} = \sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij\to t\bar{t}}$$

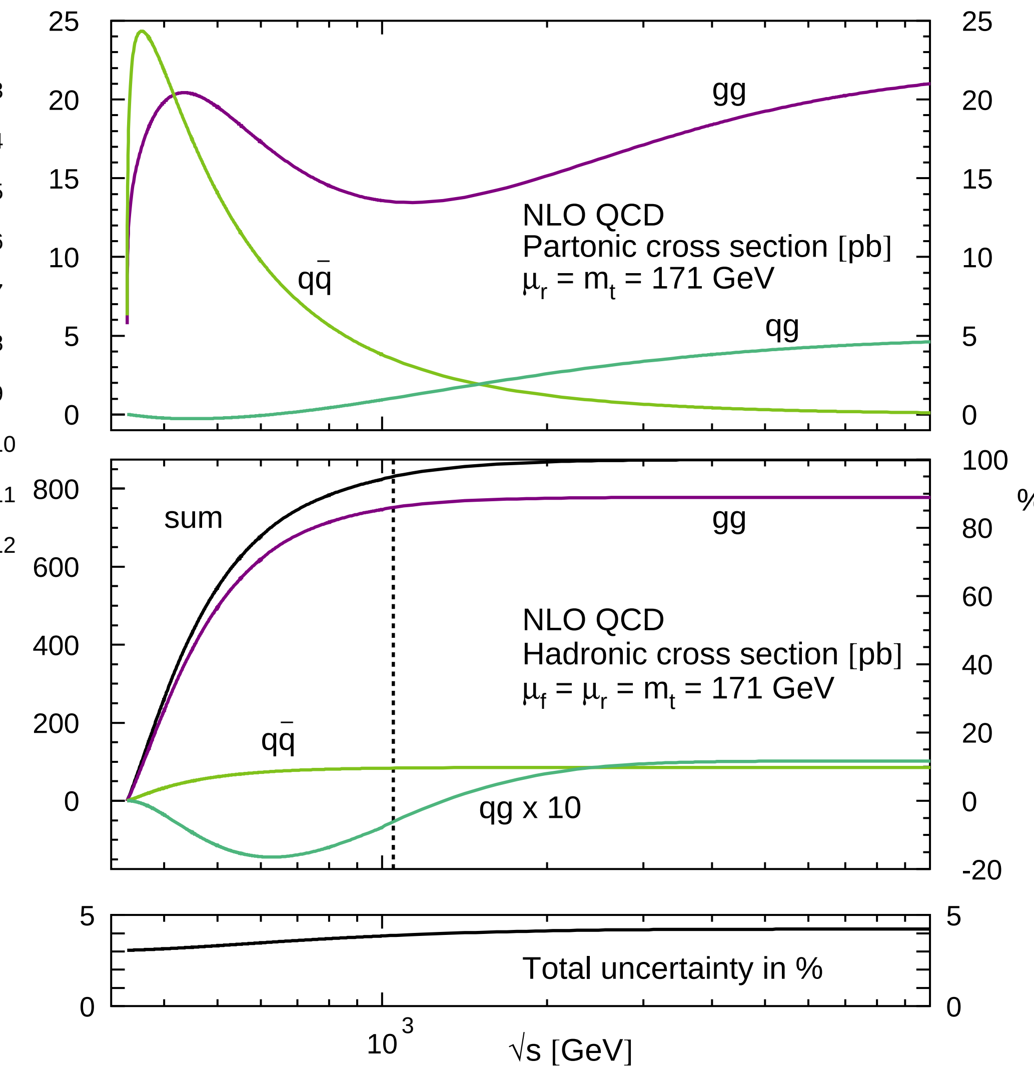




Total cross section at LHC

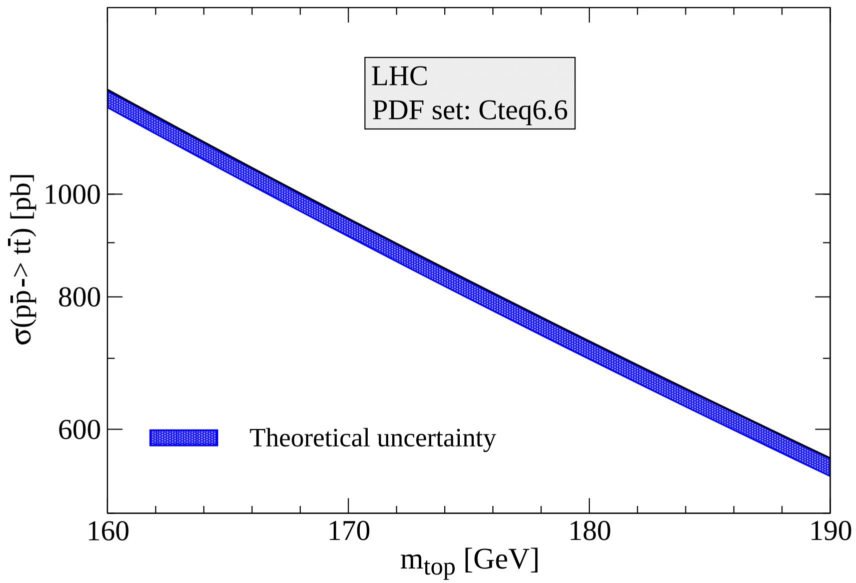
$$\sigma_{pp\to t\bar{t}} = \sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij\to t\bar{t}}$$



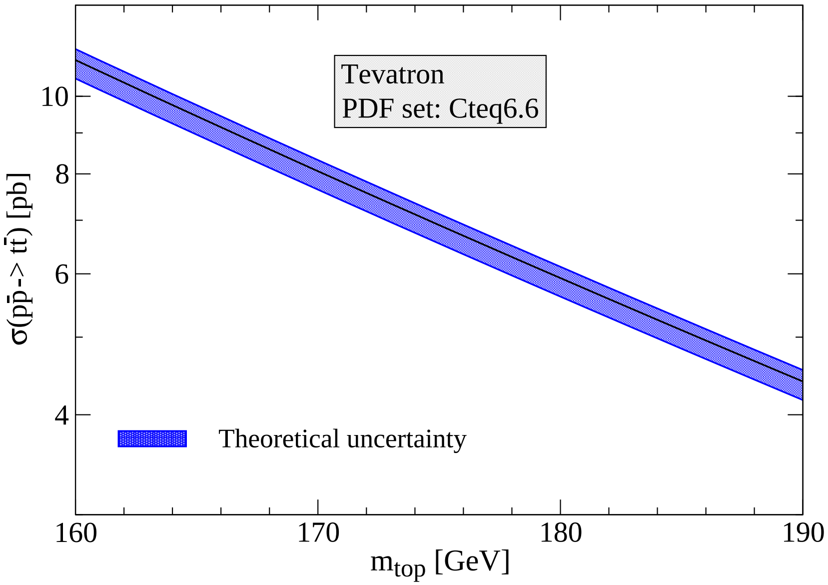


Mass dependence

- Parametrize mass dependence with a fit around $x = (m_t/\text{GeV} - 173)$
  $\sigma(\mu) = a + bx + cx^2 + dx^3 + ex^4 + fx^5 + gx^6$
  - fit precise to per mille accuracy in range $150\ \text{GeV} \leq m_t \leq 220\ \text{GeV}$
  - various scale and PDF choices: $\mu = m_t/2, m_t, 2m_t$, CTEQ6.6, MSTW2008, . . .



LHC



Tevatron

Theory improvements

- Variation of renormalization and factorization scale $\mu_R \neq \mu_F$
- Improved matching at NLO
  - consistent color-singlet and color-octet contributions at NLO Petrelli, Cacciari, Greco, Maltoni, Mangano '97;Hagiwara, Sumino, Yokoya '08
- Fits to exact NLO calculation Mitov, Czakon '08
- gq-channel included at two loops (large gq-parton luminosity at LHC)
  - leading term near threshold $\sim \beta^3 \ln^3 \beta$ (power suppressed β^2)

Scale dependence

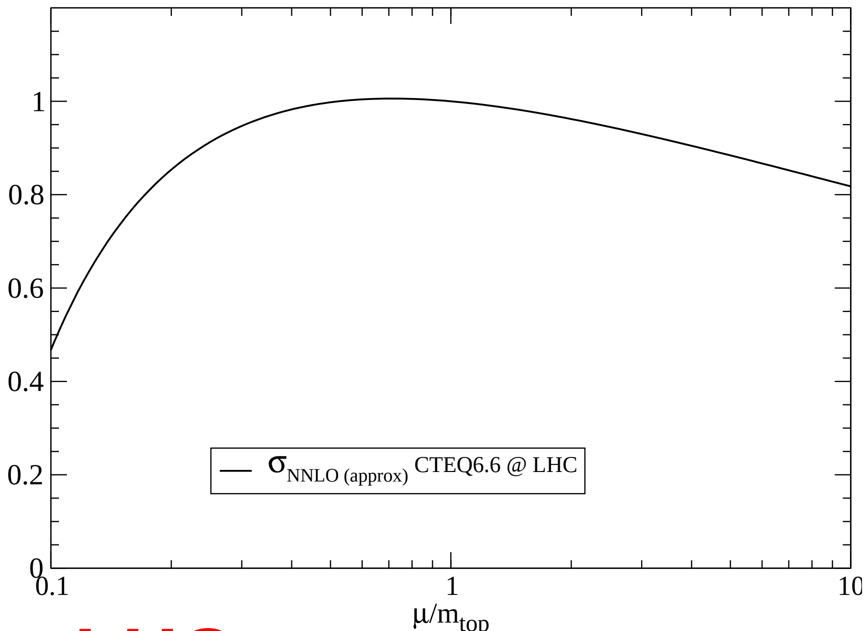
- Renormalization group methods predict all terms $L = \ln(\mu^2/m_t^2)$

$$\begin{aligned}\sigma_{t\bar{t}} &= \sigma^{(0)} + \alpha_s(\mu)\left\{\sigma^{(1)} + L\,\sigma_L^{(1)}(\sigma^{(0)}, \beta_0, P_0)\right\} \\ &+ \alpha_s^2(\mu)\left\{\sigma^{(2)} + L\,\sigma_L^{(2)}(\sigma^{(0)}, \sigma^{(1)}, \beta_0, \beta_1, P_0, P_1) + L^2\,\sigma_{L^2}^{(2)}(\sigma^{(0)}, \beta_0, P_0)\right\}\end{aligned}$$

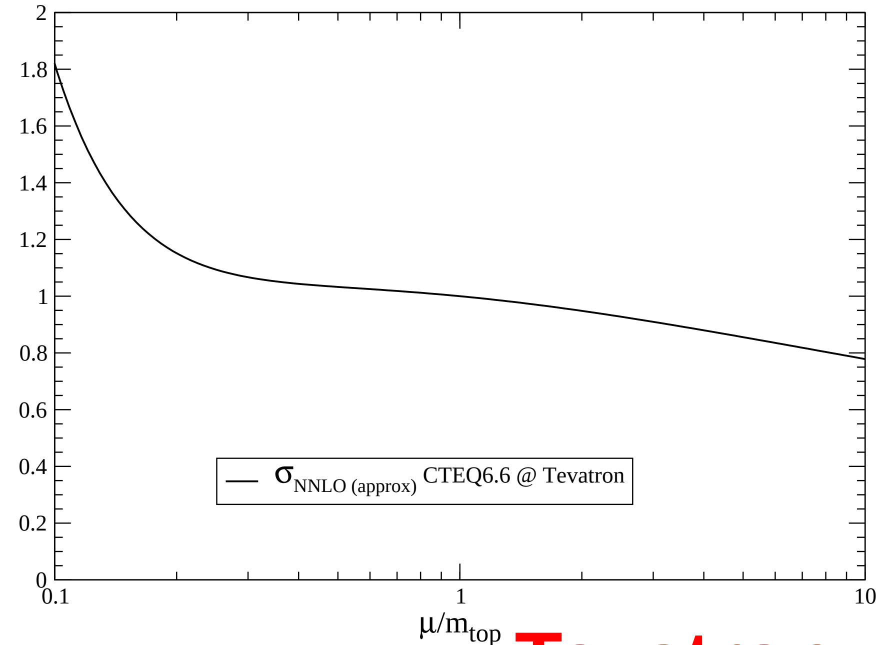
  - relax $\mu = \mu_R = \mu_F$ in study of theoretical uncertainty
  - allow for independent variation $\mu_R \neq \mu_F$

Scale dependence (I)

- Theoretical uncertainty from variation of scales $\mu = \mu_R \neq \mu_F$
  - plot with PDF set CTEQ6.6 (but largely independent on PDFs)
  - very stable predictions in range $\mu \in [m_t/2, 2m_t]$
    - $-3\% \leq \Delta\sigma \leq +0.5\%$ at LHC
    - $-4\% \leq \Delta\sigma \leq +3\%$ at Tevatron



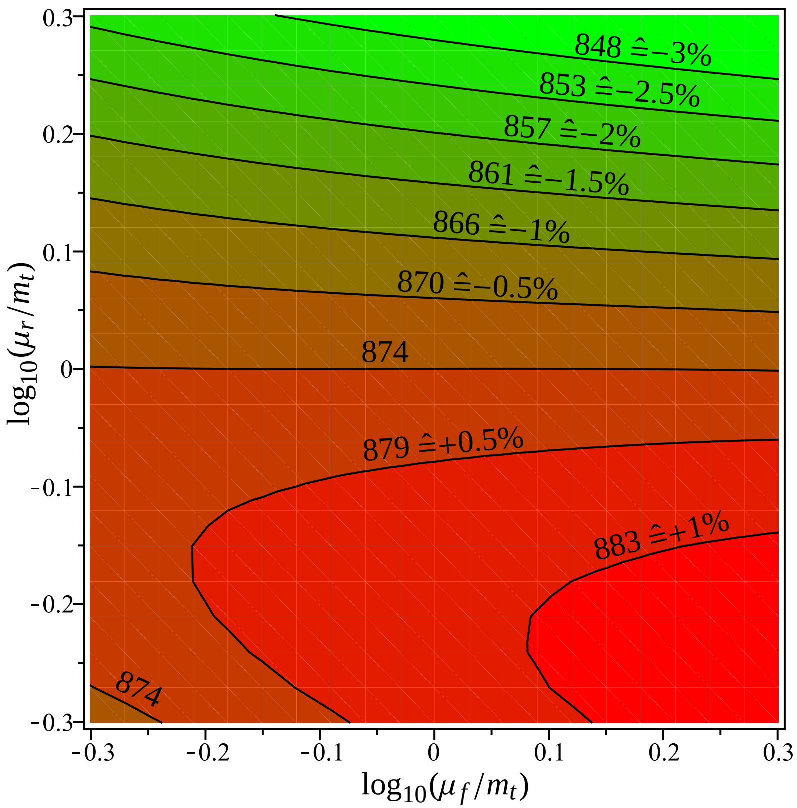
LHC

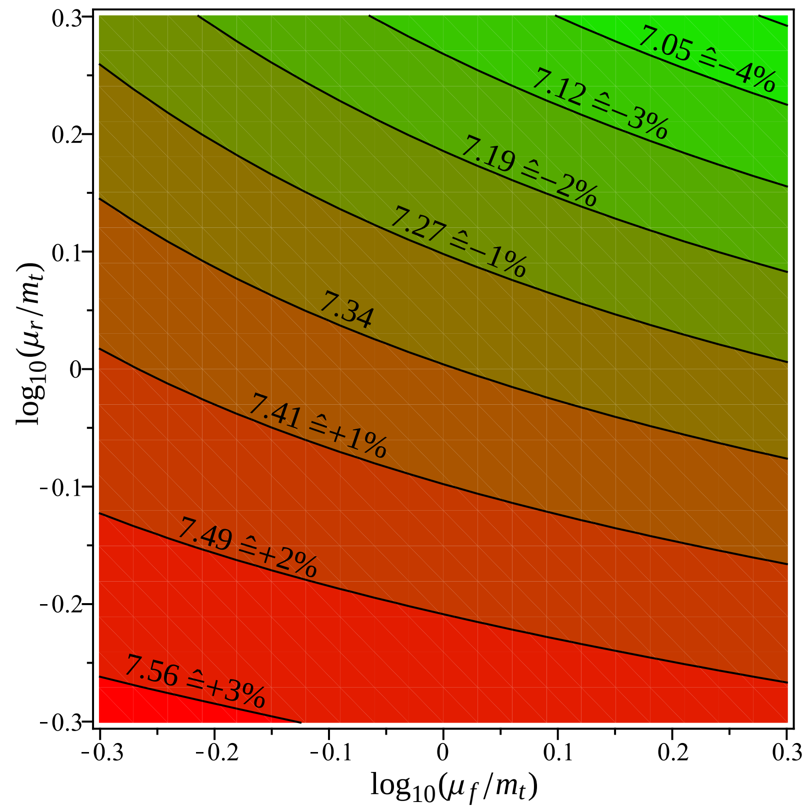


Tevatron

Scale dependence (II)

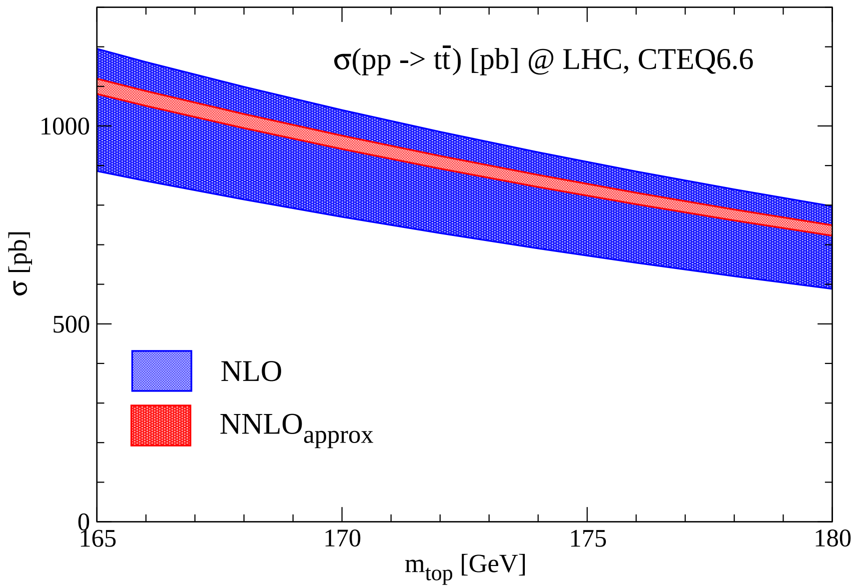
- Contour lines of total cross section for $\mu_R \neq \mu_F$
  - independent variation of renormalization and factorization scale
  - range corresponds to $\mu_R, \mu_F \in [m_t/2, 2m_t]$
  - plot with PDF set CTEQ6.6 (but largely independent on PDFs)

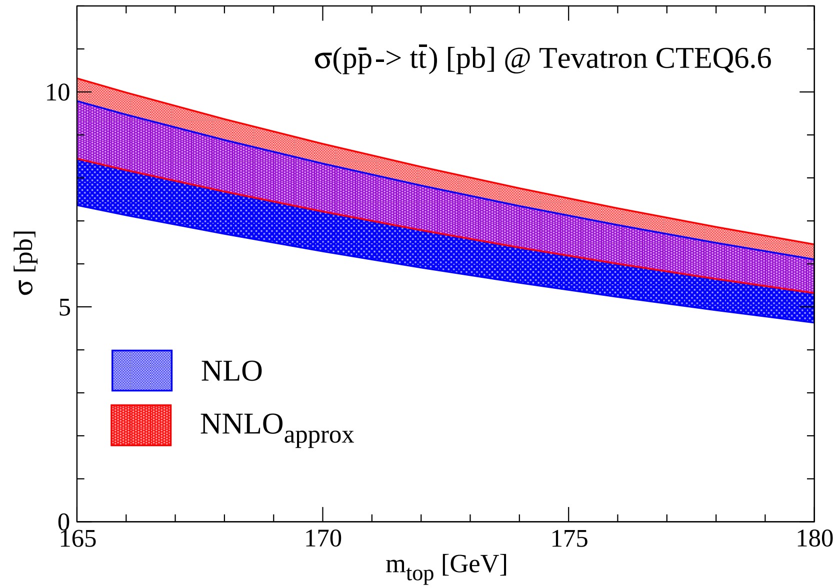




Total theory uncertainty

- NLO (with MRST2008 PDF set)
  - scale uncertainty $\mathcal{O}(10\%) \oplus$ PDF uncertainty $\mathcal{O}(5\%)$
- $\text{NNLO}_{\text{approx}}$ (with MRST2008 PDF set)
  - scale uncertainty $\mathcal{O}(3\%) \oplus$ PDF uncertainty $\mathcal{O}(2\%)$
- Theory at NNLO matches anticipated experimental precision $\mathcal{O}(10\%)$

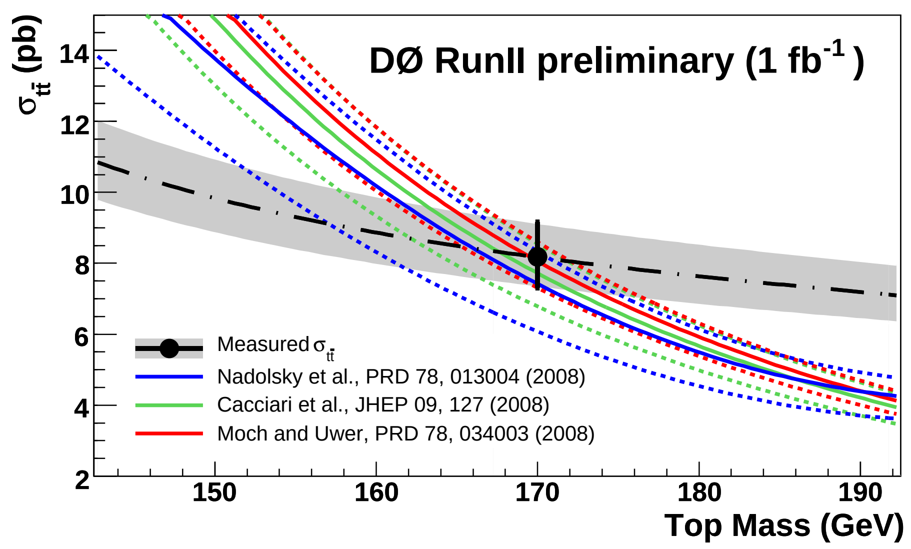


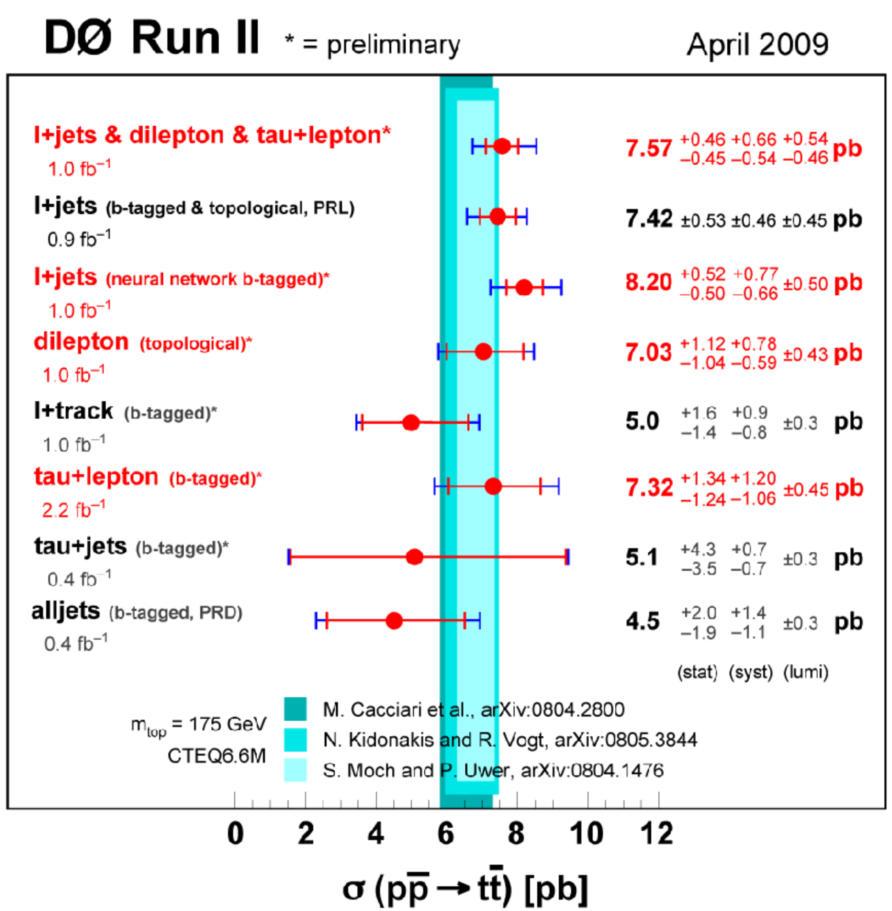


LHC **Tevatron**

Tevatron analyses

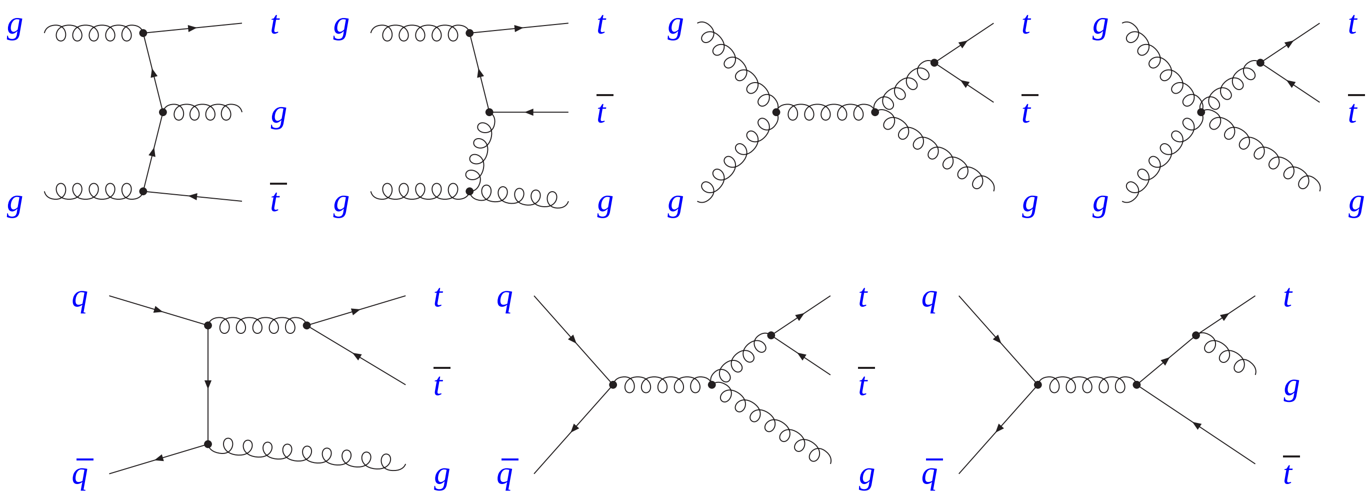
- Total cross section and different channels of Tevatron analyses (theory uncertainty band from scale variation)
- NNLO allows for precision determinations of m_t from total cross section (slope $d\sigma/dm_t$)





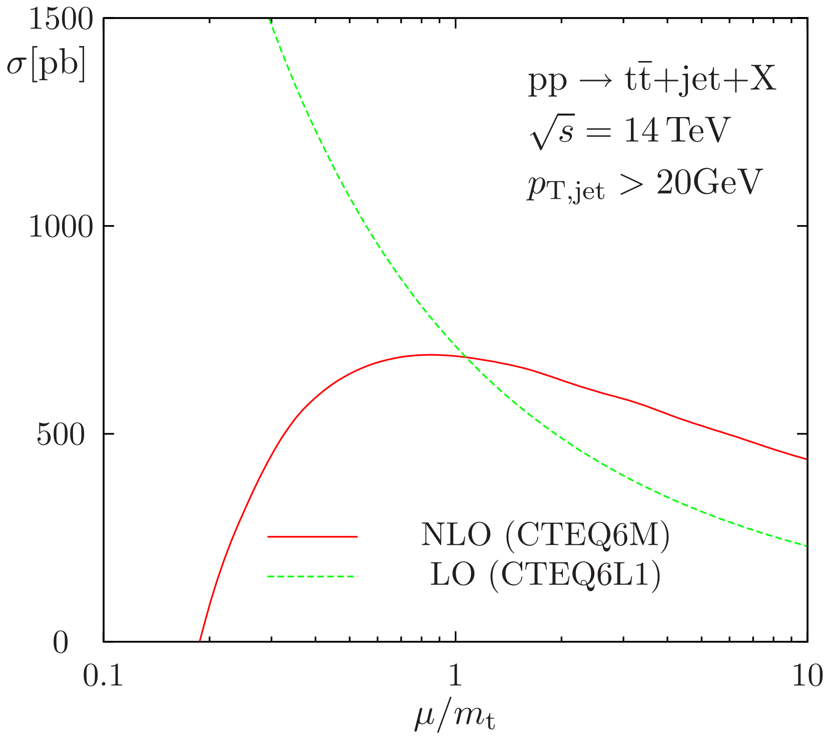
$t\bar{t}+$ jet production (I)

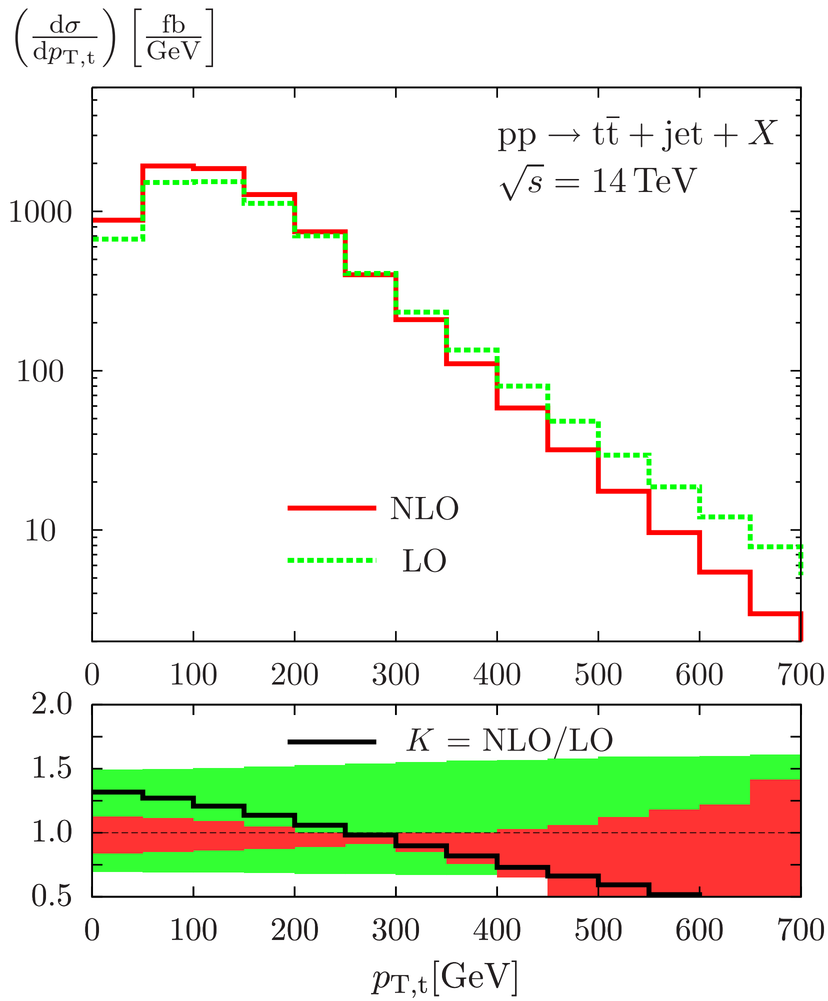
- LHC: large rates for production of $t\bar{t}$-pairs with additional jets
  - sample Feynman diagrams for $t\bar{t}+$ jet production at LO



- NLO corrections to $t\bar{t}+$jet production are part of NNLO corrections for inclusive $t\bar{t}$ production
  - at scale $\mu_R = \mu_F = m_t$ corrections are almost zero
  - threshold resummation captures dominant contributions

$t\bar{t}+$ jet production (II)

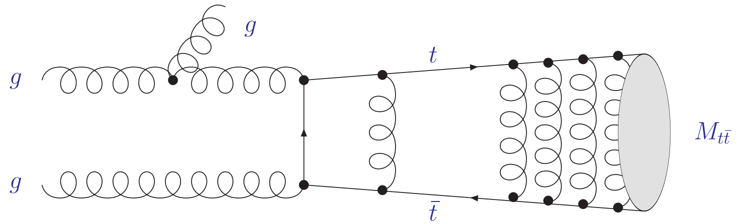




- Impressive state-of-the-art NLO QCD result Dittmaier, Uwer, Weinzierl '07-'08
  - much improved scale dependence of total rate
  - transverse-momentum distributions of top-quark $p_{T,t}$ along with K-factor and scale variation $m_t/2 \leq \mu \leq 2m_t$

Coulomb corrections

- Heavy quark production very close to threshold
  - resummation of Coulomb corrections $\sim 1/\beta$ to all orders (non-relativistic QCD)
  - NRQCD factorization Bodwin, Braaten, Lepage '95
- Much work (theory and phenomenology) for ILC
  - fixed center-of-mass energy S allows threshold scan at $\sqrt{S} \sim 2m_t$
  - dominant color-singlet production $\rightarrow t\bar{t}\left({}^3S_1^{[1]}\right)$
- Effects on top-mass measurement at LHC Hagiwara, Sumino, Yokoya '08
- Detailed study in NRQCD assembling existing knowledge at NLO/NLL
  Kiyo, Kühn, S.M., Steinhauser, Uwer '08
  - complete NLO NRQCD result Petrelli, Cacciari, Greco, Maltoni, Mangano '97 (corrections by Hagiwara, Sumino, Yokoya '08)
  - NLL resummation Cacciari '99

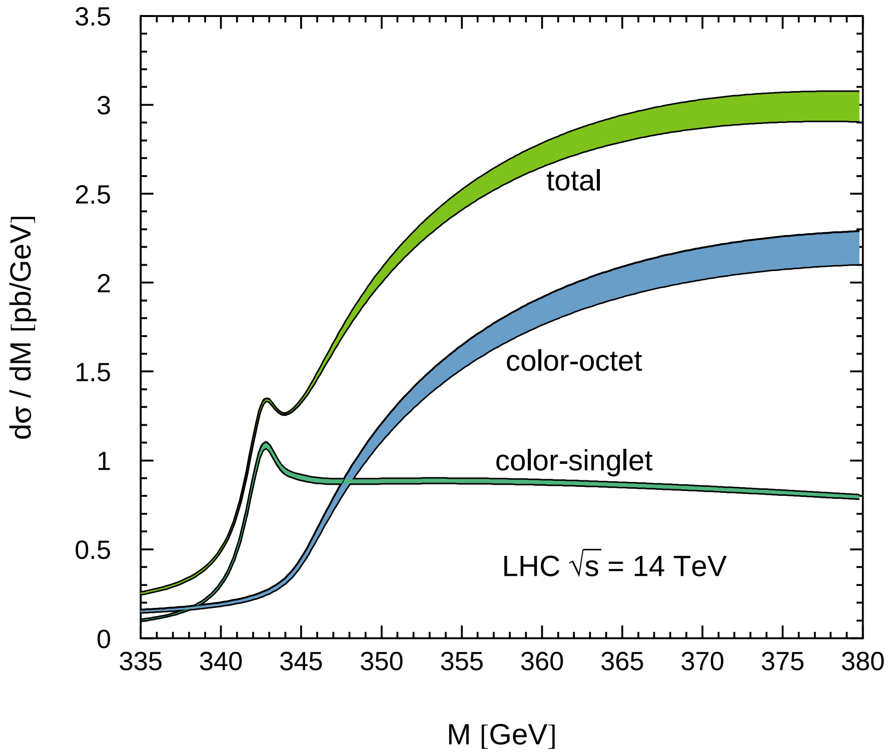


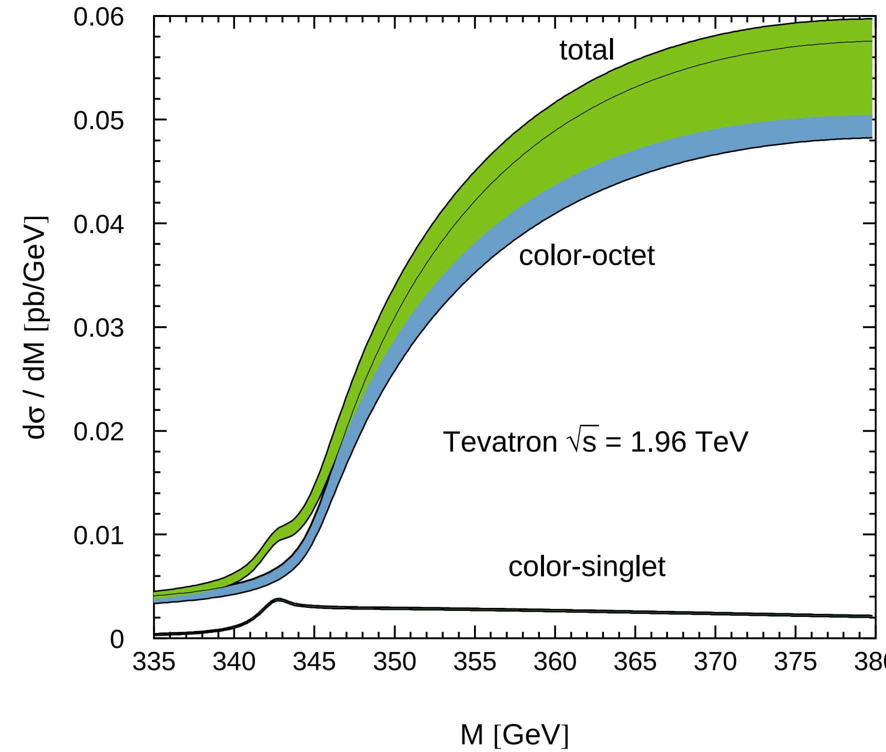
Coulomb corrections

- Recall master equation $$\sigma_{pp\to t\bar{t}} = \sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij\to t\bar{t}}$$
- Convolution with PDFs $f_i \otimes f_j$
  - top-quark pairs produced as color-singlets and color-octets $\to t\bar{t}\left({}^{2s+1}S_J^{[1,8]}\right)$
  - threshold at $M_{t\bar{t}} \sim 2m_t$ with $M_{t\bar{t}} = (p_t + p_{\bar{t}})^2$
- NRQCD factorization of partonic cross section into $\hat{\sigma}_{ij\to t\bar{t}} = F_{ij\to T} \otimes G(M_{t\bar{t}})$
  - free $t\bar{t}$ production rate F
  - evolution factor into “boundstate” (Green’s function) G
- Differential kinematics $\dfrac{d\hat{\sigma}_{ij\to t\bar{t}}}{dM_{t\bar{t}}^2} = F_{ij\to T} \times \Im G^{[1,8]}(M_{t\bar{t}})$
  - factorization of soft-collinear dynamics (real emission radiation)
  - matching at NLO and NLL resummation

Invariant mass distristribution

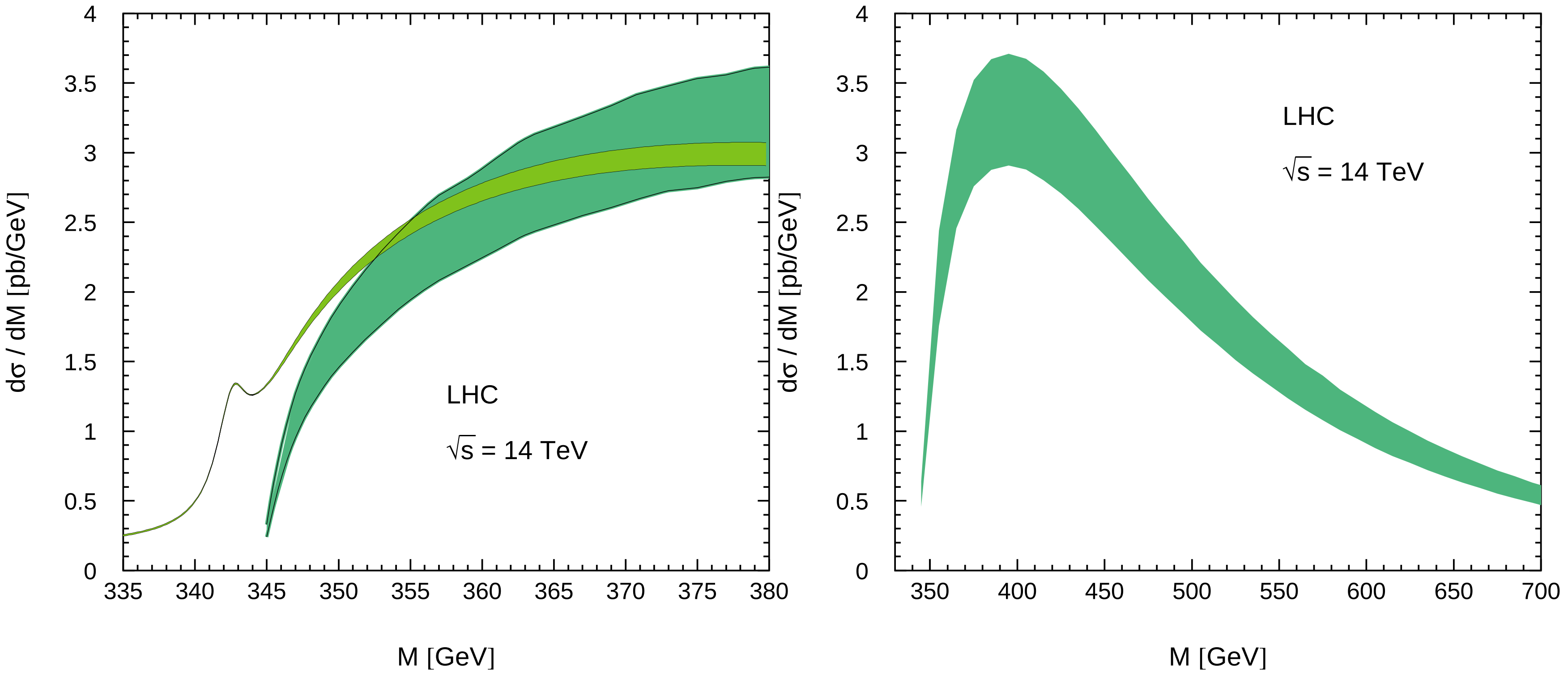
- $d\sigma/dM_{t\bar{t}}$ at LHC driven by large gluon luminosity
  - $gg \to t\bar{t}\left({}^1S_0^{[1]}\right)$ dominates
- $d\sigma/dM_{t\bar{t}}$ at Tevatron with small bound state effects
  - $q\bar{q}$-channel large with only color-octet configurations only





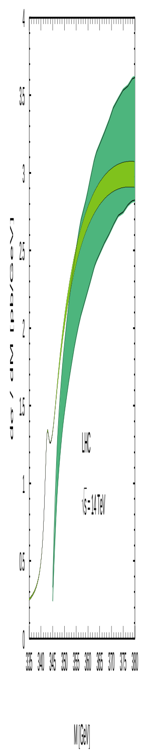
Matching to fixed order

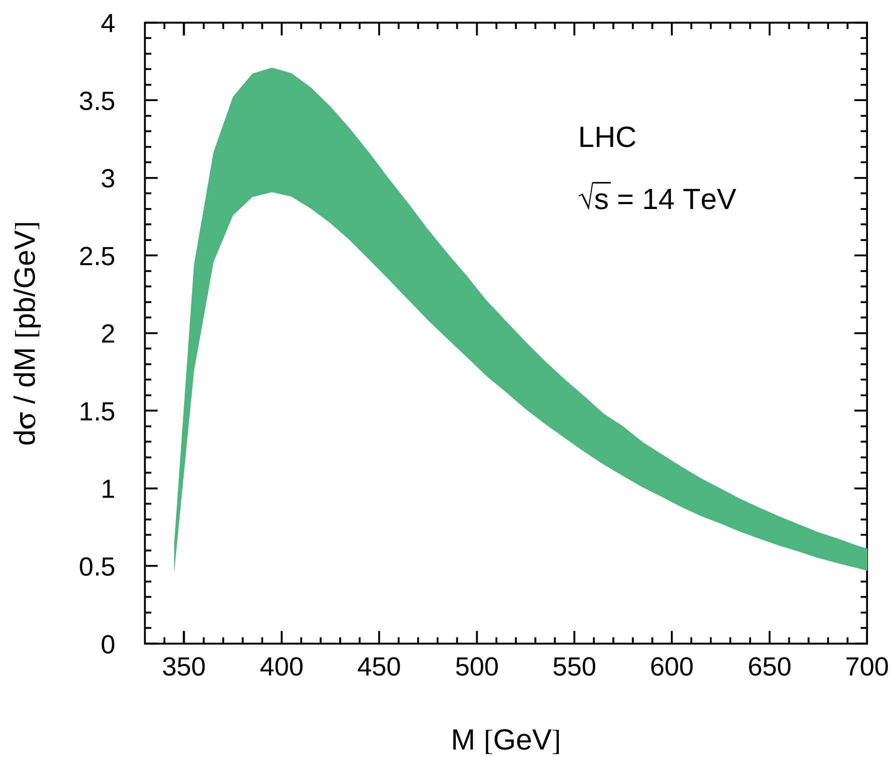
- $d\sigma/dM_{t\bar{t}}$ with at LHC
  - compare NLL resummed result in NRQCD (plain vanilla) NLO
    Mangano, Nason, Ridolfi '92
  - consistency check OK



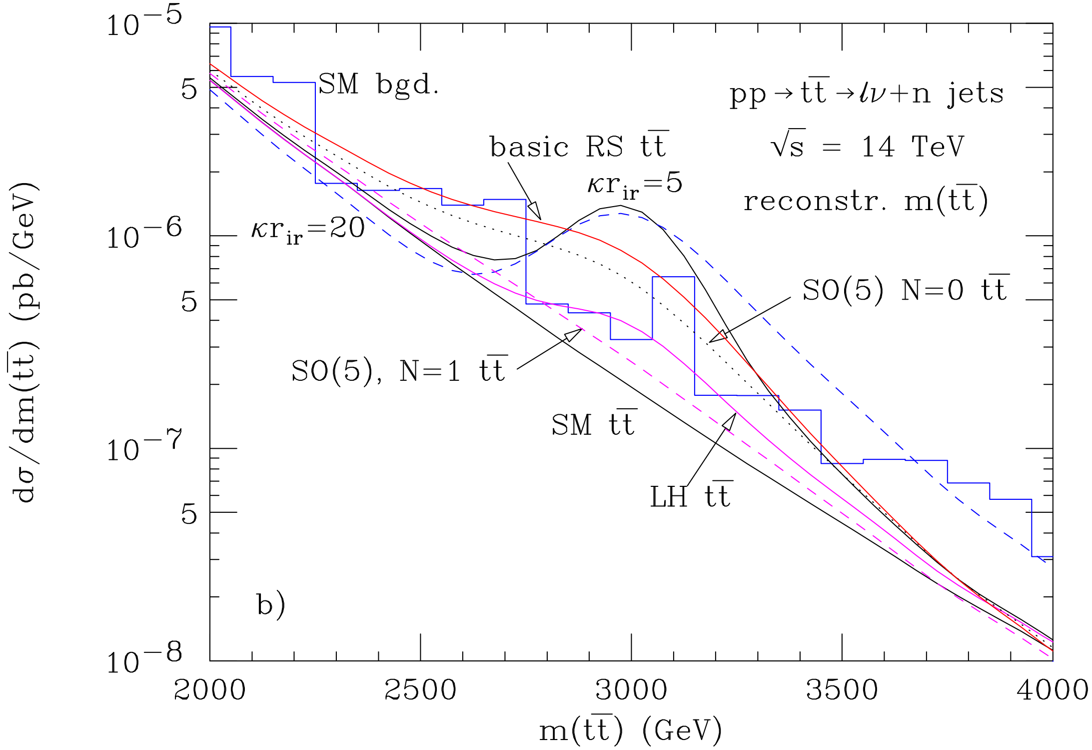
Invariant mass distristribution

- Resolution of bound state effects in $d\sigma/dM_{t\bar{t}}$ at LHC difficult (requires rather fine binning)
  - uncertainty of total cross section $\Delta\sigma \simeq \mathcal{O}(10)$ pb from
  - extrapolation of $M_{t\bar{t}}$-distribution affected by $gg \to t\bar{t}\left({}^1S_0^{[1]}\right)$





High-$M_{t\bar{t}}$ tail of invariant mass distribution



- New physics searches in high-end tail of invariant mass $M_{t\bar{t}}$-spectrum at LHC
  - Kaluza-Klein resonances (e.g. s-channel graviton exchange)
    Frederix, Maltoni '07; Baur, Orr '08

Summary

Massive amplitudes

- Factorization
  - soft and collinear limits of massive QCD amplitudes
  - relations between massless and massive amplitudes

Collider phenomenology

- Top quark theory
  - improved understanding of theory and application of new concepts
  - resummation important for Tevatron and LHC phenomenology

Higher orders

- QCD results for hard scattering at **new level of precision**