Multiparton production @ NLO with BlackHat and Sherpa

In collaboration with C. Berger, Z. Bern, L. Dixon, F. Febres Cordero, D. Forde, H. Ita, D. Kosower and D. Maître

Fermilab Theory Seminar, 10/15/2009

Based on 0907.1984 [hep-ph]
0902.2760 [hep-ph]
0709.2881 [hep-ph]
Outline

- Motivation
- Calculation of cross sections
- Techniques & implementation
  - SHERPA
  - BlackHat
- Physics Results (W+3 jet)
  - Tevatron
  - LHC
Motivation

- Many discoveries rely on precise theoretical predictions of signals and backgrounds.

**Single Top**

CDF & D0

- 5σ discovery

DØ, 2.3 fb⁻¹

Event Yield

Combination Output

CDF Run II, L = 3.2 fb⁻¹
- Single Top
- W+HF
- t̅t
- QCD+Mistag
- Other
- Data

[CDF,0903.0885]

WW / WZ

[D0, 0810.3873]

[D0,0903.0850] D0, 4.4σ evidence

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Many discoveries rely on precise theoretical predictions of signals and backgrounds

Susy search at LHC

[M.L. Mangano, 2008]

Multijet + missing ET final state

[Atlas Collaboration]
Motivation

- Tree-level (LO) predictions only qualitative estimates due to poor convergence of expansion in strong coupling
- NLO corrections can be very large (30–100% of LO)
- Matching algorithms can improve shapes, but normalization in still LO
- Scale uncertainties: (W+ n jets)

- Need NLO for reliable predictions!

<table>
<thead>
<tr>
<th>n</th>
<th>LO</th>
<th>NLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16%</td>
<td>7%</td>
</tr>
<tr>
<td>2</td>
<td>30%</td>
<td>10%</td>
</tr>
<tr>
<td>3</td>
<td>42%</td>
<td>12%</td>
</tr>
</tbody>
</table>

[Anastasiou, Dixon, Melnikov, Petriello, 2004]
## NLO wishlist (Les Houches 2009)

<table>
<thead>
<tr>
<th>Process</th>
<th>Comments</th>
<th>Motivation</th>
</tr>
</thead>
<tbody>
<tr>
<td>((V \in {Z, W, \gamma}))</td>
<td>(completed)</td>
<td></td>
</tr>
<tr>
<td><strong>pre Les Houches 2007</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. (pp \rightarrow VV\text{jet})</td>
<td>(V = Z) cases missing, (W)-decays included</td>
<td>Higgs background</td>
</tr>
<tr>
<td>2. (pp \rightarrow Higgs + 2\text{jets})</td>
<td>NLO QCD+EW to VBF</td>
<td></td>
</tr>
<tr>
<td>3. (pp \rightarrow VV V)</td>
<td>(\gamma) cases missing</td>
<td></td>
</tr>
<tr>
<td>4. (pp \rightarrow \bar{t}t \bar{b}b)</td>
<td>(m_b = 0), no (t)-decay</td>
<td>new physics background</td>
</tr>
<tr>
<td>5. (pp \rightarrow W + 3\text{jets})</td>
<td>(W)-decay included</td>
<td>background for (ttH)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>new physics background</td>
</tr>
<tr>
<td><strong>Les Houches 2007</strong></td>
<td>(in progress)</td>
<td></td>
</tr>
<tr>
<td>6. (pp \rightarrow \bar{t}t + 2\text{jets})</td>
<td>(V)-decays useful</td>
<td>relevant for (ttH)</td>
</tr>
<tr>
<td>7. (pp \rightarrow W W b\bar{b}),</td>
<td></td>
<td>relevant for (tt) benchmark process</td>
</tr>
<tr>
<td>8. (pp \rightarrow VV + 2\text{jets})</td>
<td></td>
<td>VBF (\rightarrow H \rightarrow VV)</td>
</tr>
<tr>
<td>9. (pp \rightarrow b\bar{b}b\bar{b})</td>
<td></td>
<td>Higgs and new physics signatures</td>
</tr>
<tr>
<td><strong>Les Houches 2009</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14. (pp \rightarrow W + 3\text{jets})</td>
<td>(W)-decay included</td>
<td>new physics background</td>
</tr>
<tr>
<td>15. (pp \rightarrow Wb\bar{b}\bar{b})</td>
<td>(m_b = 0) sufficient (?)</td>
<td>Higgs search</td>
</tr>
<tr>
<td>16. (pp \rightarrow jjjj)</td>
<td>leading color sufficient (?)</td>
<td>new physics background</td>
</tr>
<tr>
<td>17. (pp \rightarrow t\bar{t}t\bar{t})</td>
<td>NLO EW+QCD (completed)</td>
<td>new physics background</td>
</tr>
<tr>
<td>18. (pp \rightarrow Wjjjj)</td>
<td></td>
<td>new physics background</td>
</tr>
<tr>
<td>19. (H \rightarrow f\bar{f}f'f')</td>
<td></td>
<td>Higgs search</td>
</tr>
</tbody>
</table>

Still way to go....

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Calculation of cross sections

NLO:

\[ \sigma = \sigma^{LO} + \sigma^{NLO} \]

\[ \sigma^{NLO} = \int d\sigma^R_{m+1} + \int d\sigma^V_m \]

- Real correction:
  - Radiation of additional parton
  - Divergent in soft and collinear limits

- Virtual correction:
  - Loop amplitudes
  - Infrared poles after integration over loop momentum

Sum is free of divergences, but cancelation only after integration over phase space!
Calculation of cross sections

Construct finite integrands: Subtraction method

\[
\sigma^{NLO} = \int_{m+1} d\sigma^R - \int_{m+1} d\sigma^A + \int_m d\sigma^V + \int_{m+1} d\sigma^A
\]

\[
= \int_{m+1} (d\sigma^R - d\sigma^A) + \int_m (d\sigma^V + \int_1 d\sigma^A)
\]

Cancels soft/collinear singularities of the real correction

Simple enough to be integrated analytically over one-parton emission in dimensional regularization:

\[
\int_1 d\sigma^A_\epsilon = \epsilon^{-2}d\sigma^{(A,2)} + \epsilon^{-1}d\sigma^{(A,1)} + d\sigma^{(A,0)} + O(\epsilon)
\]

Poles cancel with virtual correction
Calculation of cross sections

Construct finite integrands: Subtraction method

- Introduce subtraction term $d\sigma^A$

$$\sigma^{NLO} = \int_{m+1} d\sigma^R - \int_{m+1} d\sigma^A + \int_m d\sigma^V + \int_{m+1} d\sigma^A$$

Integrands are finite and suitable for numerical integration over phase space
\[ \sigma = \int_{m} d\sigma^B + \int_{m+1} (d\sigma^R - d\sigma^A) + \int_{m} (d\sigma^V + \int_{1} d\sigma^A) \]
SHERPA is a full event generator, combining a number of perturbative and non-perturbative approaches to simulate high energy collisions

[TG, Hoeche, Krauss, Schoenherr, Schumann, Siegert, Winter]

Here just parts of the framework are used:
- The automated tree-level matrix element generator AMEGIC++, includes automated dipole subtraction
  [TG, Krauss]
- Phase space integration techniques
- The event generation framework and the ANALYSIS package to evaluate generated events

Now public: Sherpa v1.2 [www.sherpa-mc.de]
Automated dipole subtraction

[S. Catani, M.H. Seymour, 1997]
[TG, F. Krauss, 2007]

Subtraction term:
\[ d\sigma^A = \sum_{i \neq j \neq k} d\sigma^A_{ij,k} \]

(for \(m+1\)-parton real correction)

Momentum map:
\[ p_i, p_j, p_k \rightarrow \tilde{p}_{ij}, \tilde{p}_k \]

\[ d\sigma^A_{ij,k} = d\sigma^{LO}_{ij,k} \otimes dV_{ij,k} \]

\[ dV_{ij,k} \sim \frac{1}{p_i p_j} T_{ij} \cdot T_k \cdot V_{ij,k} \cdot d\Phi^{(1)} \]

Integrated subtraction term:
\[ \int\! d\sigma^A_{ab}(p_a, p_b) = \int\! d\sigma^B_{ab}(p_a, p_b) \otimes I(\varepsilon) \]

\[ I(\varepsilon) = -\frac{\alpha_S}{2\pi} \frac{2}{\Gamma(1-\varepsilon)} \sum_i \frac{1}{T_i^2} V_i(\varepsilon) \sum_{i \neq j} T_i \cdot T_j \left( \frac{4\pi\mu^2}{2p_i p_j} \right)^\varepsilon \]

m-parton LO-ME

Splitting operator
Convergence and consistency checks

Cutoff dependence of subtracted real correction:

\( \alpha_{\text{min}} = \min(\alpha_{\text{dipole}}) < \alpha_{\text{cut}} \)

i.e. FF–dipole:

\[ \alpha_{\text{dipole}} = y_{ij,k} = \frac{p_ip_j}{p_ip_j + p_ip_k + p_jp_k} \]

\[ \min \text{ dipole cut} = \min(\alpha_{\text{dipole}}) \]

\[ \text{e}^+\text{e}^- \rightarrow 3 \text{jets} \]

\[ \text{pp} \rightarrow 2 \text{jets} \]
Convergence and consistency checks

Cutoff dependence of subtracted real correction:

\[ \frac{1}{\sigma} \frac{d\sigma}{d(\log(\alpha_{\text{min}}))} \]

- \( e^+e^- \rightarrow 2 \text{ jets} \)
- \( e^+e^- \rightarrow 3 \text{ jets} \)
- \( e^-p \rightarrow e^- + \text{jet} \)
- \( p\bar{p} \rightarrow 2 \text{ jets} \)
Convergence and consistency checks

Consistency check with modified dipole terms: $d\sigma^A' = d\sigma^A \theta(\alpha - y)$

\[ e^+e^- \rightarrow 3\text{ jets} \]

\[ pp \rightarrow W + 3 \text{ jets} \]
Comparisons with other codes

- With M. Seymour’s code DISENT:
  \[ e^- e^+ \rightarrow 3 \text{jets}, \quad e^- p \rightarrow e^- + 2 \text{jets} \]
  - Explicit comparison of dipole terms
  - Comparison of the insertion operators I, K, P
  - Checks of integrated results for real (subtracted) correction and integrated dipole terms

- Comparison of dipole terms for \( pp \rightarrow ZZ + \text{jet} \) with a code by N. Kauer

- Checks of full NLO results for a number of processes with MCFM
Automatic Dipole Subtraction in SHERPA

\[ \sigma = \int_m d\sigma^B + \int_{m+1} \left[ d\sigma^R - d\sigma^A \right] + \int_m d\sigma^V + \int_1 d\sigma^A \]

**Interface:** 2 steps

1. **Initialization**
   Agreement about:
   - Required processes
   - Model parameters
   - Renormalization schemes
   - Sampling parameters
   - ...

1-loop codes, i.e.

- BlackHat
- Rocket
Automatic Dipole Subtraction in SHERPA

\[ \sigma = \int_m d\sigma^B + \int_{m+1} \left[ d\sigma^R - d\sigma^A \right] + \int_m \left[ d\sigma^V + \int_1 d\sigma^A \right] \]

Interface: 2 steps

2. Run time

\[ \left| M_{(1\text{-loop})} (\{p_i\}) \right|^2 \]

\[ = a\varepsilon^0 + b\varepsilon^{-1} + c\varepsilon^{-2} \]

1–loop codes, i.e.

BlackHat

Rocket

\[ \ldots \]
**Automatic Dipole Subtraction in SHERPA**

\[
\sigma = \int_m^\infty d\sigma^B + \int_{m+1} [d\sigma^R - d\sigma^A] + \int_m^\infty d\sigma^V + \int_1^\infty d\sigma^A
\]

**Interface:** 2 steps

\[ |M_{(1\text{-loop})}(\{p_i\})|^2 d\phi \]

**Interface details:** see (still preliminary)

Les Houches 2009 accord

Goal: Automating computation of QCD one-loop amplitudes

Use of unitarity techniques

- Work with gauge invariant on-shell objects
- Allows recursive calculation
- Better scaling with increasing number of external legs
  (compared to traditional Feynman diagrammatic approach)

C++ framework
Main technique: unitarity bootstrap

Decomposition of one-loop amplitudes into coefficients of scalar integrals and rational terms:

\[ A = R + C \]

\[ C = \sum_i b_i + \sum_i c_i + \sum_i d_i \]

- The coefficients \( b_i, c_i, d_i \) can be computed in \( d=4 \) dimensions using generalized unitarity
- The rational term is computed separately using on-shell recurrence relations
Integral coefficients from unitarity

- **Unitarity cut:** \( \frac{1}{p^2 - m^2 + i0} \to 2\pi \delta(p^2 - m^2) \)

- **Apply cuts on both side of the equation**

\[
\begin{align*}
R + \sum_i b_i + \sum_i c_i + \sum_i d_i & = b \\
+ \sum_i b_i & = c \\
+ \sum_i c_i & = d
\end{align*}
\]
On-shell recursions for rational terms

- Construct rational part recursively from on-shell tree-amplitudes ($T$) and rational pieces of one-loop amplitudes ($L$)

$$R = \sum_{T} + \sum_{L} + \sum_{T} + \text{spurious poles}$$

- "spurious poles" to cancel with unphysical poles in the cut part
  - Extracted numerically from the cut part (somewhat tricky and possible source for numerical inaccuracies, but works...)
Amplitudes and numerical stability

- Quantify numerical precision: \( x = \log_{10} \left( \frac{|A_{num} - A_{target}|}{A_{target}} \right) \)
- Improvement:
  - Perform tests to identify problematic phase space points
  - Recalculate those points with higher precision

\( \rightarrow \) Increases average computation time only by few %

\[
\text{Double precision tail}
\]

\[
\text{Improved treatment}
\]
W + jets physics...
Important background process to
- SM physics: Higgs, single top, diboson, …
- New physics

NLO at cross section level
- MCFM: $W+0/1/2$ jets
  [Campbell, Ellis, 2002]
- $W+3$ jet, Leading color
  [Berger, Bern, Dixon, Cordero, Forde, TG, Ita, Kosower, Maître, 2009]
  [Ellis, Melnikov, Zanderighi, 2009]
  [arXiv:0907.1984]

Full color
$W + n \text{ jets} \oplus \text{Tevatron}$

matched LO predictions

$\sigma_{\text{Data}} / \sigma_{\text{Theory}}$

integrated luminosity: $320 \text{ pb}^{-1}$

NLO predictions with MCFM

[CDF: PRD 77 011108, arXiv:0711.4044]
# W + n jet @ Tevatron

## Total Rates:

<table>
<thead>
<tr>
<th># of jets</th>
<th>CDF</th>
<th>LO</th>
<th>LC NLO</th>
<th>NLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$53.5 \pm 5.6$</td>
<td>$41.40^{+7.59}_{-5.94}$</td>
<td>$58.3^{+4.6}_{-4.6}$</td>
<td>$57.83^{+4.36}_{-4.00}$</td>
</tr>
<tr>
<td>2</td>
<td>$6.8 \pm 1.1$</td>
<td>$6.16^{+2.41}_{-1.58}$</td>
<td>$7.81^{+0.54}_{-0.91}$</td>
<td>$7.62^{+0.62}_{-0.86}$</td>
</tr>
<tr>
<td>3</td>
<td>$0.84 \pm 0.24$</td>
<td>$0.796^{+0.488}_{-0.276}$</td>
<td>$0.908^{+0.044}_{-0.142}$</td>
<td>$0.882^{+0.057}_{-0.138}$</td>
</tr>
</tbody>
</table>

Results from [Ellis, Melnikov, Zanderighi, 0906.1445]: (W+3 jets)

- LC: $1.01^{+0.05}_{-0.17}$
- LC* $R$: $0.91^{+0.05}_{-0.15}$

Jet algorithm: SISCone

- $E_T^{jet} > 25\text{GeV}$
- $E_T^e > 20\text{GeV}$
- $E_T^{miss} > 30\text{GeV}$
- $|\eta^{jet}| < 2$
- $|\eta^e| < 1.1$
- $M_T^W > 20\text{GeV}$

[CDF: PRD 77 011108, arXiv:0711.4044]

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“LC NLO” neglects subleading–color terms $\sim 1/N_C^2$ and contributions from closed fermion loops $\sim N_f/N_C$ in the finite part of one–loop amplitudes (which is reweighted by LO/LC LO)

All other pieces (born, real, subtraction) are computed in full color

Difference to full color in $W+1/2/3$ jets is just about 1–3%, which is much more expensive to compute

Our approach: use MC sampling techniques and compute the “difference” at a much lower rate
\[ \mu_r = \mu_f = \sqrt{M_W^2 + p_{T,W}^2} \]

\[ \mu / 2 \ldots 2 \mu \]
$\mu_r = \mu_f = \sqrt{M_W^2 + p_{T,W}^2}$

$\mu / 2 \ldots 2 \mu$
$\mu_r = \mu_f = \sqrt{M_W^2 + p_{T,W}^2}$

$\mu/2 \cdots 2\mu$
\[ H_T = \sum_j E_{T,j} + E_T^e + E_T^{\text{miss}} \]
$W + 3 \text{ jets @ Tevatron: total cross section}$

$W + 3 \text{ jets} + X$

$\sqrt{s} = 1.96 \text{ TeV}$

$E_T^{\text{jet}} > 20 \text{ GeV}, |\eta^{\text{jet}}| < 2$

$E_T^{\text{jet}} > 20 \text{ GeV}, |\eta^{\text{jet}}| < 1.1$

$E_T > 30 \text{ GeV}, M_T > 20 \text{ GeV}$

$R = 0.4$ [siscone]

$\mu_0 = M_W = 80.419 \text{ GeV}$

BlackHat+Sherpa

$\sigma$ [pb]

K-factor

$\mu / \mu_0$

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Predictions for the LHC

Collision energy: 14 TeV

\[ E_T^{\text{jet}} > 30 \text{GeV} \]
\[ E_T^e > 20 \text{GeV} \]
\[ E_T^{\text{miss}} > 30 \text{GeV} \]

\[ |\eta^{\text{jet}}| < 3 \]
\[ |\eta^e| < 2.5 \]
\[ M_T^W > 20 \text{GeV} \]

Jet algorithm: SISCone

[G. Salam, G. Soyez]
LHC results: Total cross section

$W^{-} + 3$ jets + X

$\sqrt{s} = 14$ TeV

$\mu_0 = 2 M_W = 160.838$ GeV

$E_T^{\text{jet}} > 30$ GeV, $|\eta^{\text{jet}}| < 3$

$E_T^{\ell} > 20$ GeV, $|\eta^{\ell}| < 2.5$

$R = 0.4$ [siscone]

BlackHat+Sherpa
LHC results

- Scale choices

\[ \mu = E_T^W = \sqrt{M_W^2 + p_{T,W}^2} \]

\[ \mu = H_T = \sum_j E_{T,j}^{jet} + E_T^e + E_T^{miss} \]
LHC results

- Scale choices

\[ \sum \mu_+^2 = \sum \mu_j^2 + \mu_H^2 + \mu_E^2 + \mu_{\mu}^2 \leq 30 \text{ GeV}, \quad |\eta|^\leq 3 \]
\[ \mu_+^2 > 20 \text{ GeV}, \quad |\eta|^\leq 2.5 \]
\[ \mu_j^2 > 30 \text{ GeV}, \quad M_T^2 > 20 \text{ GeV} \]

\[ W^{-} + 3 \text{ jets} + X \quad \sqrt{s} = 14 \text{ TeV} \]

Jet e miss

\[ W^{-} + 3 \text{ jets} + X \quad \sqrt{s} = 14 \text{ TeV} \]

\[ p \rightarrow W^{-} + 3 \text{ jets} + \not{p} \]

\[ p \rightarrow W^{-} + 3 \text{ jets} + \not{p} \]

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LHC results

\[ \frac{d\sigma}{dE_T} \text{ [pb/GeV]} \]

- LO
- NLO

\[ \mu = \mu_f = H_T \]

Scale variation: \( \mu / 2 \ldots 2\mu \)
LHC results

$W^+ + 3$ jets $+ X$

$\sqrt{s} = 14$ TeV

BlackHat+Sherpa

$E_T^{\text{jet}} > 30$ GeV, $|\eta^{\text{jet}}| < 3$

$E_T^{\text{jet}} > 20$ GeV, $|\eta^{\text{jet}}| < 2.5$

$E_T^{\text{jet}} > 30$ GeV, $M_T^\ast > 20$ GeV

$R = 0.4$ [siscone]

$\mu_R = \mu_F = \hat{H}_T$
Summary and outlook

- NLO calculations with BlackHat & Sherpa
  - New efficient computational approach to one-loop QCD amplitudes
- First results for $W+3$ jet to compare to data
- LHC predictions
- Future:
  - More processes ($Z+3$ jets, ...)
  - Improvements in speed
  - Automation
Now released

SHERPA v1.2

Includes:
- Dipole subtraction
- Analysis framework for NLO events

- A revised implementation of (LO) CKKW merging to work with multiple ME generators and Parton Showers
  [Hoeche, Krauss, Schumann, Siegert, 2009]
- The ME generator COMIX
  [TG, Hoeche, 2008]
- A new Parton shower, based on CS dipoles
  [Schumann, Krauss, 2009]
- And more…

www.sherpa-mc.de
For real (subtracted) correction: \( \int_{m+1} (d\sigma^R - d\sigma^A) \) the (m+1)–parton phase space is generated directly multi–channeling over parameterizations obtained from the (Feynman diagrammatic) structure of the real ME

- Generate a channel for each diagram using a few building blocks

\[ \sim P_0 (23) P_0 (45) D(23,45) D(2,3) D(4,5) \]

- Adapt to the full structure of the integrand by relative weights of single channels and a VEGAS grid for each map

- For \( P_0 \sim s^{-\nu} \) some caution necessary when choosing exponent \( \nu \) to not introduce (integrable) singularities