A particle physicist's perspective on topological insulators.

Andreas Karch

UIUC/Fermilab, October 10/11, 2010

based on: "Fractional topological insulators in three dimensions",
with J.Maciejko, X.-L. Qi, S. Zhang

also: "A holographic fractional topological insulator" with C. Hoyos and K. Jensen
as well as work with T. Takayanagi and J. Maciejko

Outline.

- Review of topological insulators
  (focus on effective field theory)

- Fractional topological insulators
  (work with Maciejko, Qi and Zhang)

- Holographic realization
  (work with Hoyos and Jensen)

- Quantum Spin Hall Effect
  (work with Maciejko and Takayanagi)

Review of topological insulators

Effective theory on insulators.

What is the low energy description of a generic, time reversal invariant insulator?

Insulator = gapped spectrum

Low energy DOFs: only Maxwell field.

Task: Write down the most general action for E and B, with up to two derivatives, consistent with symmetries.

Low energy effective action.

Low energy DOFs: only Maxwell field.

$$S_0 = \int d^3x dt L_0 = \frac{1}{8\pi} \int d^3x dt \left(\epsilon \vec{E}^2 - \frac{1}{\mu} \vec{B}^2 \right).$$

Permittivity and Permeability.

Rotations allow one extra term.

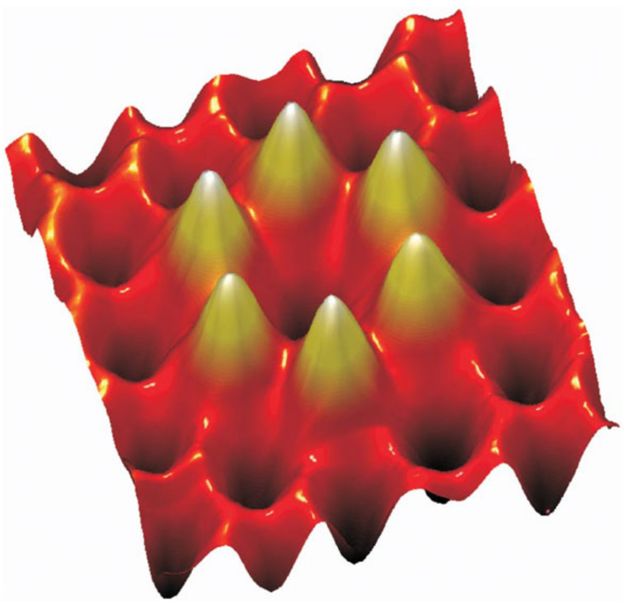
$$S_\theta = \frac{\theta}{2\pi}\frac{\alpha}{16\pi}\int d^3x dt \epsilon_{\mu\nu\rho\tau} F^{\mu\nu} F^{\rho\tau} = \frac{\theta}{2\pi}\frac{\alpha}{4\pi}\int d^3x dt \partial^\mu (\epsilon_{\mu\nu\rho\sigma} A^\nu \partial^\rho A^\tau)$$

$$= \frac{\theta\alpha}{4\pi}\int d^3 x dt (E \cdot B)$$

But: Under time reversal E is even, B odd

So naively the most general description of a time reversal invariant insulator does not allow for a theta term.

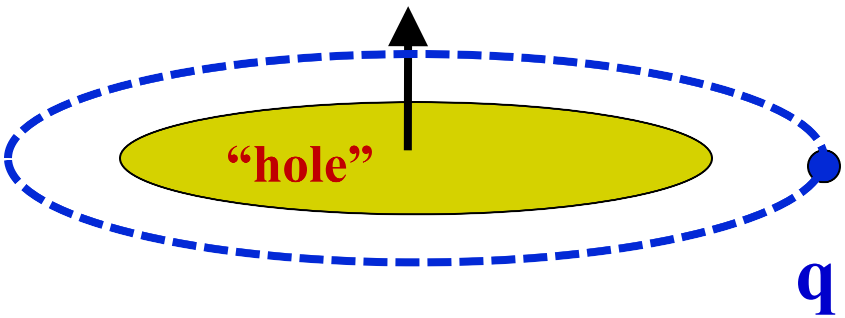
Magnetic Flux Quantization.



(from Hilgenkamp et. al., Nature **422)**

In superconductors:

$$\Phi = \int \vec{B} \cdot d\vec{S} = \oint \vec{A} \cdot d\vec{l}$$



As charge q encircles the flux, its wavefunction picks up a phase ħqΦ.

$$\Rightarrow \qquad \Phi = n\,\frac{h}{q}$$

Flux Quantization.

Dirac Quantization of magnetic charge:

$$g = n\frac{e}{2\alpha}$$

Implies quantization of magnetic flux!

$$\int_S F = g$$

On any Euclidean closed 4-manifold M:

$$\frac{\alpha}{32\pi^2}\int_M d^4x F_{\mu\nu}F_{\sigma\tau}\epsilon^{\mu\nu\sigma\tau} = N \in \mathbf{Z}$$

Flux Quantization.

- Partition function

$$Z(\theta) = \exp\left\{ i\frac{\alpha\theta}{32\pi^2}\int_M d^4x F_{\mu\nu}F_{\sigma\tau}\epsilon^{\mu\nu\sigma\tau}\right\} = e^{iN\theta}$$

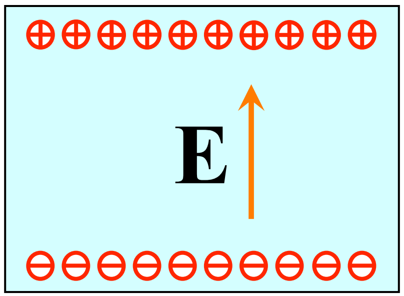
- is periodic in $\theta \rightarrow \theta+2\pi$ (Abelian version of the "Θ vacuum" (Callan, Dashen, Gross 1976, Jackiw&Rebbi, 1976))
- θ is time-reversal odd
- $\rightarrow$ time-reversal invariant insulator can have $\theta=0$ or π
- Z_2 classification

Topological Insulators

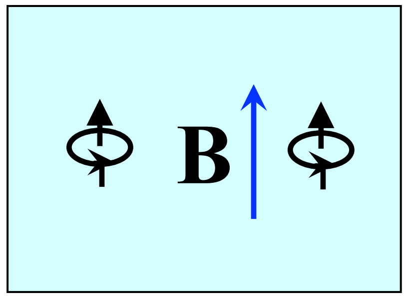
Low energy description of a T-invariant insulator described by 3 parameters: ε, μ, and:

$\theta = 0$	Topologically trivial insulators
$\theta = \pi$	Topologically non-trivial insulators

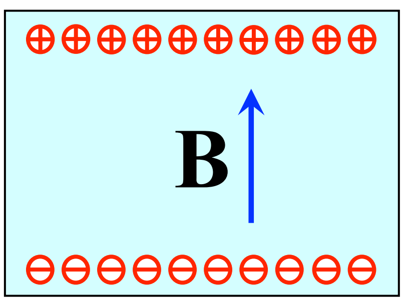
Physical Consequences.



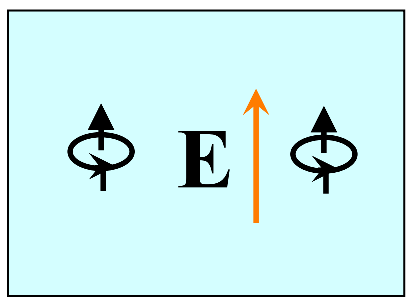
4πP=(ε-1)**E**



4πM=(1-1/μ)**B**



4πP=(αθ/2π)**B**



4πM=(αθ/2π)**E**

A topological term in the action

$$
\begin{aligned}
\nabla \cdot \mathrm{D} &= 4\pi\rho \\
\nabla \cdot \mathrm{B} &= 0 \\
\nabla \times \mathbf{E} &= -\frac{1}{c}\frac{\partial \mathrm{B}}{\partial t} \\
\nabla \times \mathrm{H} &= \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathrm{D}}{\partial t} \\
\mathrm{D} &= \mathbf{E} + 4\pi\mathrm{P} - \frac{\theta}{\pi}\alpha\mathrm{B} \\
\mathrm{H} &= \mathrm{B} - 4\pi\mathrm{M} + \frac{\theta}{\pi}\alpha\mathrm{E}
\end{aligned}
$$

Implications of Generalized Constitutive Relation.

Physics 514 Homework Set #3 Winter 2010

Due in class 1/26/10

300 pts

3. `(100 pts)` A topological insulator is a material (e.g. Bi_2Te_3) with constitutive relations

$$\vec{D} = \epsilon\vec{E} - \alpha\vec{B}, \quad \vec{H} = \frac{\vec{B}}{\mu} + \alpha\vec{E}$$

where α is the fine structure constant. For simplicity let us assume that $\epsilon \approx \epsilon_0$ and $\mu \approx \mu_0$ and only investigate the effect of the non-trivial "topological magneto-electric" effect, that is the apperance of $\vec{B}$ in $\vec{D}$ and the appearance of $\vec{E}$ in $\vec{H}$. The equations of electro- and magneto-statics are unmodified

$$\vec{\nabla} \cdot \vec{D} = \rho_e, \quad \vec{\nabla} \cdot \vec{B} = \rho_m, \quad \vec{\nabla} \times \vec{H} = \vec{j}_e, \quad \vec{\nabla} \times \vec{E} = \vec{j}_m$$

where ρ and $\vec{j}$ denote the free charge and current densities and subscripts e and m denote electric and magnetic charges respectively. We know that for physical charges $\rho_m = \vec{j}_m = 0$. Consider a planar interface between such a topological insulator at $z < 0$ and vacuum at $z > 0$.

a. Derive the boundary conditions obeyed by magnetic and electric fields at the interface (assuming no free surface charges or currents).

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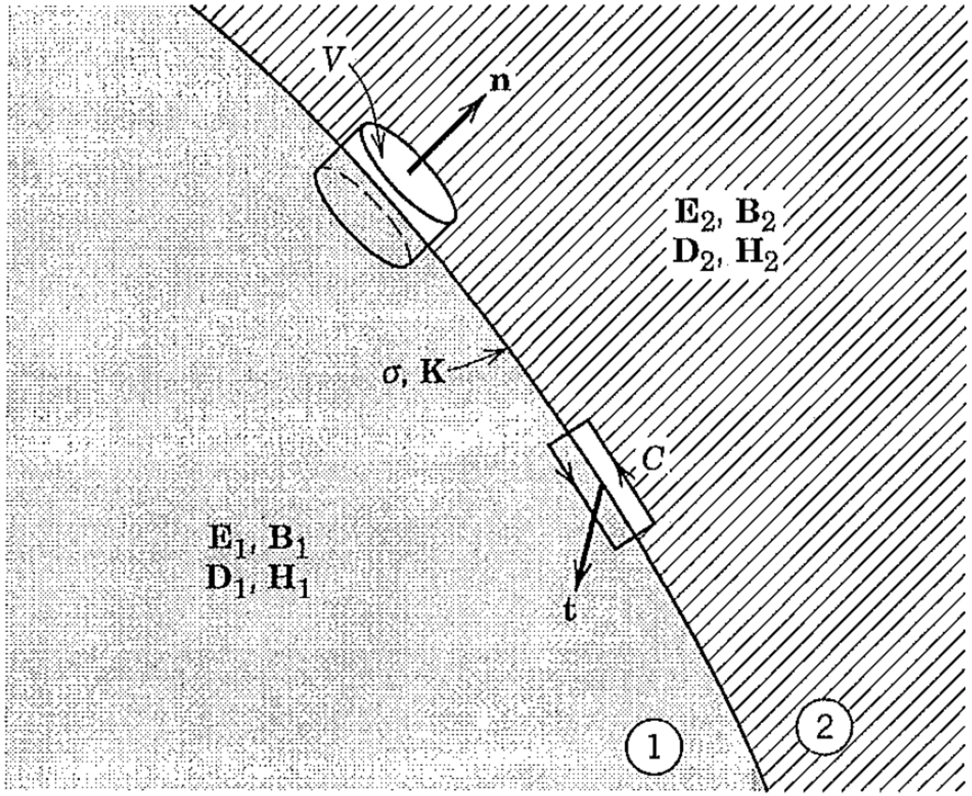
Boundary Conditions

Maxwell unmodified $\rightarrow$ BC unmodified

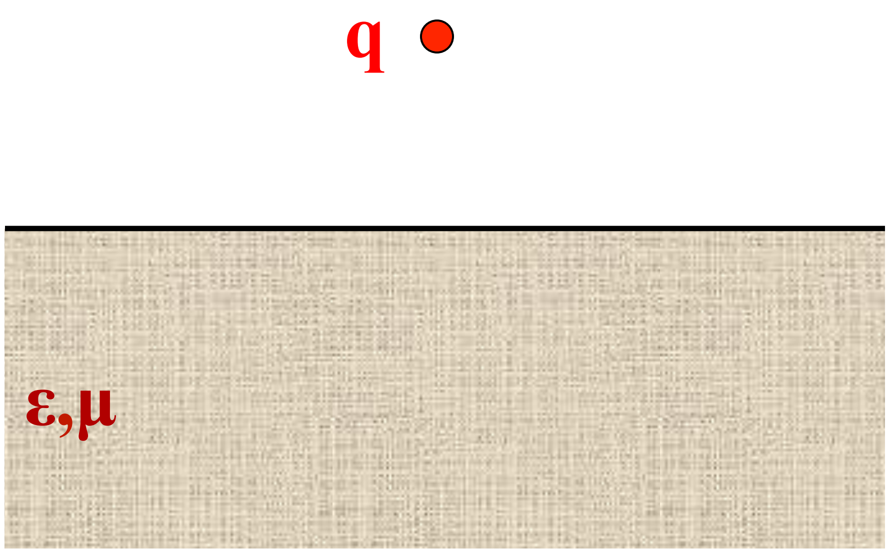


$E_{\|}$, $H_{\|}$ continuous

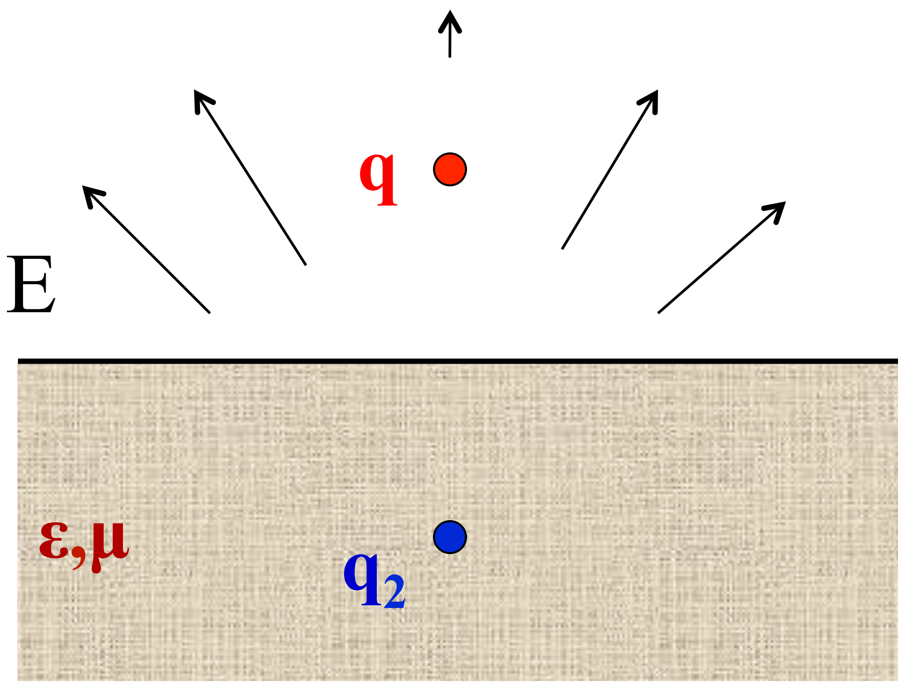
$B_{\perp}$, $D_{\perp}$ continuous

(in the absence of macroscopic surface charge or current densities)

E=?



Method of Images:

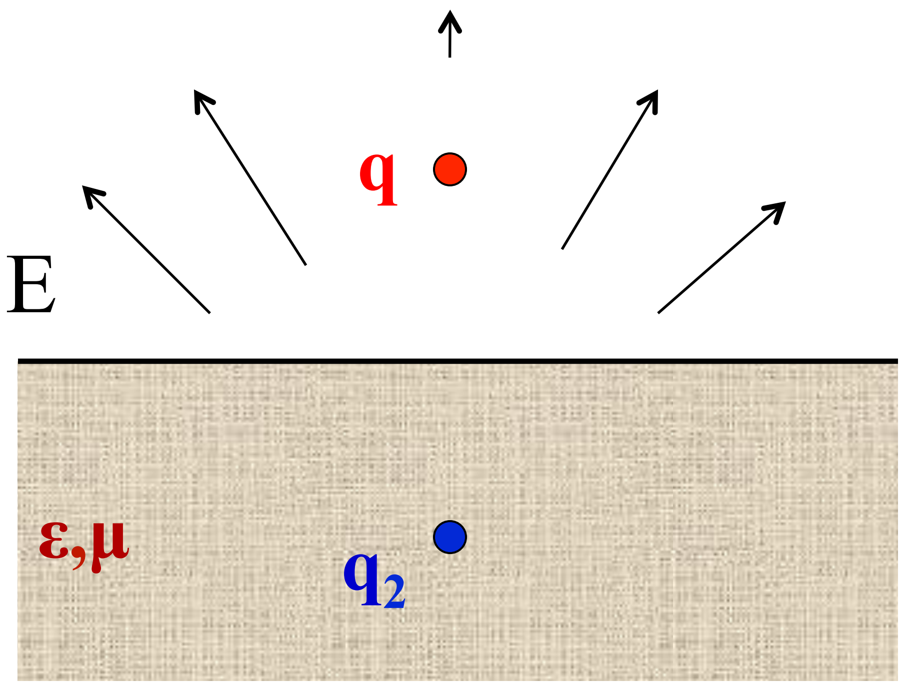


$E_{||}$, $D_{\perp}$ continuous

$$q_2 = -\left(\frac{\epsilon - \epsilon_0}{\epsilon + \epsilon_0}\right) q$$

Discontinuity in $E_{\perp}$:
microscopic surface charge density

Method of Images:



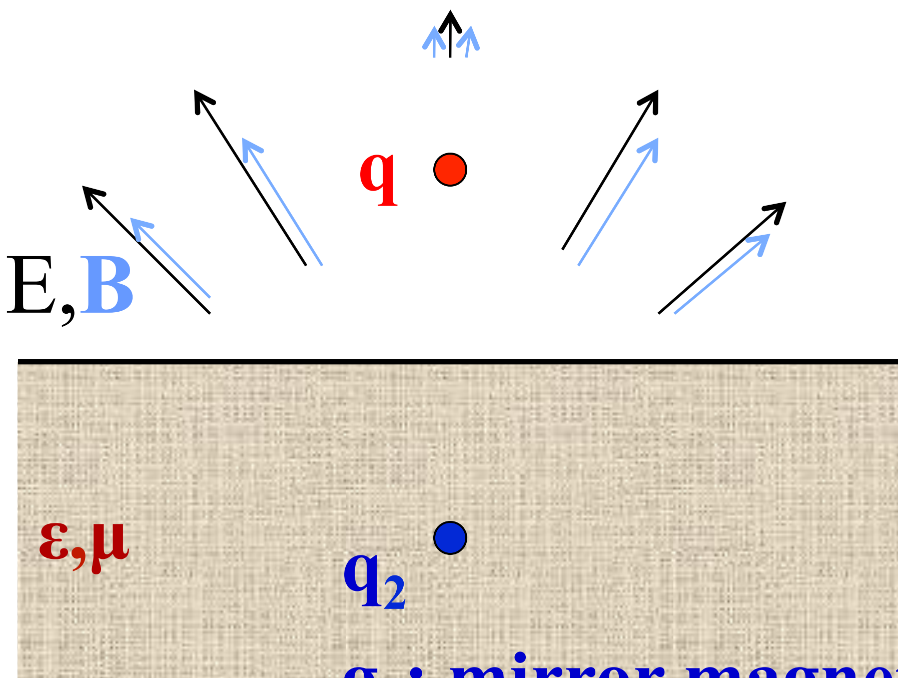
Now: $$\boxed{\vec{H} = \frac{\vec{B}}{\mu_0}}$$

$E_{\|},\ H_{\|}$ continuous
$B_{\perp},\ D_{\perp}$ continuous

$$\boxed{\vec{H} = \frac{\vec{B}}{\mu} + \frac{\alpha\theta}{\pi}\vec{E}}$$

non-zero

Method of Images:



g_2: mirror magnetic monopole charge

Now: $\boxed{\vec{H} = \frac{\vec{B}}{\mu_0}}$

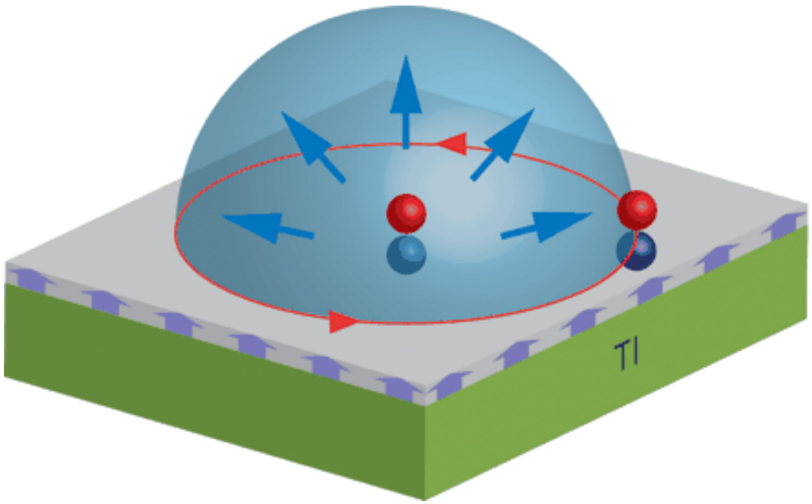
$E_{\parallel}$, $H_{\parallel}$ continuous
$B_{\perp}$, $D_{\perp}$ continuous

$$\boxed{\vec{H} = \frac{\vec{B}}{\mu} + \frac{\alpha\theta}{\pi}\vec{E}}$$

non-zero

Discontinuity in $B_{\parallel}$:
microscopic surface **current** density

Magnetic Monopoles in TI



Mirror charge of an electron is a **magnetic monopole** **(Qi, Li, Zang, Zhang *Science*)**

first pointed out by Lee and Sikivie for "axion domain walls"

$$q_1 = q_2 = \frac{1}{\epsilon_1} \frac{(\epsilon_1 - \epsilon_2)(1/\mu_1 + 1/\mu_2) - 4\alpha^2 P_3^2}{(\epsilon_1 + \epsilon_2)(1/\mu_1 + 1/\mu_2) + 4\alpha^2 P_3^2} q$$

$$g_1 = -g_2 = -\frac{4\alpha P_3}{(\epsilon_1 + \epsilon_2)(1/\mu_1 + 1/\mu_2) + 4\alpha^2 P_3^2} q$$

$$P_3 = \frac{\theta}{2\pi}$$

Maxwell has E/q ↔ B/g symmetry. So why so complicated?

Duality Covariant Mirror Charges (AK)

Duality:

$SL(2,\mathbb{Z})$

$$\begin{pmatrix} \vec{D} \\ 2\alpha\vec{B} \end{pmatrix} = \Lambda \begin{pmatrix} \vec{D}' \\ 2\alpha\vec{B}' \end{pmatrix}, \quad \begin{pmatrix} 2\alpha\vec{E} \\ \vec{H} \end{pmatrix} = (\Lambda^T)^{-1} \begin{pmatrix} 2\alpha\vec{E}' \\ \vec{H}' \end{pmatrix}$$

$$\begin{pmatrix} \rho_e \\ 2\alpha\rho_m \end{pmatrix} = \Lambda \begin{pmatrix} \rho'_e \\ 2\alpha\rho'_m \end{pmatrix}, \quad \begin{pmatrix} \vec{j}_e \\ 2\alpha\vec{j}_m \end{pmatrix} = \Lambda \begin{pmatrix} \vec{j}'_e \\ 2\alpha\vec{j}'_m \end{pmatrix}$$

Constitutive Relation:

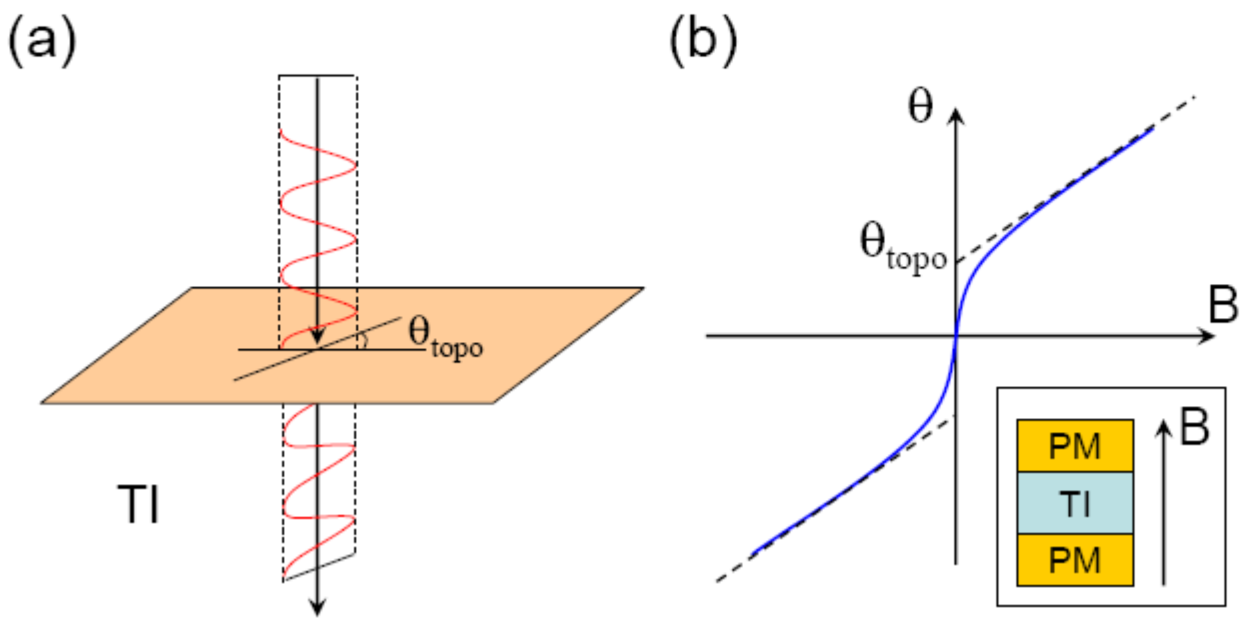
$$\begin{pmatrix} \vec{D} \\ 2\alpha\vec{B} \end{pmatrix} = \mathcal{M} \begin{pmatrix} 2\alpha\vec{E} \\ \vec{H} \end{pmatrix}$$

Mirror Charges: $\vec{q}^{(2)} = -\vec{q}^{(1)} = (\mathcal{T}+1)^{-1}(\mathcal{T}-1)\,\vec{q}.$

$$\mathcal{T} = \mathcal{M}_1\mathcal{M}_2^{-1} \qquad \mathcal{M} = \Lambda\mathcal{M}'\Lambda^T$$

$$\mathcal{T} = \Lambda\mathcal{T}'\Lambda^{-1}$$

Faraday and Kerr Effect



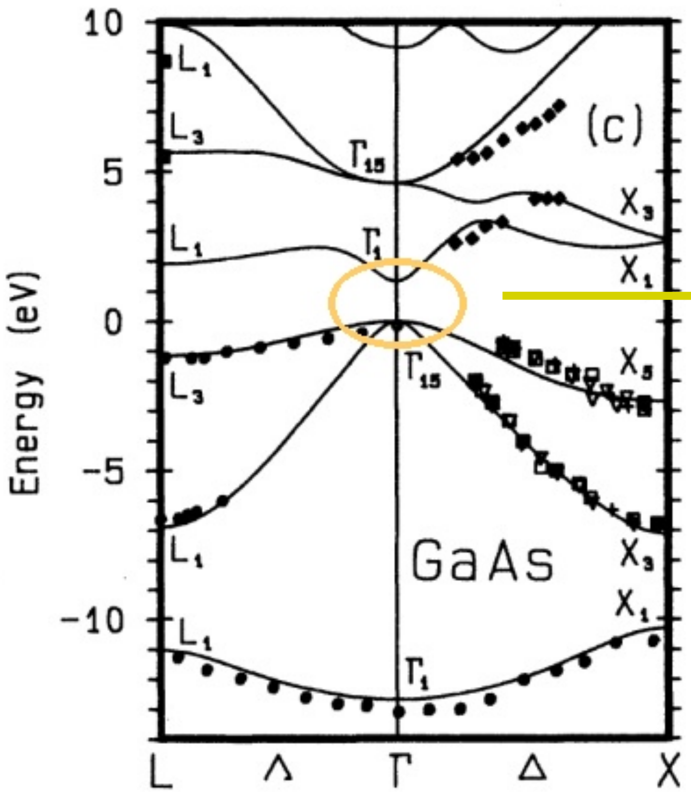
change in θ

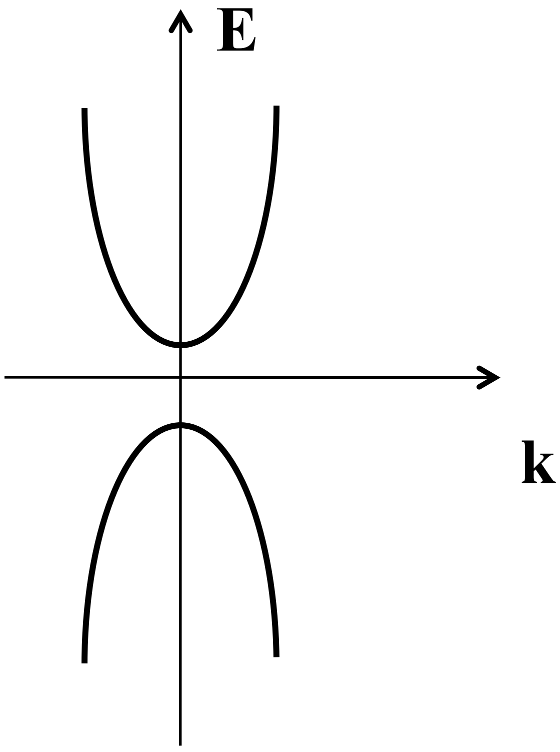
B independent contribution to Kerr/Faraday **(Qi et al)**

(Polarization of transmitted and reflected wave rotated by angle θ)

$$\theta_{\text{topo}} = \arctan \frac{2\alpha\Delta}{\sqrt{\epsilon/\mu} + \sqrt{\epsilon'/\mu'}}$$

Microscopic Model:





Spectrum of free massive Dirac fermion.

A Microscopic Model

A microscopic model: Massive Dirac Fermion.

$$\mathcal{L} = \bar{\psi}(i\partial_\mu \gamma^\mu - M)\psi$$

Time Reversal: $M \longrightarrow M^*$

Time reversal system has real mass.
Two options: positive or negative.

Chiral rotation and ABJ anomaly.

Massless theory invariant under chiral rotations:

$$\psi \to e^{-i\phi\gamma_5/2}\psi$$

Symmetry of massive theory if mass transforms:

$$M \to e^{i\phi} M$$

Phase can be rotated away! Chose M positive.

Chiral rotation and ABJ anomaly.

But in the quantum theory chiral rotation is anomalous. Measure transforms.

$$\Delta\mathcal{L} = C\alpha\frac{\phi}{32\pi^2}\,\mathrm{tr}\ \epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$

$$C = \sum\nolimits_{fields} q^2 = 1\cdot 1^2 = 1$$

Single field with unit charge.

$$\theta \to \theta - C\phi$$

Chiral rotation and ABJ anomaly.

$$\theta \rightarrow \theta - C\phi$$

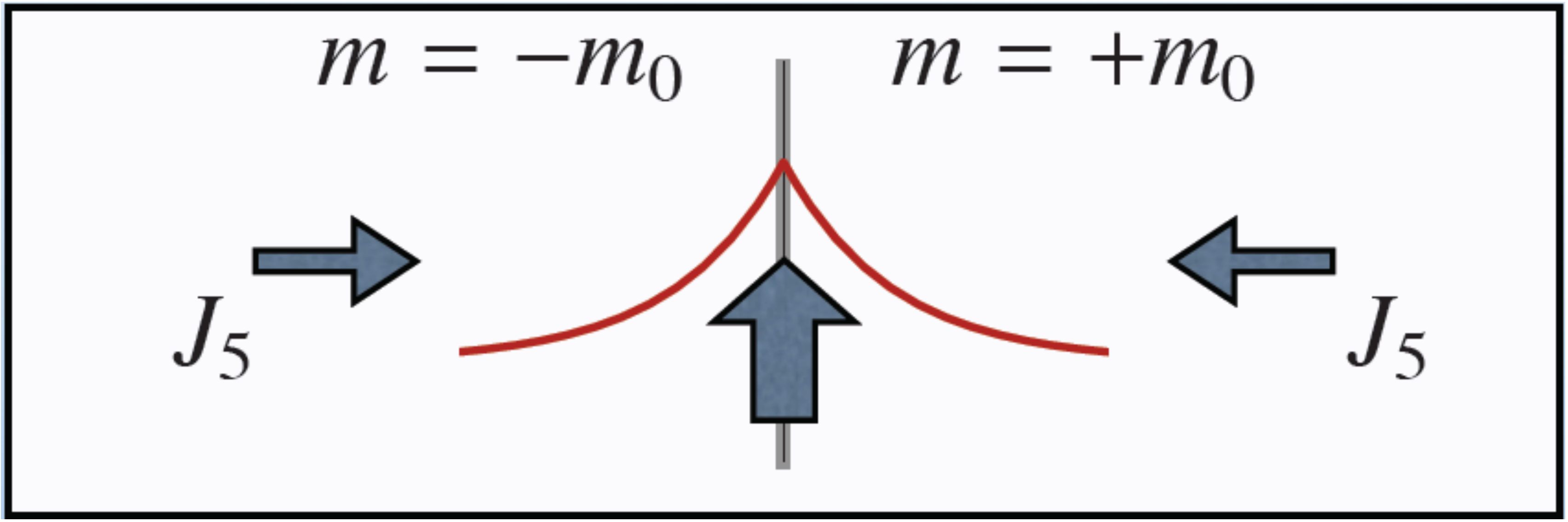
Axial rotation with $\Phi=\pi$:

- Rotates real negative mass into positive mass.
- Generates $\theta=\pi$!

Positive mass = Trivial Insulator.
Negative mass = Topological Insulator.

Localized Zero Mode on Interface.

Domain Wall has localized zero mode!



Domain Wall = TI/non-TI Interface

The boundary point of view.

Surface state stable against perturbations:

* T-invariance does not allow for mass term
* "Dirty" surface layer can only add/remove fermions in pairs! (Nielsen/Ninomiya)
* Single 2+1 fermion has parity anomaly
* Hall conductivity can be shifted by integers; half-integer contribution robust

Surface zero mode =experimental signature!

Generalizations:

Bulk topological insulator $\rightarrow$ Boundary theory With anomaly

4+1	Callan-Harvey, Kaplan	Z	axial anomaly	3+1
3+1	TRI topological insulator	Z_2	parity anomaly	2+1
2+1	Quantum Hall	Z	chiral anomaly	1+1

A Lattice Realization.

How to get $\theta = \pi$ from non-interacting electrons in periodic potential (Band-Insulator)?

Topology of Band Structure!

Define Z_2 valued topological invariant of bandstructure to distinguish trivial ("positive mass") from topologically non-trivial ("negative mass").

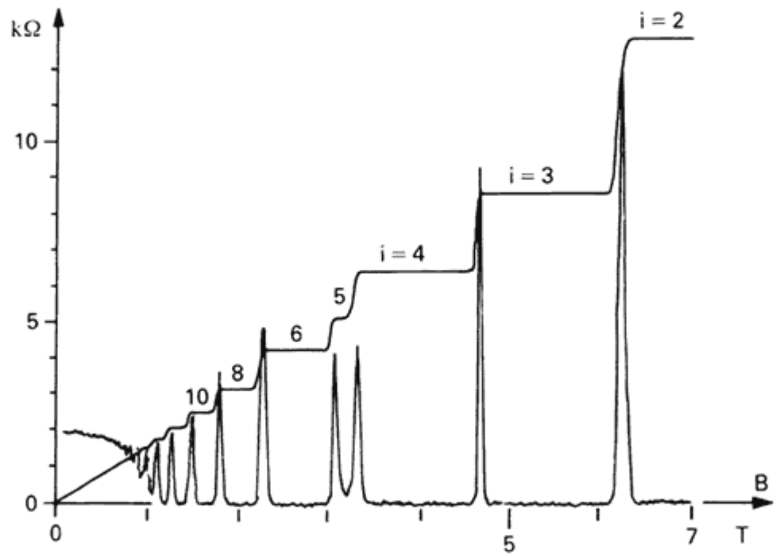
TKNN theory of Quantum Hall.

(Thouless, Kohmoto, Nightingale, den Nijs)

$$\sigma_{xy} = \frac{e^2}{h}\frac{1}{2\pi}\int dk_x \int dk_y f_{xy}(\mathbf{k})$$

$$f_{xy}(\mathbf{k}) = \frac{\partial a_y(\mathbf{k})}{\partial k_x} - \frac{\partial a_x(\mathbf{k})}{\partial k_y}$$

$$a_i(\mathbf{k}) = -i \sum_{\alpha \in \text{ occ}} \langle \alpha\mathbf{k}| \frac{\partial}{\partial k_i} |\alpha\mathbf{k}\rangle, \; i = x, y.$$



"Berry Connection" has quantized flux.

$$\sigma_{xy} = n\frac{e^2}{h}$$

Band Structure Topology.

Multi-Band-Berry-Connection.

(Qi, Hughes, Zhang)

$$\theta \equiv 2\pi P_3(\theta) = \frac{1}{16\pi^2}\int d^3\mathbf{k}\,\epsilon^{ijk}\mathrm{Tr}\{[f_{ij}(\mathbf{k}) - \frac{2}{3}i a_i(\mathbf{k})\cdot a_j(\mathbf{k})]\cdot a_k(\mathbf{k})\}$$

$$\begin{aligned} f_{ij}^{\alpha\beta} &= \partial_i a_j^{\alpha\beta} - \partial_j a_i^{\alpha\beta} + i\,[a_i, a_j]^{\alpha\beta}\,, \\ a_i^{\alpha\beta}(\mathbf{k}) &= \; -i\,\langle\alpha,\mathbf{k}|\,\frac{\partial}{\partial k_i}\,|\beta,\mathbf{k}\rangle \end{aligned}$$

θ=0	Vacuum, …
θ=π	$Bi_{1-x}Sb_x$, Bi_2Se_3, Bi_2Te_3, Sb_2Te_3

predicted: $TlBi(Sb)Te(Se, S)_2$, LaPtBi etc (Heusler compounds)

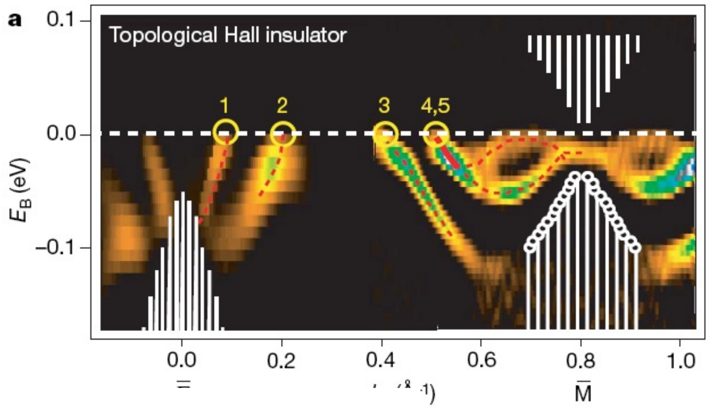
Confront with experiment.

- Based on calculation of bandstructure it is easy to predict which materials are topological insulators (needs strong LS coupling).

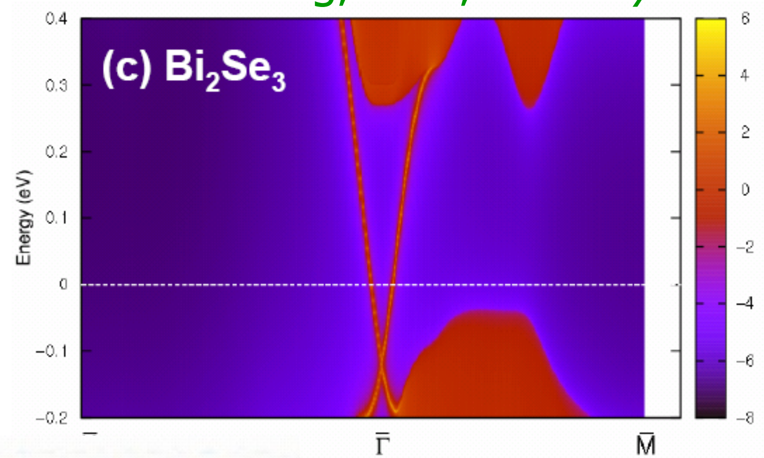
- Surface zero mode is excellent experimental signature.

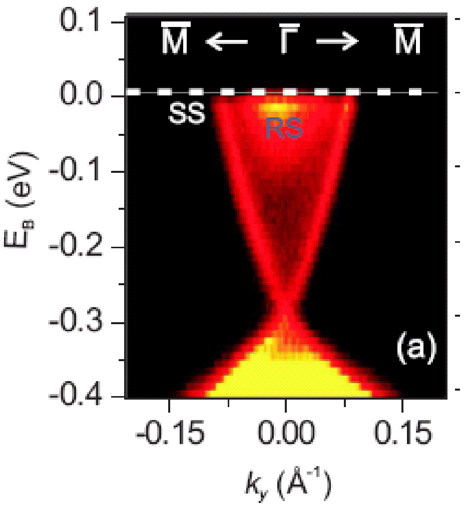
Excellent Signature!

Hasan group, *Nature* 2008

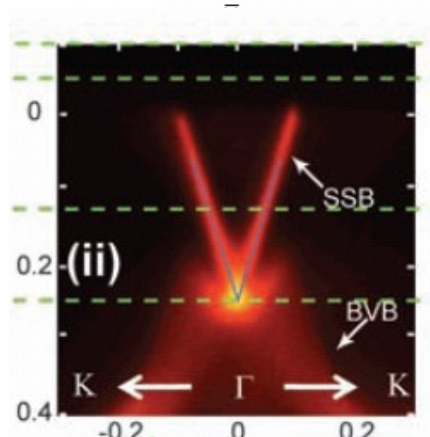


H. J. Zhang, et al, *Nat Phys* 2009

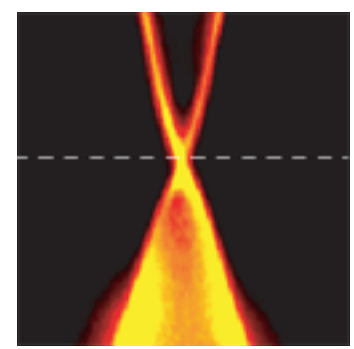




Hasan group, Bi_2Se_3 *Nat Phys* 2009



Bi_2Te_3 Chen et al *Science* 2009



$TlBiTe_2$ Kuroda et al *Phys.Rev.Lett* 2010

Summary of Strategy:

Low Energy Effective Theory:

Dirac Quantization $\longrightarrow$ $\theta = \text{Integer} \cdot \pi$

IR

Microscopic Model:

ABJ anomaly $\longrightarrow$ $\theta/\pi = \sum (\text{charge})^2$

Connection to Experiment:

Band Topology $\longrightarrow$ $\theta = \text{QHZ-invariant}$

UV

Fractional topological insulators

(work with Maciejko, Qi and Zhang)

Fractional Topological Insulators?

Recall from Quantum Hall physics:

electron

fractionalizes into m partons

e$^-$ interactions

$$\sigma_{xy} = n\frac{e^2}{h} \longrightarrow \sigma_{xy} = \frac{n}{m}\frac{e^2}{h}$$

(m odd for fermions)

Quantum Hall

Fractional Quantum Hall

Fractional Topological Insulators?

TI = half of an integer quantum hall state on the surface

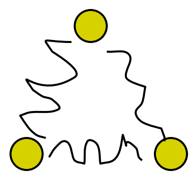
expect: fractional TI = half a fractional QHS
Hall quantum = half of 1/odd integer.

Can we get this from charge fractionalization?

Partons.

Microscopic Model:

chiral anomaly $\longrightarrow$ $\theta/\pi = \sum (\text{charge})^2$

e^- = 

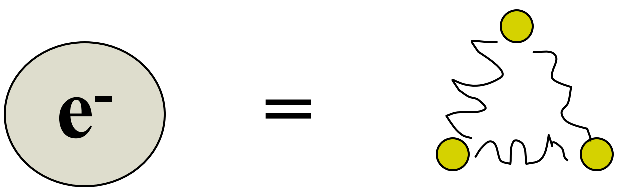
electron breaks up into m partons.

$$\theta/\pi = \sum (\text{charge})^2 = m \cdot \left(\frac{1}{m}\right)^2 = \frac{1}{m}$$

(m odd so e^- is fermion)

(if partons form a TI = have negative mass)

Partons.



To ensure that physical (= gauge invariant) states carry integer electron charge add “statistical” gauge field.

Simplest models: $U(1)^3/U(1)$ quiver gauge theory

$SU(3)$ with $N_f=1$

Relativistic Partons.

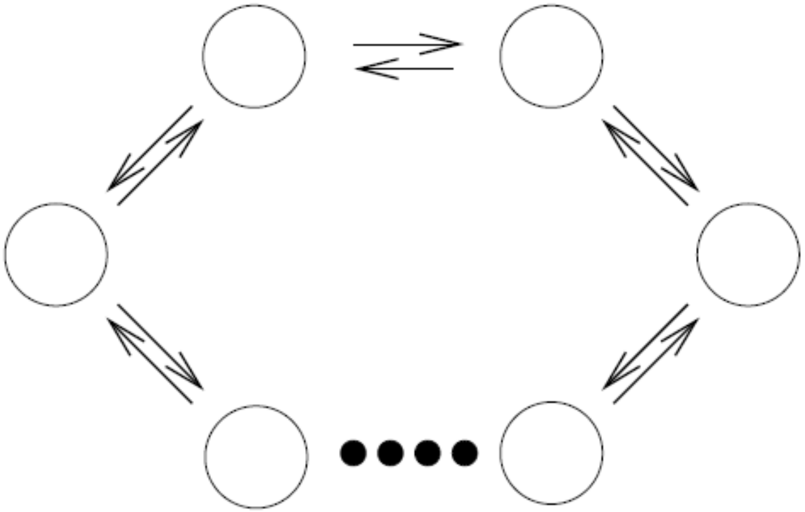
In a relativistic field theory the electron=baryon has spin m/2 in the SU(m) model.

Not a problem in non-relativistic context.

Alternative: Quiver Model

Still need m partons to be gauge invariant.

Baryons of any spin possible.



General Parton model.

$\mathcal{N}_f$ different 'flavors'

Total number of partons.

$\mathcal{N}_c^{(f)}$ partons of each flavor

$$\mathcal{N}_c^{(1)} + \mathcal{N}_c^{(2)} + \cdots + \mathcal{N}_c^{(\mathcal{N}_f)} = \text{odd}$$

$$\mathcal{N}_c^{(1)} q_1 + \mathcal{N}_c^{(2)} q_2 + \cdots + \mathcal{N}_c^{(\mathcal{N}_f)} q_{\mathcal{N}_f} = e$$

$$U(1)_{\mathrm{em}} \times \prod_{f=1}^{N_f} U(\mathcal{N}_c^{(f)})/U(1)$$

General Parton model.

$$\theta_{\text{eff}} = \left(\sum_{f=1}^{\mathcal{N}_f} \frac{\mathcal{N}_c^{(f)}}{\theta_f} \right)^{-1}$$

$$\Downarrow$$

$$\sigma_{H,s} = \frac{p}{q}\frac{e^2}{2h}, \qquad p, q \text{ odd}$$

Important dynamical question.

Is the gauge theory in a confining or deconfining phase?

We need: deconfined! Favors abelian models.
(or N=4 SYM with N=2 massive hyper)

Gapless modes present; charged fields all gapped.

Alternative: Higgsed Phase with unbroken discrete gauge invariance; groundstate degenerate .

Why not confined phase?

Need chiral symmetry to be unbroken.
("Confinement w/o chiral symmetry" breaking ok
--- but also has extra light, neutral states.)

MIT theorem (basically Dirac quantization):
Need either extra massless degrees of freedom
(e.g deconfined phase or SUSY QCD with $N_f=N_c+1$)
or degenerate groundstate (e.g. Higgs phase)

How to make a fractional TI?

Need: Strong electron/electron interactions

(so electrons can potentially fractionalize)

Strong spin/orbit coupling

(so partons can form topological insulator)

How can one tell if a given material is a fractional TI (in theory/in practice)?

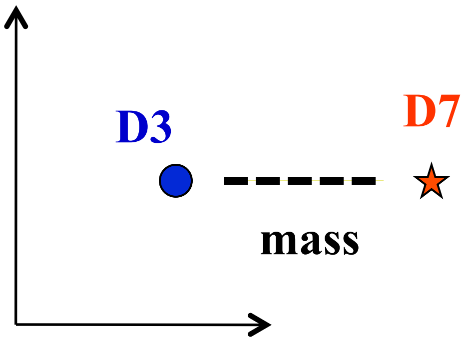
Transport! Fractional Hall + Kerr.

Holographic Realization

(work with Hoyos and Jensen)

fTI in N=4 SYM and AdS/CFT

	0	1	2	3	4	5	6	7	8	9
D3	X	X	X	X	-	-	-	-	-	-
D7	X	X	X	X	X	X	X	X		

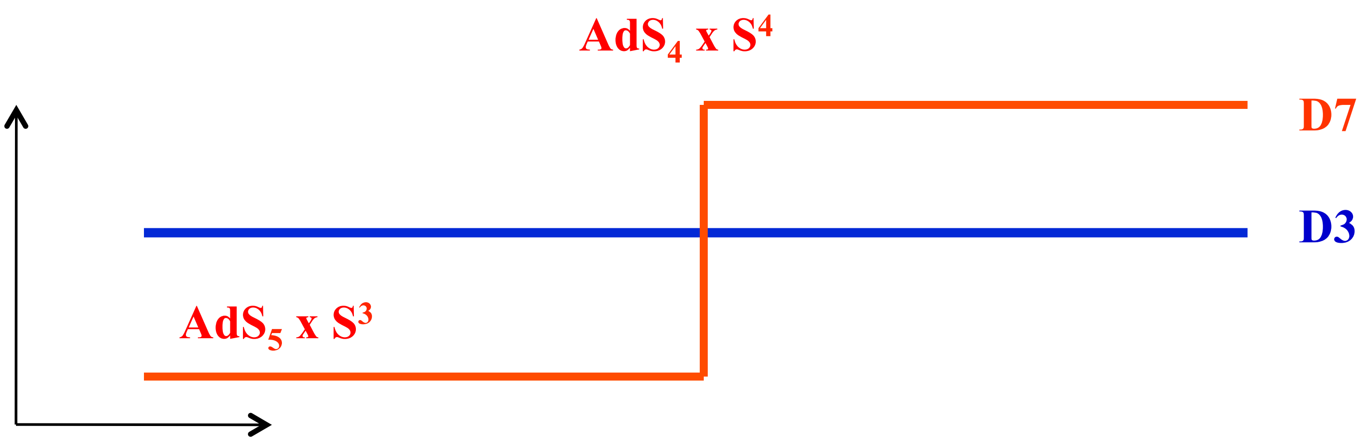


(8,9) space

T-invariant =real mass
= D7 at x_9=0, any x_8

(here this is a choice to preserve T, Takayanagi and Ryu impose orientifold that makes T-violating mass inconsistent)

holographic fTI



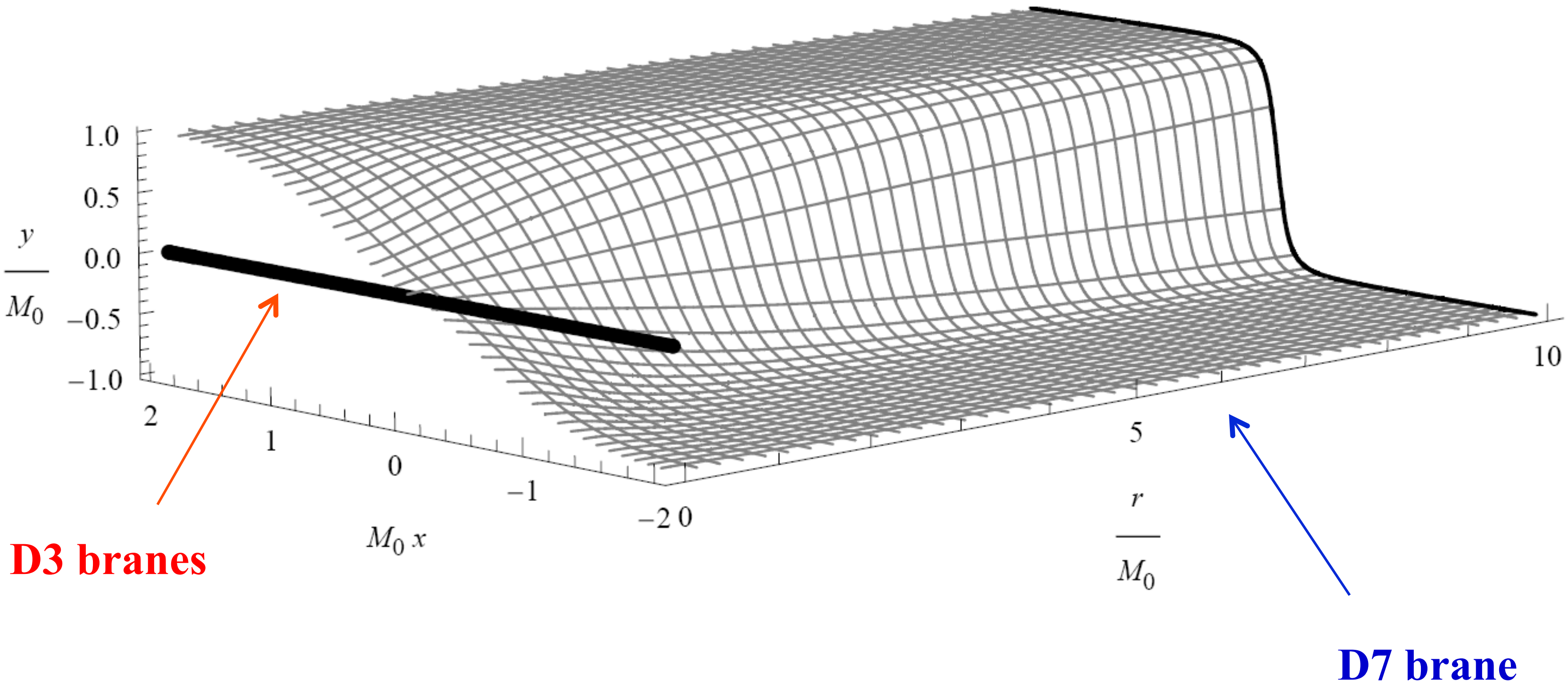
(3,8) space

$$ds^2_{S^5} = \cos^2\theta d\Omega_3^2 + d\theta^2 + \sin^2\theta d\phi^2$$

Find: $\theta(x, r)$

Smooth embedding. Approaches step at r=∞ (boundary of AdS)

holographic fTI



holographic fTI

- Other mass profiles possible; expect Hall current with filling fraction 1/(2m) for any zero crossing profile.
- This can easily be verified from AdS. Independent of details of embedding, the Hall current is uniquely determined by WZ term.

Was expected: WZ term = anomalies

Explicit Realization of a non-abelian fTI

The Quantum Spin Hall Effect

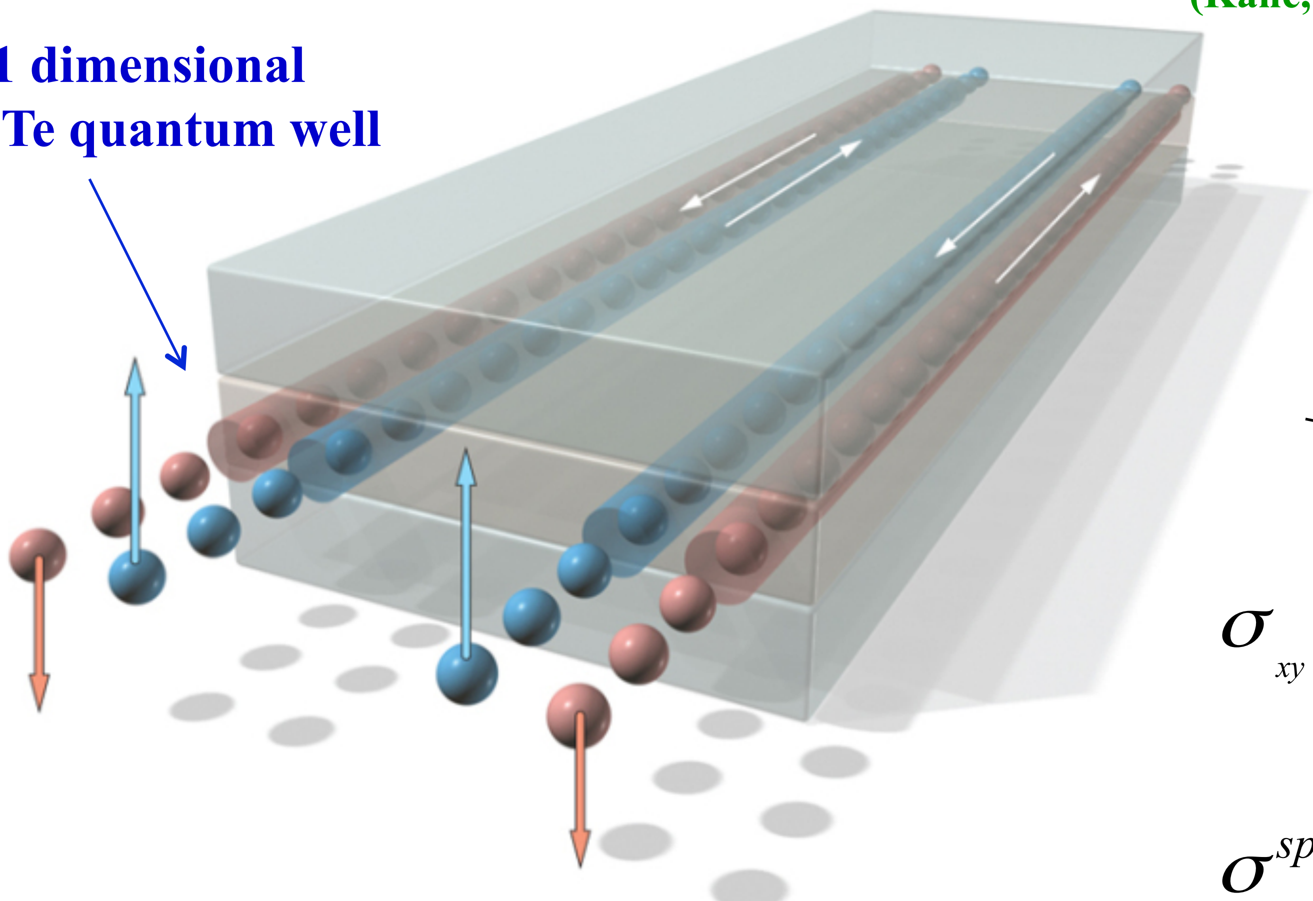
(or better: the quantum R-current-Hall effect)

(work with Maciejko and Takayanagi)

Quantum Spin Hall Effect in HgTe

(Kane, Mele; Bernevig Zhang)

2+1 dimensional
HgTe quantum well



electric field

$$\sigma_{xy} = 0$$

$$\sigma_{xy}^{spin} = \frac{e}{2\pi}$$

Continuum Description:

$$\mathcal{L} = \bar{\psi}(i\partial_\mu \gamma^\mu - M)\psi$$

2+1: only one mass M
T odd. Single fermion massless!

But: **(Jackiw-Templeton)**

opposite sign!

$$\mathcal{L}_M = M\bar{\Psi}\Psi = M(\bar{\psi}_2\psi_2 - \bar{\psi}_1\psi_1)$$

T invariant mass possible for two fermions!

Coupling to Electromagnetism:

Symmetry: $$U(2) \longrightarrow U(1)_{EM} \times U(1)_R$$

symmetry of free fermions

gauged

remaining global symmetry

$\boxed{\text{U(1)}_\text{R} \text{ plays role of spin!}}$

	$U(1)_{EM}$	$U(1)_R$	Sign(mass)
ψ_1	+1	+1	+1
ψ_2	+1	-1	-1

Integrate out fermions:

Induced CS: $$k_{ab} = \frac{1}{2} \sum_i q_{i,a} q_{i,b} \ \mathrm{sign}(M)$$

Electromagnetic: Contributions from the two fermions cancel.
No CS term. No Hall current.

Mixed EM/R: Contributions from the two fermions add. k=1.

The $A_R \wedge F_{EM}$ gives rise to "Quantum-R-Hall-Effect"

Applications.

- As in 3+1, given the continuum description it is trivial to construct low energy description of fractional quantum spin hall effect. e splits into m partons. CS term picks up factor of 1/m.

- Holographic realization in terms of probe brane system also straightforward. This time it's the D3/D5 system. Again, topological properties independent of embedding.

Summary.

- Effective field theory for fractional TI can be constructed
- Basic ingredient: fractionalization
- Effective θ follows from anomaly/Dirac quantization
- Requires strong LS coupling and strong interactions.
- Experimental signatures: transport + Kerr/Faraday
- Holographic realization straight forward