Phenomenology of $B$-meson mixing and decay constants

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Theory seminar

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1. **Introduction**

- Flavour-violating and CP-violating processes allow us to test high energy physics
  - Tests limited by precision.
  - Standard Model (SM) predictions for those observables depend on a few parameters → overconstrain those parameters.

- Test the SM

- Already several $2 - 3 \sigma$ tensions between flavour data and SM predictions

- Phenomenological goals:
  - Determination of fundamental parameters of the SM: quark masses, Cabibbo-Kobayashi-Maskawa matrix elements.
  - Unveiling New Physics (NP) effects. Even before non-SM particles directly produced at LHC.
  - Constraining NP models.
1. Introduction

# Interplay flavour physics with direct searches for new physics and electroweak precision studies

→ Which is the correct extension of the SM?
2. Neutral $B$ mixing

- $B_0$ mixing parameters determined by the off diagonal elements of the mixing matrix

\[ i \frac{d}{dt} \begin{pmatrix} |B_{s/d}(t)\rangle \\ |\bar{B}_{s/d}(t)\rangle \end{pmatrix} = \left( M^{s/d} - \frac{i}{2} \Gamma^{s/d} \right) \begin{pmatrix} |B_{s/d}(t)\rangle \\ |\bar{B}_{s/d}(t)\rangle \end{pmatrix} \]

\[ \Delta M_{s/d} \propto |M_{12}^{s/d}| \quad \Delta \Gamma_{s/d} \propto |\Gamma_{12}^{s/d}| \]

New physics can significantly affect $M_{12}^{s/d} \propto \Delta M_{s/d}$

* $\Gamma_{12}$ dominated by CKM-favoured $b \rightarrow c\bar{c}s$ tree-level decays.
2.1. Mixing parameters in the Standard Model

In the Standard Model

\[ \Delta M_q|_{\text{theor.}} = \frac{G_F^2 M_W^2}{6\pi^2} |V_{tq}^* V_{tb}|^2 \eta_2^B S_0(x_t) M_{B_s} f_{B_q}^2 \hat{B}_{B_q} \]

* Non-perturbative input

\[ \frac{8}{3} f_{B_q}^2 B_{B_q}(\mu) M_{B_q}^2 = \langle \bar{B}_q^0 | O_1 | B_q^0 \rangle(\mu) \quad \text{with} \quad O_1 \equiv [\bar{b}^i q^i]_{V-A} [\bar{b}^j q^j]_{V-A} \]

In terms of decay constants and bag parameters

\[ \xi = \frac{f_{B_s} \sqrt{B_{B_s}}}{f_{B_d} \sqrt{B_{B_d}}} \]

* Many uncertainties in the theoretical (lattice) determination cancel totally or partially in the ratio \( \Rightarrow \) very accurate calculation
2.1. Mixing parameters in the Standard Model

Experimentally: Mass differences very well measured.

\[ \Delta M_d|_{exp.} = (0.507 \pm 0.005)ps^{-1} \quad \Delta M_s|_{exp.} = (17.77 \pm 0.12)ps^{-1} \]

HFAG 09 \quad CDF

Experimentally: Decay width differences still have large errors.

\[ \left( \frac{\Delta \Gamma}{\Gamma} \right)_d = 0.010 \pm 0.037 \quad \left( \frac{\Delta \Gamma}{\Gamma} \right)_s = 0.15 \pm 0.07 \]

HFAG 09
2.2. $B_0$ mixing beyond the SM

# Comparison of experimental measurements and theoretical predictions can constraint some BSM parameters and help to understand BSM physics.

# Effects of heavy new particles seen in the form of effective operators built with SM degrees of freedom

# The most general Effective Hamiltonian describing $\Delta B = 2$ processes is

\[
\mathcal{H}_{eff}^{\Delta B=2} = \sum_{i=1}^{5} C_i Q_i + \sum_{i=1}^{3} \tilde{C}_i \tilde{Q}_i \quad \text{with}
\]

\[
Q_1^q = \left( \bar{\psi}_b^i \gamma^\nu (I - \gamma_5) \psi_q^i \right) \left( \bar{\psi}_b^j \gamma^\nu (I - \gamma_5) \psi_q^j \right)
\]

\[
Q_2^q = \left( \bar{\psi}_b^i (I - \gamma_5) \psi_q^i \right) \left( \bar{\psi}_b^j (I - \gamma_5) \psi_q^j \right) \quad Q_3^q = \left( \bar{\psi}_b^i (I - \gamma_5) \psi_q^j \right) \left( \bar{\psi}_b^j (I - \gamma_5) \psi_q^i \right)
\]

\[
Q_4^q = \left( \bar{\psi}_b^i (I - \gamma_5) \psi_q^i \right) \left( \bar{\psi}_b^j (I + \gamma_5) \psi_q^j \right) \quad Q_5^q = \left( \bar{\psi}_b^i (I - \gamma_5) \psi_q^j \right) \left( \bar{\psi}_b^j (I + \gamma_5) \psi_q^i \right)
\]

\[
\tilde{Q}_{1,2,3} = Q_{1,2,3}^q \quad \text{with the replacement} \quad (I \pm \gamma_5) \rightarrow (I \mp \gamma_5)
\]

where $\psi_b$ is a heavy b-fermion field and $\psi_q$ a light ($q = d, s$) fermion field.

• $C_i, \tilde{C}_i$ Wilson coeff. calculated for a particular BSM theory

• $\langle \bar{B}^0|Q_i|B^0 \rangle$ calculated on the lattice
2.2. $B_0$ mixing beyond the SM

# Some examples:


M. Ciuchini and L. Silvestrini, PRL 97 (2006) 021803; SUSY

Constraints on the mass insertions ($|Re(\delta^d_{23})_{RR}| < 0.4$, $|(\delta^d_{23})_{LL}| < 0.1,...$)

M. Blanke et al, JHEP 12(2006) 003; Little Higgs model with T-parity

$\Delta M_q$ can be used to test viability of the model. To constrain and test the model in detail $\Delta M_s/\Delta M_d$ and $\Delta \Gamma_q$.

Lunghi and Soni, JHEP0709(2007)053; Top Two Higgs Doublet Model

Constraints on $\beta_H$ (ratio of vev's of the two Higgs) and $m_{H^+}$

M. Blanke et al, JHEP0903(2009)001; Warped Extra Dimensional Models

Constraints on the KK mass scale: anarchic approach seems implausible, generally $M_{KK} > 20 TeV$ but can be as low as $M_{KK} \simeq 3 TeV$ (moderate fine tunning).
2.2. $B_0$ mixing beyond the SM

# Some examples:

W. Altmannshofer et al, 0909.1333; SUSY flavor models

Identify useful flavour observables ($S_{\psi\phi}$, $B_s \rightarrow \mu^+\mu^-$, ...) to exclude some SUSY models and/or distinguish them from LHT and RS models. Updated analysis of bound on flavor violating terms in the SUSY soft sector.
2.2. \( B_0 \) mixing beyond the SM

# NP effects in \( B^0 - \bar{B}^0 \) mixing can be parametrized by

\[
\langle B^0_q|H_{\text{eff}}|\bar{B}^0_q \rangle = A_q^{SM} + A_q^{NP} = C_{B_q}^{0} e^{2i\phi B_q^0} A_q^{SM}
\]

* The mixing phase also governs mixing-induced CP violation in exclusive channels like \( B_s \to J/\psi\phi \).
2.3. Unitarity Triangle analyses

# For $V_{ub}^*V_{ud} + V_{cb}^*V_{cd} + V_{tb}^*V_{td} = 0 \rightarrow \text{CKM unitarity triangle}$.

Can use the following set of parameters

\[
\lambda \equiv |V_{us}|, \ |V_{cb}|, \ R_t \text{ and } \beta
\]

where

\[
R_t \equiv \left| \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} \right| = \sqrt{(1 - \bar{\rho})^2 + \bar{\eta}^2} = \frac{1}{\lambda} \left| \frac{V_{td}}{V_{cb}} \right|,
\]

$V_{td} = |V_{td}|e^{-i\beta}$

# Within the SM and CMFV

\[
R_t = \xi \frac{1}{\lambda} \sqrt{\frac{m_{B_s^0}}{m_{B_d^0}}} \sqrt{\frac{\Delta M_{B_d^0}}{\Delta M_{B_s^0}}} \sin 2\beta = S_{\psi K_S}
\]
2.3. Unitarity Triangle analyses

* Mixing-induced CP asymmetry

\[ A_{\psi K_S} = \frac{\Gamma(\bar{B}_d^0(t) \to \psi K_S) - \Gamma(B_d^0(t) \to \psi K_S)}{\Gamma(\bar{B}_d^0(t) \to \psi K_S) + \Gamma(B_d^0(t) \to \psi K_S)} \simeq S_{\psi K_S} \sin(\Delta M t) - C_{\psi K_S} \cos(\Delta M t) \]

# In the presence of NP those relations are modified by

\[ R_t = \xi \frac{1}{\sqrt{m_{B_0^q}}} \sqrt{\frac{\Delta M_{B_d^0}}{\Delta M_{B_{s,d}^0}}} \sqrt{\frac{C_{B_0^q}}{C_{B_d^0}}} \sin(2\beta + 2\phi_{B_0^0}) = S_{\psi K_S} \]

with the NP parameters defined as

\[ \langle B_q^0|H_{eff}^q|\bar{B}_q^0 \rangle = A_q^{SM} + A_q^{NP} = C_{B_q^0} e^{2i\phi_{B_q^0}} A_q^{SM} \]
Most observations in the flavour sector are consistent with SM expectations but ... 

... there are currently several $2 - 3\sigma$ tensions that may indicate New Physics.
3.1. Tension in the CKM unitarity triangle

**UT fit**: Global fit to the CKM unitarity triangle using experimental and theoretical constraints.

*2 − 3σ* tension in the CKM description

* Tension is between the three most precise constraints: the $K^0 - \bar{K}^0$ mixing parameter $\epsilon_K$, the ratio of mass differences $\Delta M_{B_s}/\Delta M_{B_d}$ describing $B^0 - \bar{B}^0$ mixing and $\sin(2\beta)$.


** Degree of tension depend on $|V_{cb}|$

$$|V_{cb}^{\text{exc. (latt. average)}}| = (38.6 \pm 1.2) \times 10^{-3} \quad |V_{cb}^{\text{inc.}}| = (41.6 \pm 0.6) \times 10^{-3}$$
3.1. Tension in the CKM unitarity triangle

2 – 3σ tension in the CKM description

** Independent of (controversial) |V_{ub}|

|V_{ub}^{exc. \ (latt. \ average)}| = (3.42 \pm 0.37) \times 10^{-3} \quad |V_{ub}^{inc.}| = (4. - 4.5) \times 10^{-3}

* If we assume no NP at tree-level at current precision
  → tension can be a sign of NP either in K^0 or B^0 mixing.

** Current data prefer NP in K^0 mixing.

* Constraints from ε_K, ΔM_d/ΔM_s, and |V_{ub}/V_{cb}| limited by lattice errors for |V_{cb}|_{excl.}, ξ, and |V_{ub}|_{excl.}
3.1. Tension in the CKM unitarity triangle

E. Lunghi and A. Soni, arXiv:0912.0002: UT analysis without using semileptonic decays

* $|V_{ub}|$ and $|V_{cb}|$ inclusive and exclusive disagree by $\approx 2\sigma$

$\rightarrow$ eliminate the $|V_{cb}|$ constraint from the analysis in favor of

$$f_{B^0_s}\sqrt{\hat{B}_{B^0_s}} \text{ or } Br(B \rightarrow \tau\nu) \times f_{B_d}^{-2}$$

* 1.8$\sigma$ tension observed. Slight preference for NP in $B^0_d$ mixing.

* Improvement in $f_{B^0_s}\sqrt{\hat{B}_{B^0_s}}$ and $f_B$ will help a lot to identify the origin of the tension.
3.2. Mixing in the $B_s$ system: the $S_{J/\psi\phi}$ asymmetry

* Lenz and Nierste, JHEP 06, 072 (2007)*

# Study the mixing-induced CP asymmetry.

$$A_{J/\psi\phi} = \frac{\Gamma(\bar{B}_s^0(t) \to J/\psi\phi) - \Gamma(B_s^0(t) \to J/\psi\phi)}{\Gamma(B_s^0(t) \to J/\psi\phi) + \Gamma(\bar{B}_s^0(t) \to J/\psi\phi)} = S_{J/\psi\phi} \sin(\Delta M t) - C_{J/\psi\phi} \cos(\Delta M t)$$

# $B_s$ mixing phase $\beta_s$ responsible for this asymmetry in the SM

$$\langle B_s | H_{eff}^S | \bar{B}_s \rangle = A_s^S e^{-2i\beta_s}$$

$$(S_{J/\psi\phi})_{SM} = \sin(2|\beta_s|) = \sin \left(2 \left| \arg \left( \frac{-V_{ts}V_{tb}^*}{V_{cs}V_{cb}} \right) \right| \right) \approx 0.04$$

# World average based on flavour-tagged analyses of $B_s \to J/\psi\phi$ decays in CDF and DØ is 2.2σ different from SM predictions

$$(S_{J/\psi\phi})_{exper.} \approx 0.81^{+0.12}_{-0.32}$$

* Expect improvements of experimental measurements of $S_{J/\psi\phi}$ asymmetry in CDF, DØ, LHCb, ATLAS and CMS.
3.2. Mixing in the $B_s$ system: the $S_{J/ψφ}$ asymmetry

* Possible new phases in $B_s$ decays would lead to correlated effects between $ΔB = 2$ processes and $b → s$ decays

\[
(S_{J/ψφ}) = sin(2|β_s| - 2φ_{B_0})
\]

→ need to improve measurements of CP-violation in $b → s$ penguin decays.

# Enhancement of the asymmetry can be found in RSc and GMSSM. Also supersymmetric flavour models with significant right-handed curr. Buras, arXiv:0910.1032
3.3. Measurement of $\text{Br}(B_{s,d} \to \mu^+\mu^-)$

One of the main targets of flavour physics is measuring the highly suppressed decay $\text{Br}(B_s \to \mu^+\mu^-)$.

* CDF (DØ) bounds $\text{Br}(B_s \to \mu^+\mu^-) \leq 3.3(5.3) \times 10^{-8}$, 
  $\text{Br}(B_d \to \mu^+\mu^-) \leq 1 \times 10^{-8}$

* The SM prediction for these branching ratios is given by

$$\text{Br}(B_q \to \mu^+\mu^-)_{SM} = \tau_{B_q} \frac{G_F^2}{\pi} \frac{\eta_Y^2}{\alpha} \left( \frac{\alpha}{4\pi \sin^2 \theta_W} \right)^2 f_{B_q}^2 m_\mu^2 m_{B_q}|V_{tb}V_{tq}|^2 Y^2(x_t)$$

** Uncertainty dominated by error in $f_{B_q}$: 9-11%

* The most precise way of extracting these branching ratios is from (in the SM)

$$\frac{\text{Br}(B_q \to \mu^+\mu^-)}{\Delta M_q} = \tau(B_q) \frac{6\pi}{\eta_B} \frac{\eta_Y}{\alpha} \left( \frac{\alpha}{4\pi M_W \sin^2 \theta_W} \right)^2 m_\mu^2 \frac{Y^2(x_t)}{S(x_t)} \frac{1}{\hat{B}_q}$$

** Uncertainty dominated by error in $\hat{B}_q$: 5-9%
3.3. Measurement of $Br(B_{s,d} \rightarrow \mu^+\mu^-)$

* CDF (DØ) bounds $Br(B_s \rightarrow \mu^+\mu^-) \leq 3.3(5.3) \times 10^{-8}$, 
  $Br(B_d \rightarrow \mu^+\mu^-) \leq 1 \times 10^{-8}$

* Using lattice determinations of $\hat{B}_q$ HPQCD, PRD80 (2009) 014503

  $\rightarrow Br(B_s \rightarrow \mu^+\mu^-) = (3.19 \pm 0.19) \times 10^{-9}$ and
  $Br(B_d \rightarrow \mu^+\mu^-) = (1.02 \pm 0.09) \times 10^{-10}$

** An error of 0.14 in $Br(B_s \rightarrow \mu^+\mu^-)$ is coming from $\hat{B}$ uncertainty.

* Scalar operators in the effective hamiltonian can enhance branching ratios to current experimental bounds (example: Higgs penguin).

* In some models there is a strong correlation between $Br(B_q \rightarrow \mu^+\mu^-)$ and $\Delta M_{B_q^0}$ (example: MSSM with MFV and large $\tan\beta$.)

** Testing the correlation predicted by those kind of models needs a reduction of errors in the theoretical prediction for $\Delta M_{s}^{SM}$ 
  $\rightarrow$ need smaller lattice errors for the non-perturbative inputs.
3.3. Measurement of $Br(B_{s,d} \rightarrow \mu^+\mu^-)$

# Tests of MFV: In the SM model and CMFV models, the following model independent relation hold with $r = 1$ Buras, PLB566 (2003) 115

\[
\frac{Br(B_s \rightarrow \mu^+\mu^-)}{Br(B_d \rightarrow \mu^+\mu^-)} = \frac{\hat{B}_d}{\hat{B}_s} \frac{\tau(B_s)}{\tau(B_d)} \frac{\Delta M_s}{\Delta M_d} r
\]

Any deviation from this relation ($r \neq 1$) would indicate NP effects.

Supersymmetry, little Higgs models, extra space dimensions ... discussed in Buras, arXiv:0910.1032

**LHT:** $0.3 \leq r \leq 1.6$, **RSc:** $0.6 \leq r \leq 1.3$

* LHCb can reach the SM level for this branching ratio.
3.4. $B \rightarrow \tau \nu$ leptonic decay

1.9$\sigma$ discrepancy between $f_B$ values from lattice (HPQCD and FNAL/MILC) and experiment (using $V_{ub}$ from lattice QCD).

A. Kronfeld, PHENO '09

$$\text{Br}(B^+ \rightarrow \tau^+ \nu)_{SM} = \frac{G_F^2 m_B m_{\tau}^2}{8\pi} \left(1 - \frac{m_{\tau}^2}{m_{B^+}^2}\right)^2 f_{B^+}^2 |V_{ub}|^2 \tau_B^+$$

* Differences: Fermion discretization describing $b$ quarks.

HPQCD 09, PRD80(2009)014503: NRQCD.

3.4. $B \to \tau \nu$ leptonic decay

2.4$\sigma$ discrepancy between SM prediction for $\mathcal{B}(B \to \tau \nu)$ from UT fit (relies on several lattice inputs $f_{B^0_d,s}, \hat{B}_{B^0_d,s}, f_{B^0_d,s} \sqrt{\hat{B}_{B^0_d,s}}$) and experimental average BaBar, Belle

**CKM fitter**, Moriond 09, Beauty 09

$\mathcal{B}_{\text{exp}}(B^+ \to \tau^+ \nu) = (1.73 \pm 0.35) \times 10^{-4}$

$\mathcal{B}_{\text{fit}}(B^+ \to \tau^+ \nu) = (0.80^{+0.16}_{-0.11}) \times 10^{-4}$

# Alternative extraction of SM prediction

$$\mathcal{B}(B^+ \to \tau^+ \nu)_{SM} = \frac{3\pi}{4\eta_B S_0(x_t) \hat{B}^0_d m^2_{\tau}} \frac{m^2_{\tau}}{m^2_W} \left(1 - \frac{m^2_{\tau}}{m^2_{B^+}}\right)^2 \left|\frac{V_{ub}}{V_{td}}\right|^2 \tau^+_B$$

with $\left|\frac{V_{ub}}{V_{td}}\right| = \left(\frac{1}{1 - \chi^2}\right)^2 \frac{1 + R_t^2 - 2 R_t \cos \beta}{R_t^2}$

$$\mathcal{B}(B^+ \to \tau^+ \nu)_{SM} = (0.80 \pm 0.12) \times 10^{-4}$$
3.4. $B \rightarrow \tau \nu$ leptonic decay

# Discrepancy can be due to charged Higgs, but not a natural explanation. Could increase or decrease SM $\mathcal{B}r$.

* Most MSSM scenarios would lead to a suppression of the branching fraction.

* Example: Limits in the 2HDM $\tan \beta - m_{H^+}$ plane

$$(m_{H^+} / \tan \beta > 3.3 \text{ GeV})$$

# Reducing experimental errors will be difficult at LHCb. Good prospects for a super-B factory
4. Lattice calculation of $B^0$ mixing parameters and decay constants

# Hints of discrepancies between SM expectations and some flavour observables (see, for example, A. Buras, talk at EPS-HEP 2009 or R. Van de Water, plenary talk at Lattice 2009)

These analyses depend on several theoretical inputs including:

$$f_{B^0_q} \sqrt{\hat{B}_{B^0_q}}, \ f_{B^0_q}, \text{ and the SU}(3) \text{ breaking mixing parameter } \xi:$$

# Comparison of $\Delta M$ and $\Delta \Gamma$ with experiment also provides bounds for NP effects

# Bag parameters $B_{B_s}$ and $B_{B_d}$ can be used for theoretical predictions of $\mathcal{B} \tau(B \rightarrow \mu^+ \mu^-)$ and $\mathcal{B} \tau(B^+ \rightarrow \tau^+ \nu)$
4.1. Some details of the lattice formulations and simulations

**HPQCD**, PRD80 (2009) 014503

**Unquenched**: Fully incorporate vacuum polarization effects

**MILC** $N_{f}^{sea} = 2 + 1$

**Asqtad action**: improved staggered quarks $\Rightarrow$ errors $O(a^2\alpha_s), O(a^4)$

* good chiral properties
* accessible dynamical simulations

**NRQCD**: Non-relativistic QCD improved through $O(1/M^2), O(a^2)$ and leading relativistic $O(1/M^3)$

* Simpler and faster algorithms to calculate $b$ propagator

**Improved gluon action**

* For further reduction of discretization errors
4.1. Some details of the lattice formulations and simulations: Parameters of the simulation

# Lattice spacing: Two different values $a \simeq 0.12 \text{ fm}, 0.09 \text{ fm}$. Extracted from $\Upsilon$ 2S-1S splitting.

# Bottom mass: Fixed to its physical value from $\Upsilon$ mass.

# Light masses: We work with full QCD points ($m_{\text{valence}} = m_{\text{sea}}$).
   * Strange mass: Very close to its physical value (from Kaon masses).
   * up, down masses: six different values ($m_{\pi}^{\text{min.}} \simeq 230\text{MeV}$)
     $\rightarrow$ chiral regime

# Renormalization and matching to the continuum: One-loop.

$$ < O_1 >^\text{MS} \propto (1 + \rho_{LL} \alpha_s) < O_1 >^\text{latt.} + \rho_{LS} \alpha_s < O_2 >^\text{latt} $$

with $O_1 = [\bar{b} \gamma_\mu (1 - \gamma_5) q] [\bar{b} \gamma_\mu (1 - \gamma_5) q]$ and $O_2 = [\bar{b} (1 - \gamma_5) q] [\bar{b} (1 - \gamma_5) q]$. 
4.1. Some details of the lattice formulations and simulations

# Need 3-point (for any $\hat{Q} = Q_X, Q_X^1$) and 2-point correlators

\[ C^{(4f)}(t_1, t_2) = \sum_{\vec{x}_1, \vec{x}_2} \langle 0 | \Phi_{\bar{B}_q}(\vec{x}_1, t_1) \left[ \hat{Q} \right] (0) \Phi_{\bar{B}_q}^\dagger(\vec{x}_2, -t_2) | 0 \rangle \]

\[ C^{(B)}(t) = \sum_{\vec{x}} \langle 0 | \Phi_{\bar{B}_q}(\vec{x}, t) \Phi_{\bar{B}_q}^\dagger(0, 0) | 0 \rangle \]

• $\Phi_{\bar{B}_q}(\vec{x}, t) = \bar{b}(\vec{x}, t) \gamma_5 q(\vec{x}, t)$ is an interpolating operator for the $B_q^0$ meson.

* Use smearing functions $\phi$ to increase overlap with the ground state

\[ \Phi_{\bar{B}_q}(t) = \bar{b}(\vec{x}_2, t) \gamma_5 \phi(\vec{x}_2 - \vec{x}_1) q(\vec{x}_1, t) \]
4.1. Some details of the lattice formulations and simulations

We carried out **simultaneous** fits of the 3-point and 2-point correlators using Bayesian statistics to the forms \( \rightarrow \) extract \( \langle O_X \rangle \) and \( f_{B_5(d)} \).

\[
C^{(4f)}(t_1, t_2) = \sum_{j,k=0}^{N_{\text{exp}}-1} A_{jk} \zeta_i \zeta_j (-1)^{j \cdot t_1} (-1)^{k \cdot t_2} e^{-E_B^{(j)}(t_1-1)} e^{-E_B^{(k)}(t_2-1)}
\]

\[
C^B(t) = \sum_{j=0}^{N_{\text{exp}}-1} \zeta_j (-1)^{j \cdot t} e^{-E_B^{(j)}(t-1)}
\]
4.2. Results: \( f_{Bq} \sqrt{M_{Bq}B_{Bq}} \)

\[ f_{B_s} \sqrt{\hat{B}_{B_s}} = 266(6)(17)\text{MeV} \]

\[ f_{B_d} \sqrt{\hat{B}_{B_d}} = 216(9)(12)\text{MeV} \]

**Chiral+continuum extrapolations:** NLO Staggered CHPT.

* accounts for NLO quark mass dependence.

* accounts for light quark discretization effects through \( \mathcal{O}\left(\alpha_s^2 a^2 \Lambda_{QCD}^2\right) \)

→ remove the dominant light discretization errors
4.2. Results: \( \xi = \sqrt{\frac{M_{B_s}}{M_{B_d}}} \)

\[
\xi = \frac{f_{B_s} \sqrt{B_{B_s}}}{f_{B_d} \sqrt{B_{B_d}}} = 1.258(25)(21) \quad \Rightarrow \quad \left| \frac{V_{td}}{V_{ts}} \right| = 0.214(1)(5)
\]

* Previous value used in UT fits and another analyses (HPQCD/JLQCD):
\[
\xi = 1.20 \pm 0.06
\]
4.2. Results: \( \xi \sqrt{\frac{M_{B_s}}{M_{B_d}}} \)

4.2. Results: \( f_{B_q} \sqrt{M_{B_q}} \)

\[
\begin{align*}
\frac{f_{B_s}}{f_{B_d}} &= 1.226(26) \\

f_{B_s} &= 231(15) \text{MeV} \\
f_{B_d} &= 190(13) \text{MeV} \\

\end{align*}
\]

* To be compared with preliminary FNAL/MILC, PoS LATTICE 2008, 278 (2008)

\[
\begin{align*}
f_{B_s} &= 243(11) \text{MeV} \\
f_{B_d} &= 195(11) \text{MeV} \\
\end{align*}
\]
5. Impact of up-to-date lattice averages on UT.

Laiho, Lunghi & Van de Water, arXiv:0910.2928

* $2 - 3\sigma$ tension

* If we assume no NP at tree-level at current precision → tension can be a sign of NP either in $K^0$ or $B^0$ mixing.

** Current data prefer NP in $K^0$ mixing.

* Constraints from $\varepsilon_K$, $\Delta M_d/\Delta M_s$, and $|V_{ub}/V_{cb}|$ limited by lattice errors for $|V_{cb}|_{excl.}$, $\xi$, and $|V_{ub}|_{excl.}$.
5. Impact of up-to-date lattice averages on UT

When lattice QCD uncertainties become smaller

* Lattice QCD errors are reduced to 1% keeping central values.

* Use only exclusive $|V_{cb}|$.

Could see NP with a high significance!
5.1. Hints of New Physics in neutral $B$ mixing

**CKMfitter:** $\langle B^0_q | M_{12}^{SM+NP} | \bar{B}^0_q \rangle = \Delta_{qNP}^{NP} \langle B^0_q | M_{12}^{SM} | \bar{B}^0_q \rangle$ V. Tisserand, 0905.1572

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**1.9σ** : Tension driven by the exp. measurement $(2\beta_s, \Delta \Gamma_s)$.

* Tree-level mediated decays through a Four Flavor Change $(b \to q_i \bar{q}_j q_k)$ are SM

* NP effects in oscillation parameters, weak phases, semi-leptonic asymmetries and $B$ lifetime differences parametrized through $\Delta$

**2.1σ** : Tension between $\sin(2\beta)$ and $|V_{ub}|_{\tau\nu}$
6. Future plans for lattice analyses of $B^0$ mixing and decay constants

- Reduction of errors for $f_{Bq}$, $f_{Bq} \sqrt{\overline{B_{Bq}}}$, $\xi$: smaller lattice spacing ($a = 0.06, 0.045$), more statistics, improved renormalization methods, improved actions, better fitting and smearing methods ...

- Calculation of matrix elements needed for $\Delta \Gamma_q$ Lenz and Nierste, JHEP0706 (2007) 072

$$\left( \frac{\Delta \Gamma}{\Gamma} \right) = \left( \frac{1}{245\text{MeV}} \right)^2 \left[ 0.170 \left( f_{Bq}^2 \overline{B_{Bq}} \right) + 0.059 R^2 \left( f_{Bq}^2 \overline{B_S} R^2 \right) - 0.044 f_{Bq}^2 \right]$$

* Useful to impose constraints on BSM building, M. Blanke et al, LHT

* Allows a theoretical prediction for

$$(A_{SL}^s)_{SM} \equiv \frac{\Gamma(\overline{B}_s^0 \rightarrow l^+X) - \Gamma(B_s^0 \rightarrow l^-X)}{\Gamma(\overline{B}_s^0 \rightarrow l^+X) + \Gamma(B_s^0 \rightarrow l^-X)} = \text{Im} \left( \frac{M_{12}^s}{\Gamma_{12}^s} \right)$$

$$(A_{SL}^s)_{SM} \sim 10^{-5} \text{ Lenz and Nierste, JHEP 06 (2007) 072}$$
6. Future plans for lattice analyses of $B^0$ mixing and decay constants to compare with the value of the asymmetry in the presence of NP

Z. Ligeti, M. Papucci and G. Perez, PRL 97 (2006) 101801

\[ A_{SL}^s = -\frac{\Delta \Gamma_s}{\Delta M_s} \frac{S_{\psi\phi}}{C_{B_0^s}} \simeq -(2.6 \pm 1.0) \times 10^{-3} \frac{S_{\psi\phi}}{C_{B_0^s}} \]

** Even $S_{\psi\phi} \simeq 0.1$ would lead to an order of magnitude enhancement relative to SM.

* Some preliminary results HPQCD
6. Future plans for lattice analyses of $B^0$ mixing and decay constants

# Calculation of matrix elements corresponding to operators that only appear in BSM theories.

* Only quenched calculation available Becirevic et al, JHEP 04 (2002) 025

* Straightforward extension of previous calculation
  $\rightarrow$ FNAL/MILC: work in progress

# Analysis of short-distance contributions to $D^0 - \bar{D}^0$ mixing

* Also provides strong constraints on BSM physics E. Golowich, J. Hewett, S. Pakvasa and A. Petrov, PRD 76 (2007)

* FNAL/MILC already working on extending their calculation to $D^0 - \bar{D}^0$ mixing
7. More conclusions

# High precision measurements/calculations of low energy observables allow to indirectly probe very short-distances.

* Test SM and BSM theories

* Learning about the flavour structure of the new physics.
## Error budget for $B^0$ mixing parameters

<table>
<thead>
<tr>
<th>source of error</th>
<th>$f_{B_s} \sqrt{\hat{B}_{B_s}}$</th>
<th>$f_{B_d} \sqrt{\hat{B}_{B_d}}$</th>
<th>$\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>stat. + chiral extrap.</td>
<td>2.3</td>
<td>4.1</td>
<td>2.0</td>
</tr>
<tr>
<td>residual $a^2$ extrap. uncertainty</td>
<td>3.0</td>
<td>2.0</td>
<td>0.3</td>
</tr>
<tr>
<td>$r_1^{3/2}$ uncertainty</td>
<td>2.3</td>
<td>2.3</td>
<td>—</td>
</tr>
<tr>
<td>$g_{B^*B\pi}$ uncertainty</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$m_s$ and $m_b$ tuning</td>
<td>1.5</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>operator matching</td>
<td>4.0</td>
<td>4.0</td>
<td>0.7</td>
</tr>
<tr>
<td>relativistic corr.</td>
<td>2.5</td>
<td>2.5</td>
<td>0.4</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>6.7</strong></td>
<td><strong>7.1</strong></td>
<td><strong>2.6</strong></td>
</tr>
</tbody>
</table>
5.2.2 Value of the UT angle $\sin(2\beta)$

The value of the UT angle $\sin(2\beta)$ obtained from $b \to q\bar{q}s$ penguin decays is lower than from tree-level $b \to c\bar{c}s$ (expected to be less sensitive to NP).

* For example, $\sin(2\beta)_{B \to \phi K^0}$ is $1.3\sigma$ from tree-level average (including, for example, $\sin(2\beta)_{\Psi K_S}$).

* This tension can not be resolved at LHCb (only some clues from $B_S \to \phi\phi$). Need Super Belle at KEK and Super-B machine at Frascati.

* Need better measurements of $b \to q\bar{q}s$ penguin decays.
5.2.4 The $f_{D_s}$ puzzle

R. van de Water (Lattice09)

- 2008 $3.6\sigma$ discrepancy in $f_{D_s}$ between HPQCD and experiment. Agreement in $f_K$, $f_\pi$, $f_D$, $m_D$, $m_{D_s}$, $\frac{2m_{D_s} - m_{\eta_c}}{2m_D - m_{\eta_c}}$.
- 2009 $2.3\sigma$ discrepancy between lattice(average)-exper.(average) New CLEO, BaBar and FNAL/MILC results.

* Leptonic decays occurs at tree level → disagreement difficult to accommodate in BSM.

* Models with charged Higgs or leptoquarks can work Kronfeld and Dobrescu → signal in $D \to K(\pi)\ell\nu$. 
5.2.4 The $f_{D_s}$ puzzle

* Lattice calculations.

** HPQCD has redetermined the scale that converts lattice quantities to physical units $r_1$. New value will make their value lower by $1 - 1.5\sigma \rightarrow$ disagreement under $2\sigma$. Update soon.

** Include effects of sea charm since errors are around 1%

** Need lattice results with different fermion formulations.

* Some experimental issues.

** Experiment uses $|V_{cs}| = |V_{ud}|$.

** Better understanding of radiative corrections.

* BES-III should measure $f_D$ and $f_{D_s}$ with $\sim 1\%$ precision.
5.2.4 The $f_{D_s}$ puzzle

Andreas Kronfeld
5.2.6 Clarification of $\mu$ anomalous magnetic moment, 

$(g - 2)_\mu$ anomaly

The measured $(g - 2)_e$ is in excellent agreement with SM but measured $(g - 2)_\mu$ is significantly larger ($3.1\sigma$) than predicted.

* Hadronic (non-perturbative) contributions to $(g - 2)_\mu$ make the comparison of data and theory a bit problematic.

* New experiments are being designed to reduce the experimental error by a factor of 5.


* Example: Confirmation of exper. measurements $\rightarrow$ favour the MSSM over LHT.
2.2. $B_0$ mixing beyond the SM

# Some examples:

Isidori, Nir and Perez, Ann. Rev. Nucl. Part. Sci 2010:

Bounds on representative dimension-six $\Delta F = 2$ operators.

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{SM} + \sum \frac{c_i^{(d)}}{\Lambda^{(d-4)}} O_i^{(d)}$$

<table>
<thead>
<tr>
<th>Operator</th>
<th>Bounds on $c_{ij}$ ($\Lambda = 1$ TeV)</th>
<th>Observables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\bar{c}_L \gamma^\mu u_L)^2$</td>
<td>$5.6 \times 10^{-7}$ $1.0 \times 10^{-7}$</td>
<td>$\Delta m_D$; $</td>
</tr>
<tr>
<td>$(\bar{c}_R u_L)(\bar{c}_L u_R)$</td>
<td>$5.7 \times 10^{-8}$ $1.1 \times 10^{-8}$</td>
<td>$\Delta m_D$; $</td>
</tr>
<tr>
<td>$(\bar{b}_L \gamma^\mu d_L)^2$</td>
<td>$3.3 \times 10^{-6}$ $1.0 \times 10^{-6}$</td>
<td>$\Delta m_{B_d}$; $S_{B_d \to \psi K}$</td>
</tr>
<tr>
<td>$(\bar{b}_R d_L)(\bar{b}_L d_R)$</td>
<td>$8.8 \times 10^{-7}$ $2.6 \times 10^{-7}$</td>
<td>$\Delta m_{B_d}$; $S_{B_d \to \psi K}$</td>
</tr>
<tr>
<td>$(\bar{b}_L \gamma^\mu s_L)^2$</td>
<td>$6.0 \times 10^{-5}$ $6.0 \times 10^{-5}$</td>
<td>$\Delta m_{B_s}$</td>
</tr>
<tr>
<td>$(\bar{b}_R s_L)(\bar{b}_L s_R)$</td>
<td>$1.6 \times 10^{-5}$ $1.6 \times 10^{-5}$</td>
<td>$\Delta m_{B_s}$</td>
</tr>
</tbody>
</table>
Effects of NP in some flavour observables involving neutral meson mixing parameters

(Altmannshofer et al., arXiv:0909.1333 and Buras, EPS-HEP 2009)

<table>
<thead>
<tr>
<th></th>
<th>AC</th>
<th>RVV2</th>
<th>AKM</th>
<th>$\delta$LL</th>
<th>FBMSSM</th>
<th>LHT</th>
<th>RS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^0 - \bar{D}^0$</td>
<td>★★★</td>
<td>★</td>
<td>★</td>
<td>★</td>
<td>★</td>
<td>★★★★</td>
<td>?</td>
</tr>
<tr>
<td>$S_{\psi\phi}$</td>
<td>★★★</td>
<td>★★★★</td>
<td>★★★★</td>
<td>★</td>
<td>★</td>
<td>★★★★</td>
<td>★★★★</td>
</tr>
<tr>
<td>$S_{\phi K_S}$</td>
<td>★★★</td>
<td>★★</td>
<td>★</td>
<td>★★★★</td>
<td>★★★★</td>
<td>★</td>
<td>?</td>
</tr>
<tr>
<td>$B_s \to \mu^+\mu^-$</td>
<td>★★★</td>
<td>★★★★</td>
<td>★★★★</td>
<td>★★★★</td>
<td>★★★★</td>
<td>★</td>
<td>★</td>
</tr>
</tbody>
</table>

AC = SUSY flavour model with right-handed currents

RVV2 = SUSY flavour model with right-handed currents

AKM = SUSY flavour model with right-handed currents

$\delta$LL = SUSY flavour model with only left-handed currents

FBMSSM = flavour blind MSSM

LHT = Little Higgs models with T-parity.

RS = Randall Sundrum models