Differential Distributions from Soft-collinear Effective Theory

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arXiv:0911.4135

Fermilab theory seminar, April 15, 2010

Outline

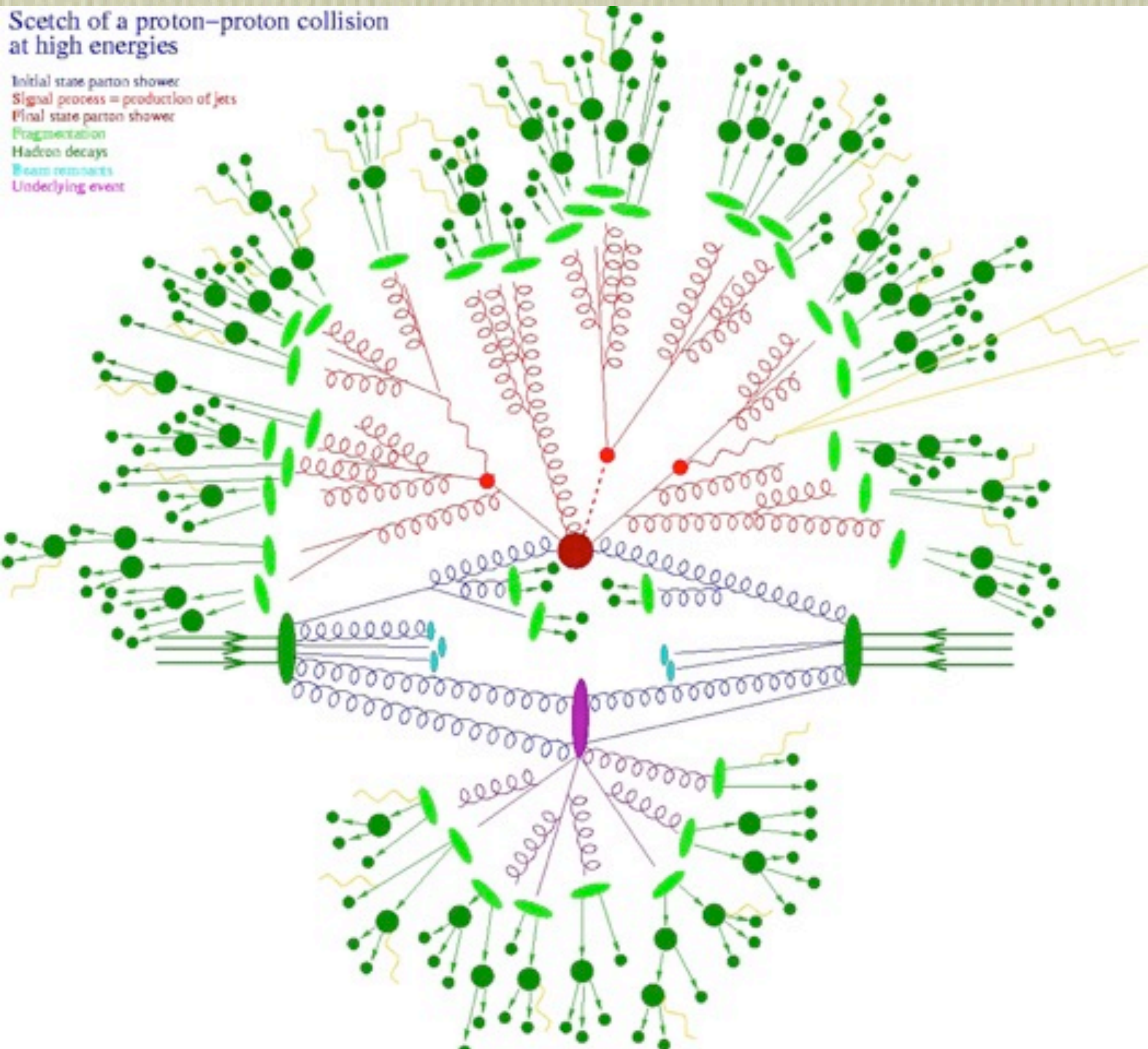
- Introductory Remarks
- Phenomenological motivations
- Collins-Soper-Sterman approach to low-p_T resummation
- Soft-collinear effective theory approach

  -Factorization and resummation formula:

$$\frac{d^2\sigma}{dp_T^2 dY} \sim \underbrace{H}_{\text{RG evolution}} \otimes \underbrace{\mathcal{G}^{ij}}_{\text{Soft-collinear emissions}} \otimes f_i \otimes \underbrace{f_j}_{\text{PDFs}}$$

- Conclusions

Factorization

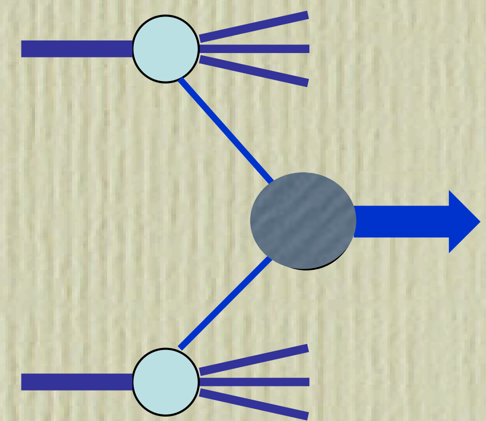


- New physics at hard scale; M_H for example
- Parton shower evolution from M_H to Λ_{QCD}
- Final state hadronization at Λ_{QCD}
- Parton distribution functions at Λ_{QCD}
- Multiple parton interactions, hadron decays, ...

- How do we make sense of this environment?

Factorization!

Factorization



$$d\sigma = \sum_{i,j} d\sigma_{ij}^{\text{part}} \otimes f_i(\xi_a) \otimes f_j(\xi_b)$$

Calculable in pQCD

Extracted from data

- Separates perturbative and non-perturbative scales
- Turns pQCD into a predictive framework in complicated hadron collider environments
- Factorization is not obvious, and is often difficult to prove

$$\text{lepton} + A \to \text{lepton}' + X$$
$$e^+ + e^- \to A + X$$
$$A + B \to V + X$$
$$A + B \to \text{jet} + X$$
$$A + b \to \text{heavy quark} + X$$

Factorization
"expected to hold"

Collins-Soper-Sterman, 2004 review

Resummation

- Fully inclusive Drell-Yan:

$$d\sigma = \sum_{i,j} d\sigma_{ij}^{\text{part}} \otimes f_i(\xi_a) \otimes f_j(\xi_b)$$

Lives at the hard scale

Live at non-perturbative scale

RG evolve to hard scale

- Large logarithms of hard and non-perturbative scales arise ➡ Resummation needed

- Resummation done by evaluating PDFs at the hard scale after renormalization group running (DGLAP)

Resummation

- In the presence of final state restrictions:

$$d\sigma = \sum_{i,j} d\sigma_{ij}^{\text{part}} \otimes f_i(\xi_a) \otimes f_j(\xi_b)$$

Multiple disparate scales involved.

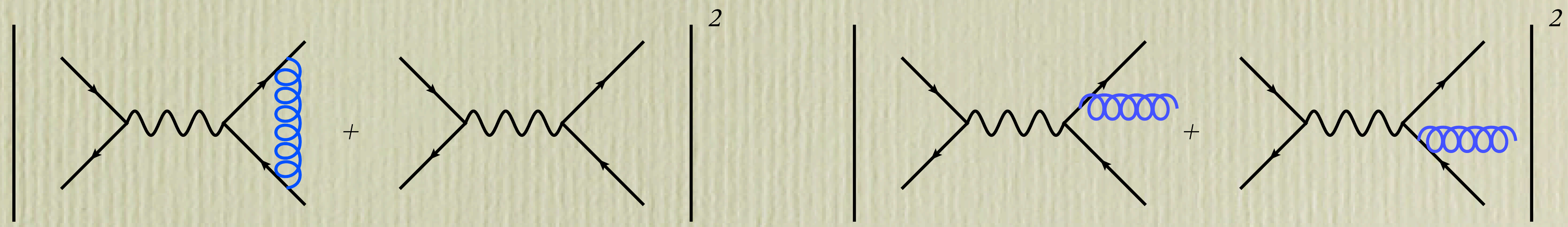
Live at non-perturbative scale.

Additional resummation needed.

- Example: low transverse momentum distribution in Drell-Yan, Higgs production

Why do logs arise from final state restrictions?

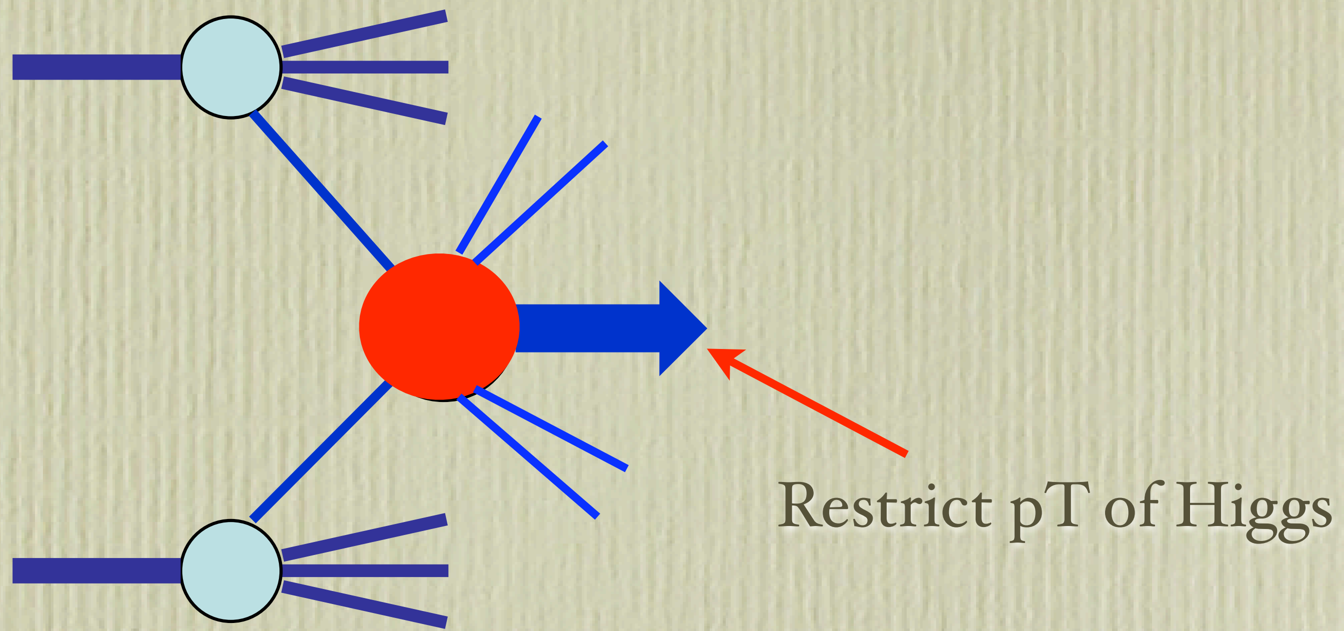
- Fully inclusive electron-positron annihilation:



Cancellation of infrared divergences between virtual and real graphs. → Infrared Safety!

- Incomplete cancellation of IR divergences in presence of final state restrictions gives rise to large logarithms of restricted kinematic variable

Example: Higgs low pT Restriction



$$pp \rightarrow h + X$$

- We restrict the transverse momentum of the Higgs:

$$m_h \gg p_T \gg \Lambda_{QCD}$$

- This in turn restricts the transverse momentum of final state radiation, giving rise to logs of p_T

- Such p_T restrictions can be studied for any color neutral particle. We use Higgs production as an illustrative example.

Low pT Region

- The schematic perturbative series for the pT distribution for pp → h+X

$$\frac{1}{\sigma}\frac{d\sigma}{dp_T^2} \simeq \frac{1}{p_T^2}\left[A_1\alpha_S \ln\frac{M^2}{p_T^2} + A_2\alpha_S^2 \ln^3\frac{M^2}{p_T^2} + \ldots + A_n\alpha_S^n \ln^{2n-1}\frac{M^2}{p_T^2} + \ldots\right]$$

Large Logarithms spoil perturbative convergence

- Resummation of large logarithms required

- Low p_T resummation has been studied in great detail

(Dokshitzer, Dyakonov, Troyan; Parisi, Petronzio; Curci et al.; Davies, Stirling; Collins, Soper, Sterman; Arnold, Kauffman; Berger, Qiu; Ellis, Ross,Veseli; Ladinsky, Yuan; Bozzi, Catani, de Florian, Grazzini,....)

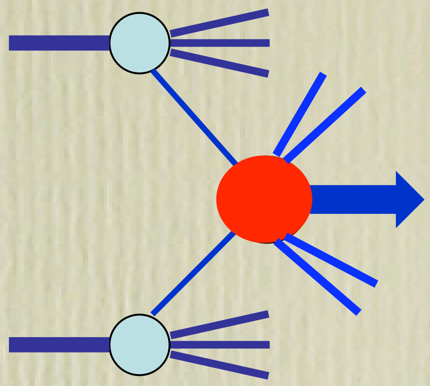
Why restrict the final state?

- Often one needs to make cuts to enhance the signal over background.

- The low transverse momentum region is important for Higgs searches in the range

$$130\ \mathrm{GeV} < m_h < 180\ \mathrm{GeV}$$

Higgs Search at the LHC



- For the Higgs mass range:

$$130 \text{ GeV} < m_h < 180 \text{ GeV}$$

- Higgs search channel:

$$gg \to h \to W^+W^- \to \ell^+\nu\ell^-\bar{\nu}$$

- Large background from:

$$pp \to t\bar{t} \to bW^+\bar{b}W^- \to \ell^+\nu\ell^-\bar{\nu} + \text{jets}$$

- Background elimination requires jet vetoes:

veto events with jets of $p_T > 20$ GeV

Jet Veto enhances signal to background ratio

LHC 14 TeV		Accepted event fraction		
reaction $pp \to X$	$\sigma \times BR^2$ [pb]	cut 1-3	cut 4-6	cut 7
$pp \to H \to W^+W^-$ ($m_H = 170$ GeV)	1.24	0.21	0.18	0.080
$pp \to W^+W^-$	7.4	0.14	0.055	0.039
$pp \to t\bar{t}$ ($m_t = 175$ GeV)	62.0	0.17	0.070	0.001
$pp \to Wtb$ ($m_t = 175$ GeV)	≈6	0.17	0.092	0.013

(Dittmar, Dreiner)

Collins-Soper-Sterman Formalism

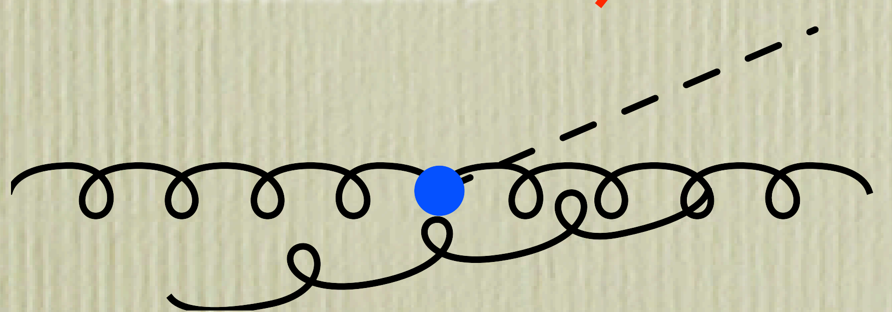
CSS Formalism

$$A(P_A) + B(P_B) \to C(Q) + X\,, \qquad C = \gamma^*, W^\pm, Z, h$$

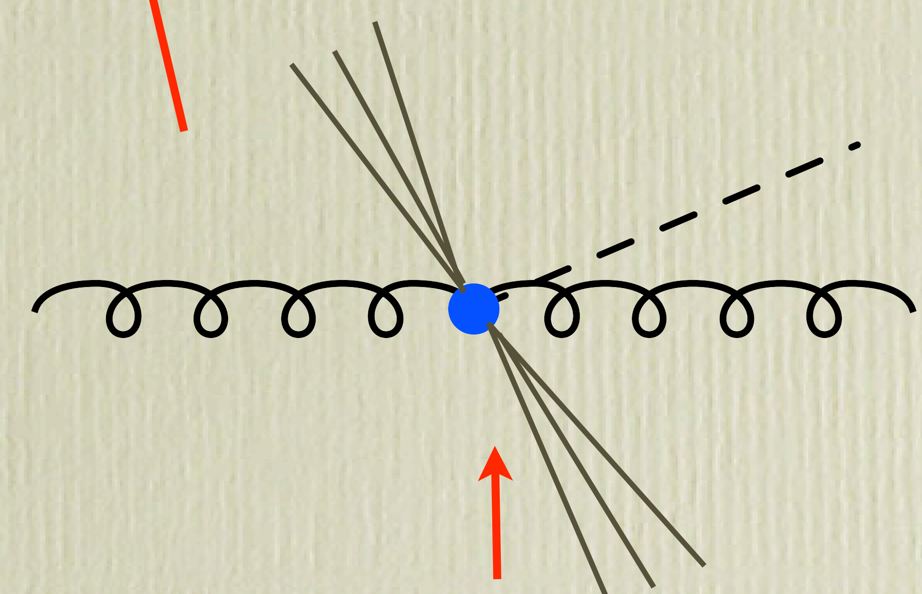
- The transverse momentum distribution in the CSS formalism is schematically given by:

$$\frac{d\sigma_{AB\to CX}}{dQ^2\,dy\,dQ_T^2} = \frac{d\sigma^{(\text{resum})}_{AB\to CX}}{dQ^2\,dy\,dQ_T^2} + \frac{d\sigma^{(\text{Y})}_{AB\to CX}}{dQ^2\,dy\,dQ_T^2}$$

Most singular contribution



Soft or collinear gluon emissions



Contributions from hard jets

CSS Formalism

Focus of this talk

$$\frac{d\sigma_{AB\to CX}}{dQ^2\,dy\,dQ_T^2} = \frac{d\sigma^{(\mathrm{resum})}_{AB\to CX}}{dQ^2\,dy\,dQ_T^2} + \frac{d\sigma^{(\mathrm{Y})}_{AB\to CX}}{dQ^2\,dy\,dQ_T^2}$$

- Singular as at least Q_T^{-2} as $Q_T \to 0$
- Important in region of small Q_T
- Treated with resummation

- Less singular terms (integrable without distributions)
- Important in region of large Q_T

CSS Formalism

- The CSS resummation formula takes the form:

PDF

Perturbatively calculable

$$\frac{d^2\sigma}{dp_T\,dY} = \sigma_0 \int \frac{d^2 b_\perp}{(2\pi)^2} e^{-i\vec{p}_T\cdot\vec{b}_\perp} \sum_{a,b} \left[C_a \otimes f_{a/P}\right](x_A, b_0/b_\perp) \left[C_b \otimes f_{b/P}\right](x_B, b_0/b_\perp)$$
$$\times \exp\left\{ \int_{b_0^2/b_\perp^2}^{\hat{Q}^2} \frac{d\mu^2}{\mu^2} \left[\ln\frac{\hat{Q}^2}{\mu^2} A(\alpha_s(\mu^2)) + B(\alpha_s(\mu^2)) \right] \right\}.$$

Sudakov Factor

Coefficients with well defined perturbative expansions

Why b-space?

- Both matrix elements and phase space simplify in soft-emission limit

Eikonal approximation (soft photons):

$$\mathcal{M}_n \propto g^n \mathcal{M}_0 \left\{ \frac{p_1 \cdot \epsilon_1 \ldots p_1 \cdot \epsilon_n}{p_1 \cdot k_1 \ldots p_1 \cdot k_n} + (-1)^n \frac{p_2 \cdot \epsilon_1 \ldots p_2 \cdot \epsilon_n}{p_2 \cdot k_1 \ldots p_2 \cdot k_n} \right\}$$

Phase space:

$$d\Pi_n \propto \nu(k_{T1})\, d^2k_{T1} \ldots \nu(k_{Tn})\, d^2k_{Tn}\ \overbrace{\delta^{(2)}\left(\vec{p}_T - \sum_i \vec{k}_{Ti}\right)}^{\text{sum to Higgs } p_T}$$

$$\nu(k_T) = k_T^{-2\epsilon} \ln\left(\frac{s}{k_T^2}\right)$$

- Would be independent except for phase-space constraint; Fourier transform to b-space accomplishes this

$$\int \frac{d^2b}{(2\pi)^2}\ \mathrm{e}^{-i\vec{b}\cdot\vec{p}_T} \int d^2k_{T1}\, f(k_{T1}) \ldots d^2k_{Tn}\, f(k_{Tn})\, \delta^{(2)}\left(\vec{p}_T - \sum_i \vec{k}_{Ti}\right)$$

$$= \int \frac{d^2b}{(2\pi)^2}\, \mathrm{e}^{-i\vec{b}\cdot\vec{p}_T} \left[\tilde{f}(b)\right]^n, \quad \tilde{f}(b) = \int d^2k_T\, \mathrm{e}^{i\vec{b}\cdot\vec{k}_T} f(k_T)$$

CSS Formalism

$$\frac{d^2\sigma}{dp_T\,dY} = \sigma_0 \int \frac{d^2b_\perp}{(2\pi)^2} e^{-i\vec{p}_T\cdot\vec{b}_\perp} \sum_{a,b} \left[C_a \otimes f_{a/P}\right](x_A, b_0/b_\perp) \left[C_b \otimes f_{b/P}\right](x_B, b_0/b_\perp)$$
$$\times\ \exp\left\{ \int_{b_0^2/b_\perp^2}^{\hat{Q}^2} \frac{d\mu^2}{\mu^2} \left[\ln\frac{\hat{Q}^2}{\mu^2} A(\alpha_s(\mu^2)) + B(\alpha_s(\mu^2)) \right] \right\}.$$

Landau Pole

- The integration over the impact parameter introduces a Landau pole
- Must specify a treatment of the Landau pole for *any* value of p_T

Landau-pole prescriptions

- Introduce cutoff for the large b region by evaluating at the point (Collins, Soper 1982)

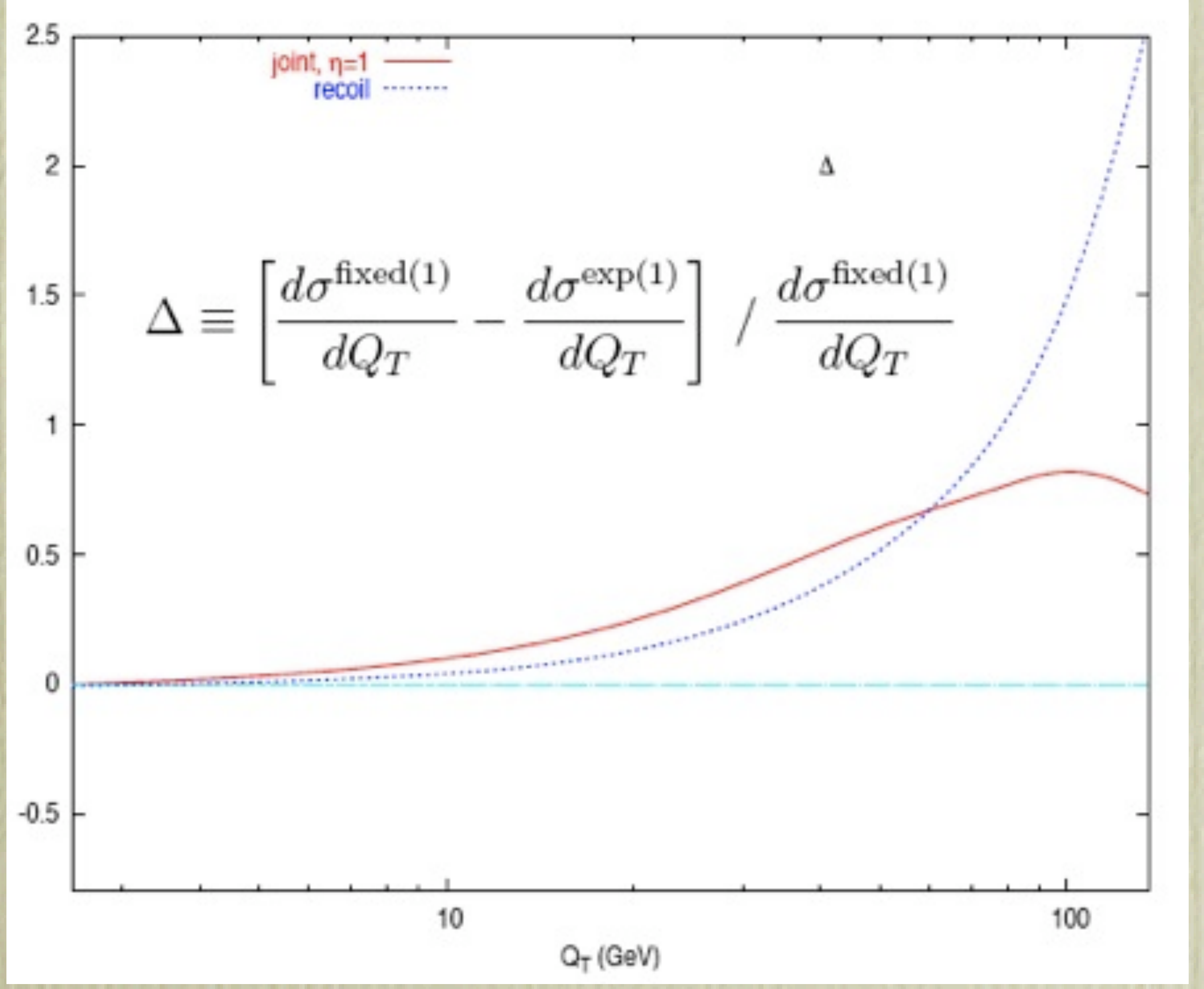
$$b_* = \frac{b}{\sqrt{1 + (b/b_{max})^2}}$$

- "Minimal prescription:" deform b-contour to avoid singularities (Catani, Mangano, Nason, Trentadue 1996; Laenen, Sterman, Vogelsang 2000)

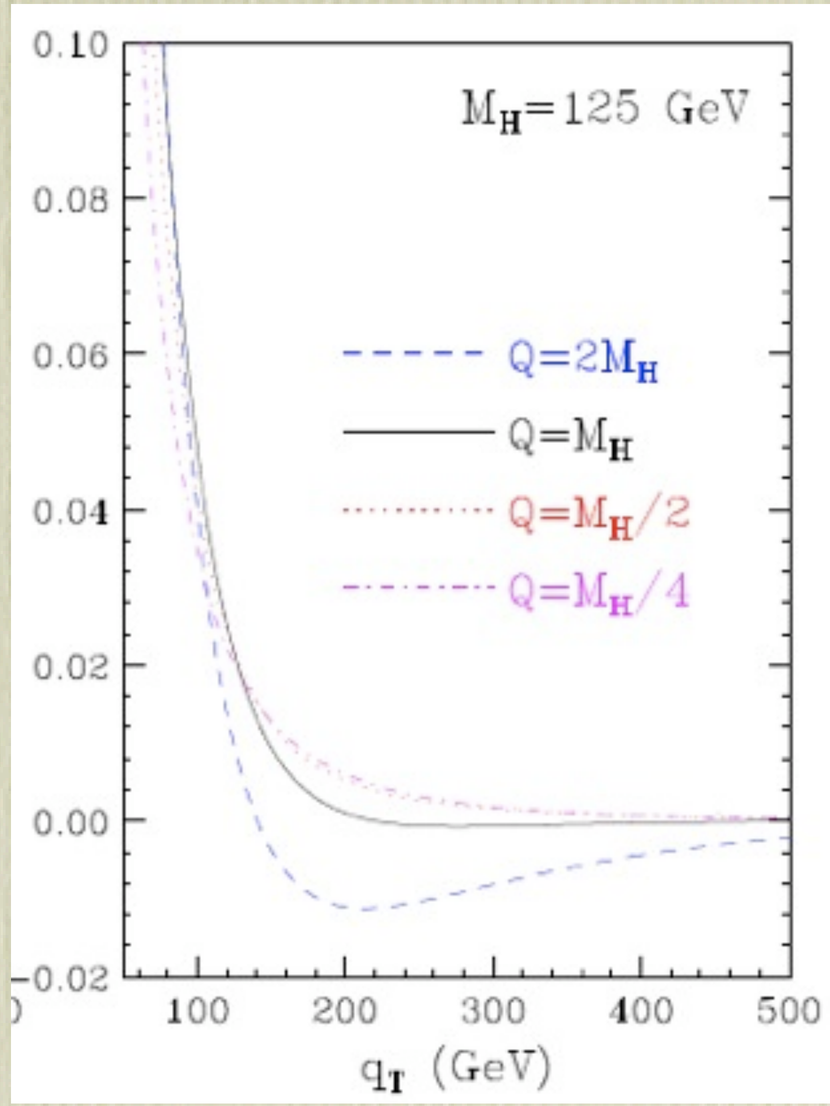
$$b = [\cos\phi \pm i \sin\phi]\, t$$

Matching to fixed-order

- Resummed exponent in b-space, fixed-order in p_T space ⇒ leads to difficulties in matching



Kulesza, Sterman, Vogelsang 2003



Bozzi, Catani, de Florian, Grazzini 2005

EFT Approach

Effective Field Theory (EFT)

- EFTs allow one to:
  - deal with one scale at a time
  - formulate problem in an expansion of the ratio of disparate scales
  - factorize effects from each scale
  - perform resummation via the renormalization group equations of EFT operators
- Low transverse momentum distribution has the scales

$$m_h \gg p_T \gg \Lambda_{QCD}$$

- The most singular pT emissions recoiling against the Higgs are soft and collinear emissions whose dynamics may be addressed in Soft-Collinear Effective Theory (SCET)

(Bauer, Fleming, Luke, Pirjol, Stewart)

SCET at the LHC

- Study of SCET at the LHC, particularly for differential quantities, is still in its infancy
  - threshold resummation for inclusive Drell-Yan, Higgs, ttbar (Becher, Neubert et al.)
  - Factorization at the LHC with cuts on hadronic final states (Stewart, Tackmann, Waalewijn)
- Still determining what objects appear in SCET factorization theorems at the LHC; gain knowledge of how to apply SCET to hadronic collisions from this study

EFT framework

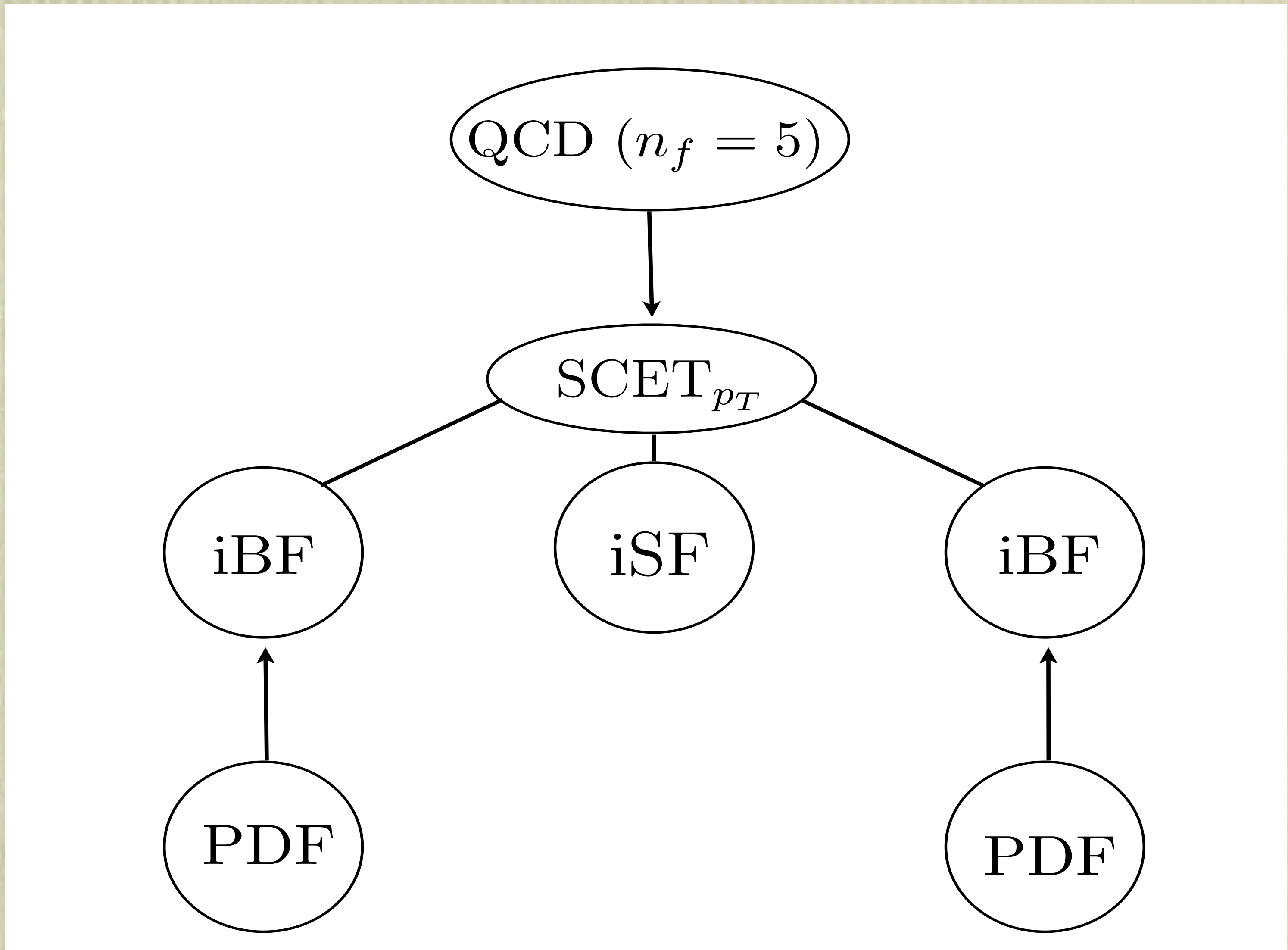
$$\mathrm{QCD}(n_f = 6) \to \mathrm{QCD}(n_f = 5) \to \mathrm{SCET}_{p_T} \to \mathrm{SCET}_{\Lambda_{QCD}}$$

Top quark integrated out.

Matched onto SCET.

Soft-collinear factorization.

Matching onto PDFs.



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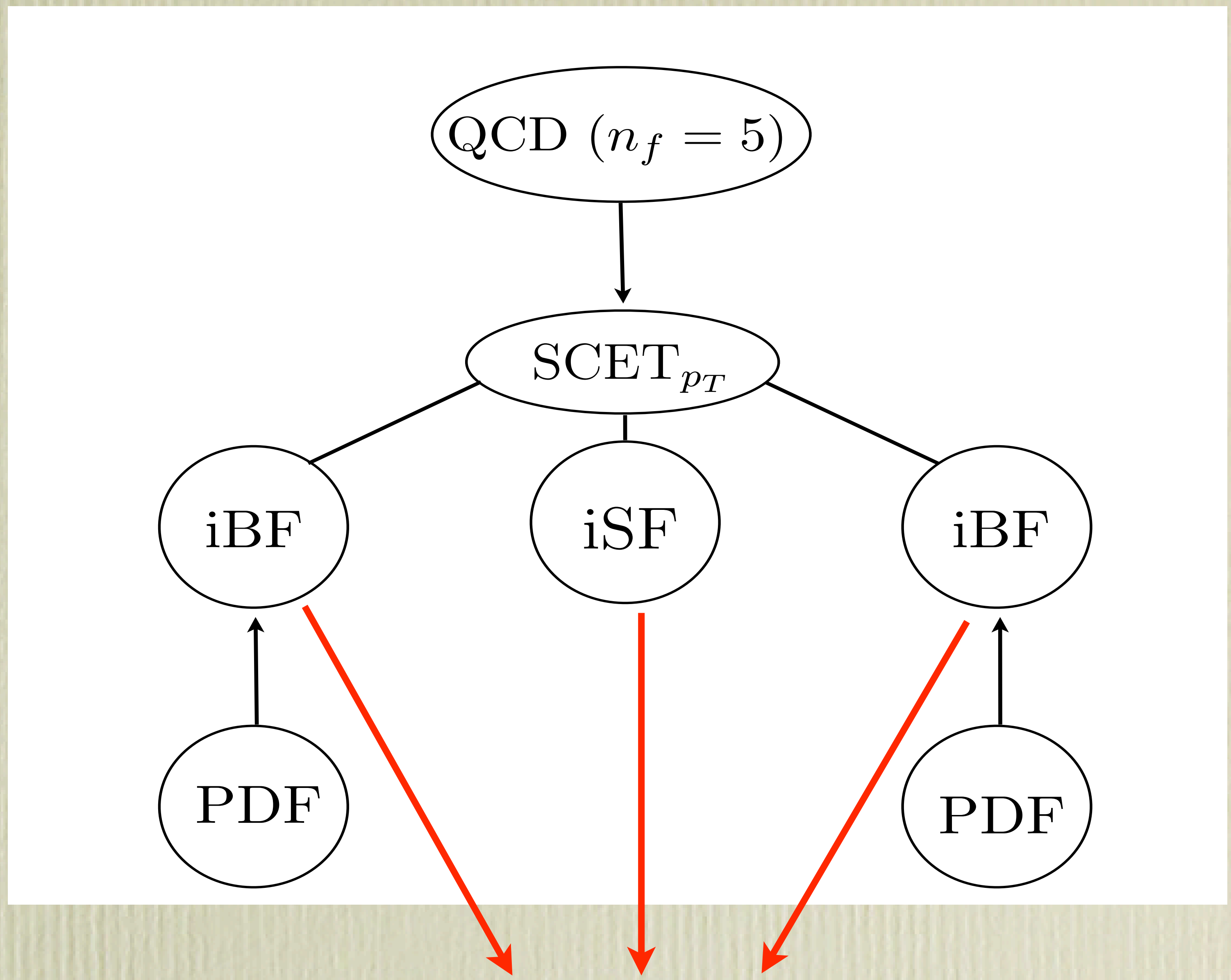


Newly defined objects describing soft and collinear pT emissions

iBF = impact-parameter Beam Function
iSF = inverse Soft Function

SCET Factorization Formula

- Factorization formula derived in SCET in schematic form:

$$\frac{d^2\sigma}{dp_T^2\,dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

Hard function.

Sums logs of m_h/p_T

Transverse momentum function.

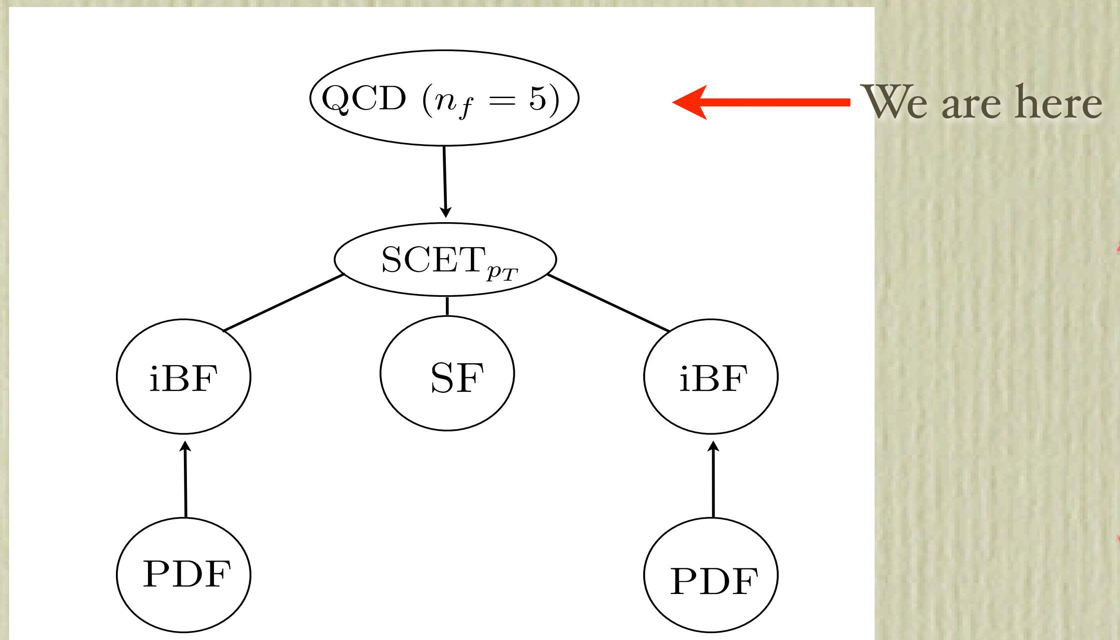
Evaluated at pT scale.

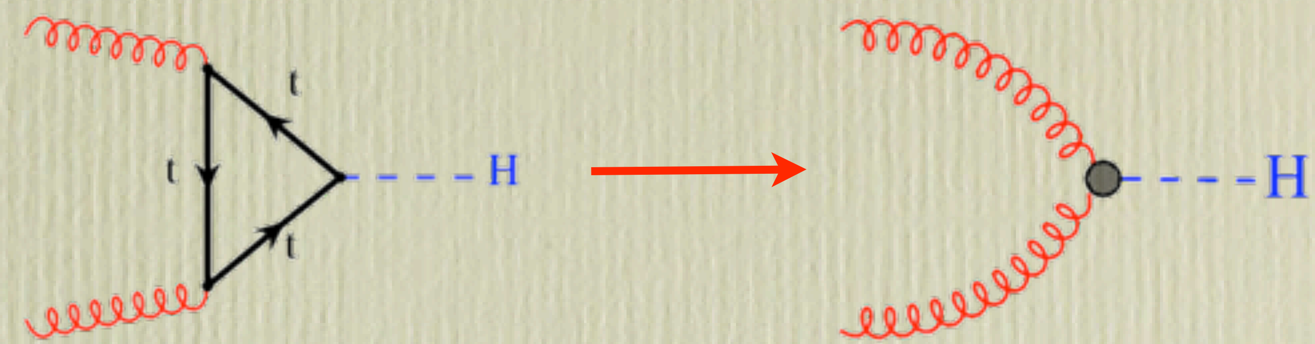
PDFs.

RG evolved to pT scale

- All objects are field theoretically defined
- Large logarithms are summed via RG equations in EFTs
- Formulation is free of Landau poles

Integrating out the top





- Leading term in the Higgs effective interaction with gluons:

$$\mathcal{L}_{m_t} = C_{GGh} \frac{h}{v} G^a_{\mu\nu} G^{\mu\nu}_a \quad , \qquad C_{GGh} = \frac{\alpha_s}{12\pi} \left\{ 1 + \frac{11}{4} \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2) \right\}$$

SCET in a NutShell

- Effective theory with soft and collinear degrees of freedom:

$$p^\mu \equiv (p^+, p^-, p_\perp)$$

$$p_n \sim m_h(\eta^2, 1, \eta), \quad p_{\bar{n}} \sim m_h(1, \eta^2, \eta), \quad p_s \sim m_h(\eta, \eta, \eta),$$

- Well defined power counting:

$$\eta \sim \frac{p_T}{m_h}$$

Corresponds to soft and collinear modes with transverse momentum of order p_T

- Soft and collinear fields are distinguished and are decoupled at leading order in η

- Soft and Collinear gauge invariance restricts the form of SCET operators that can appear

Soft-Collinear Decoupling

(Bauer, Fleming, Stewart, Pirjol)

- The SCET Lagrangian with a power counting scheme as:

$$\mathcal{L}_{SCET} = \mathcal{L}^{(0)}_{SCET} + \mathcal{L}^{(1)}_{SCET} + \mathcal{L}^{(2)}_{SCET} + \cdots$$

- At leading order the soft and collinear modes are decoupled:

$$\mathcal{L}^{(0)}_{SCET} = \mathcal{L}^{(0)}_{\text{coll.}} + \mathcal{L}^{(0)}_{\text{soft}}$$

- Soft and collinear interactions produce off-shell modes that must be integrated out:

$$p_{hc} \sim p_c + p_s \sim Q(\eta^2, 1, \eta) + Q(\eta, \eta, \eta) \sim Q(\eta, 1, \eta)$$

$$p_{hc}^2 \sim Q^2 \eta \gg p_c^2, p_s^2$$

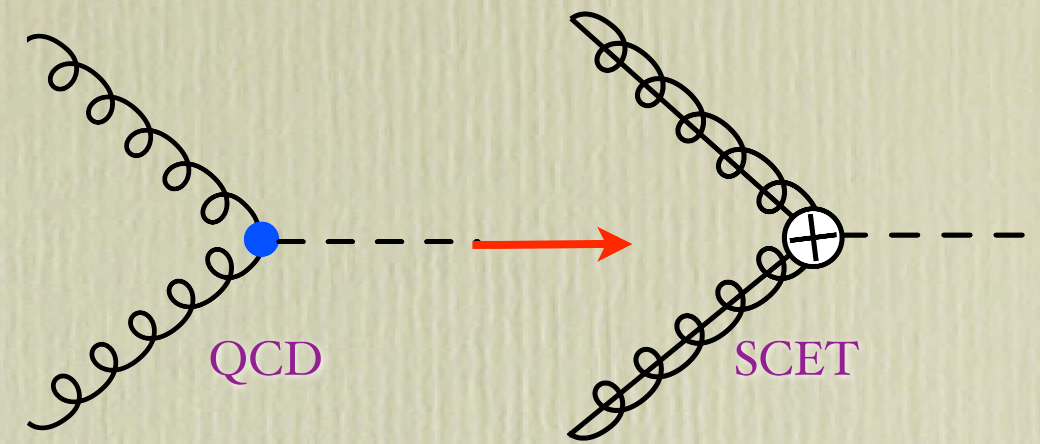
- Decoupling property allows for factorization of matrix elements

$$\langle \mathcal{O}_{\text{SCET}} \rangle \to \langle \mathcal{O}_{coll.} \rangle \langle \mathcal{O}_{soft} \rangle$$

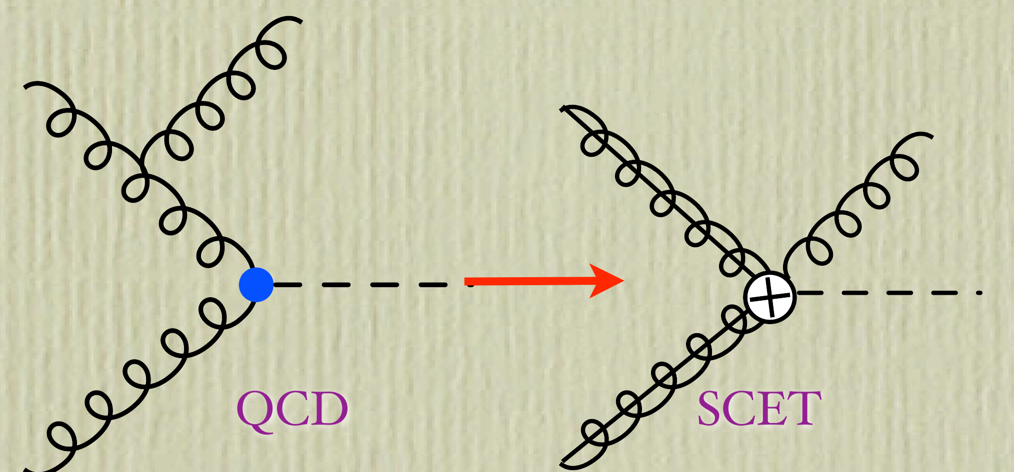
Matching onto SCET

- Matching equation:

$$O_{QCD} = \int d\omega_1 \int d\omega_2 \, C(\omega_1, \omega_2) \, \mathcal{O}(\omega_1, \omega_2)$$

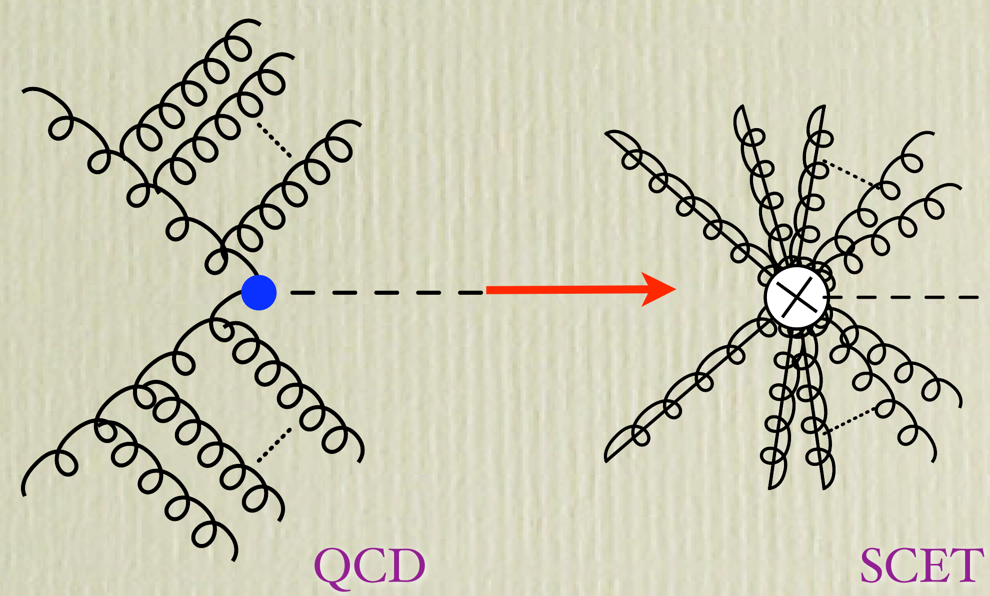


Tree level matching (EFT graphs scale-less in dim-reg ⇒ finite part of virtual corrections



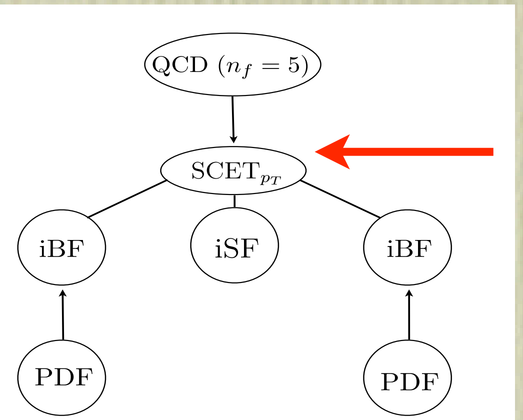
Matching real emission graphs

Soft and collinear emissions build into Wilson lines determined by soft and collinear gauge invariance



- Effective SCET operator:

$$\mathcal{O}(\omega_1, \omega_2) = g_{\mu\nu} h \, T\{\mathrm{Tr}\Big[S_n (g B^{\mu}_{n\perp})_{\omega_1} S_n^{\dagger} S_{\bar{n}} (g B^{\nu}_{\bar{n}\perp})_{\omega_2} S_{\bar{n}}^{\dagger}\Big]\}$$



We are here

SCET Cross-Section

- SCET differential cross-section:

$$\frac{d^2\sigma}{du\,dt} = \frac{1}{2Q^2}\left[\frac{1}{4}\right]\int \frac{d^2p_{h_\perp}}{(2\pi)^2}\int \frac{dn\cdot p_h d\bar{n}\cdot p_h}{2(2\pi)^2}(2\pi)\theta(n\cdot p_h+\bar{n}\cdot p_h)\delta(n\cdot p_h\bar{n}\cdot p_h-\vec{p}_{h_\perp}^{\,2}-m_h^2)$$
$$\times\ \delta(u-(p_2-p_h)^2)\delta(t-(p_1-p_h)^2)\sum_{\text{initial pols.}}\sum_X \left|C(\omega_1,\omega_2)\otimes\langle hX_nX_{\bar{n}}X_s|\mathcal{O}(\omega_1,\omega_2)|pp\rangle\right|^2$$
$$\times\ (2\pi)^4\delta^{(4)}(p_1+p_2-P_{X_n}-P_{X_{\bar{n}}}-P_{X_s}-p_h),$$

- Schematic form of SCET cross-section:

$$\frac{d^2\sigma}{dp_T^2dY} \sim \int PS\,|C\otimes\langle\mathcal{O}\rangle|^2$$

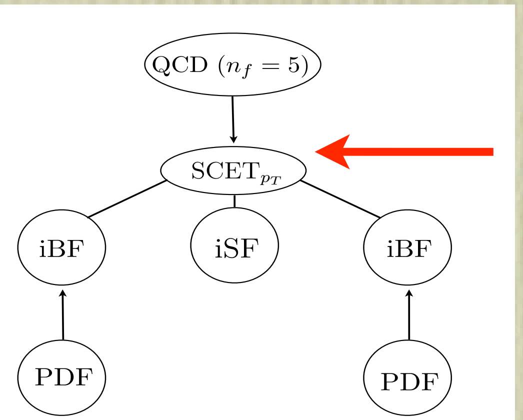
Phase space integrals.

Hard matching coefficient.

SCET matrix element.

Factorize using soft-collinear decoupling

Factorization in SCET



We are here

$$\frac{d^2\sigma}{dp_T^2 dY} \sim \int PS \, |C \otimes \langle \mathcal{O} \rangle|^2$$

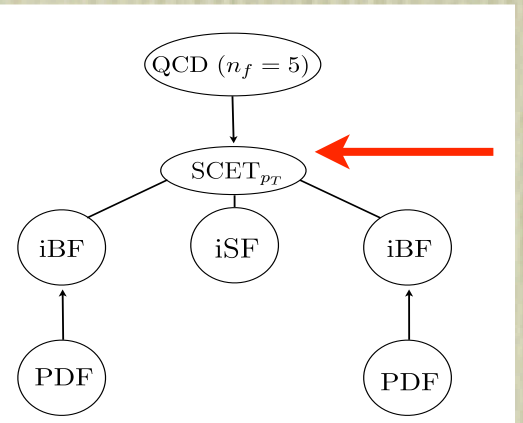
Factorize cross-section using soft-collinear decoupling in SCET

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes B_n \otimes B_{\bar{n}} \otimes S$$

Hard matching coefficient squared

Decoupled collinear and soft functions

Factorization in SCET



We are here

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes B_n \otimes B_{\bar{n}} \otimes S$$

Hard function

Physics of hard scale.
Sums logs of mh/pT.

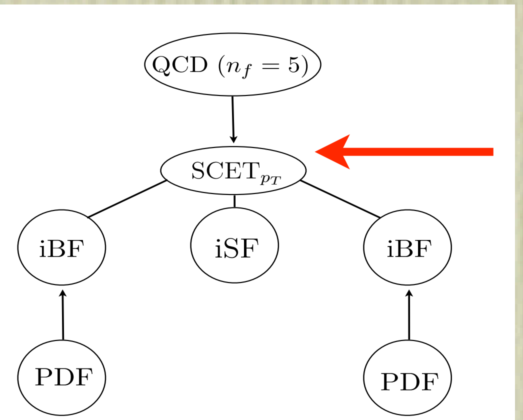
Impact-parameter Beam Functions (iBFs)

Describes collinear pT emissions

Soft function

Describes soft pT emissions

Factorization in SCET



We are here

- Factorization formula in full detail:

Hard

$$\frac{d^2\sigma}{du\,dt} = \frac{(2\pi)}{(N_c^2-1)^2 8Q^2}\int dp_h^+ dp_h^- \int d^2k_h^\perp \int \frac{d^2b_\perp}{(2\pi)^2} e^{-i\vec{k}_h^\perp\cdot\vec{b}_\perp}$$

$$\times\, \delta\left[u-m_h^2+Qp_h^-\right]\delta\left[t-m_h^2+Qp_h^+\right]\delta\left[p_h^+p_h^- - \vec{k}_{h\perp}^2 - m_h^2\right]\int d\omega_1 d\omega_2 |C(\omega_1,\omega_2,\mu)|^2$$

$$\times \int dk_n^+ dk_{\bar n}^-\, B_n^{\alpha\beta}(\omega_1,k_n^+,b_\perp,\mu)\, B_{\bar n\alpha\beta}(\omega_2,k_{\bar n}^-,b_\perp,\mu)\, S(\omega_1-p_h^- - k_{\bar n}^-, \omega_2 - p_h^+ - k_n^+, b_\perp,\mu)$$

n-collinear iBF

bn-collinear iBF

Soft

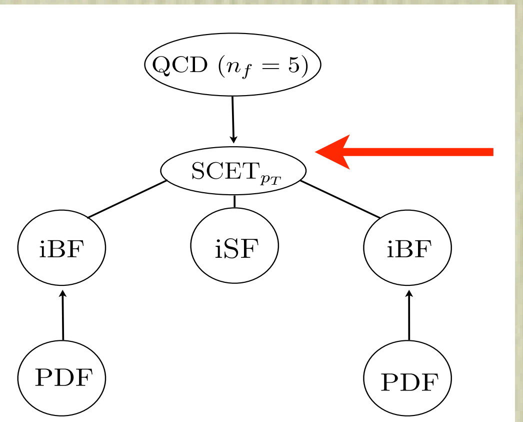
- iBFs and soft functions field theoretically defined as the fourier transform of:

$$J_n^{\alpha\beta}(\omega_1,x^-,x_\perp,\mu) = \sum_{\text{initial pols.}} \langle p_1|\left[gB^A_{1n\perp\beta}(x^-,x_\perp)\delta(\bar{\mathcal{P}}-\omega_1)gB^A_{1n\perp\alpha}(0)\right]|p_1\rangle$$

$$J_{\bar n}^{\alpha\beta}(\omega_1,y^+,y_\perp,\mu) = \sum_{\text{initial pols.}} \langle p_2|\left[gB^A_{1n\perp\beta}(y^+,y_\perp)\delta(\bar{\mathcal{P}}-\omega_2)gB^A_{1n\perp\alpha}(0)\right]|p_2\rangle$$

$$S(z,\mu) = \langle 0|\bar{T}\left[\mathrm{Tr}\left(S_{\bar n}T^D S_{\bar n}^\dagger S_n T^C S_n^\dagger\right)(z)\right] T\left[\mathrm{Tr}\left(S_n T^C S_n^\dagger S_{\bar n} T^D S_{\bar n}^\dagger\right)(0)\right]|0\rangle$$

Factorization in SCET



We are here

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes B_n \otimes B_{\bar{n}} \otimes S$$

iBFs are proton matrix elements
and sensitive to the
non-perturbative scale

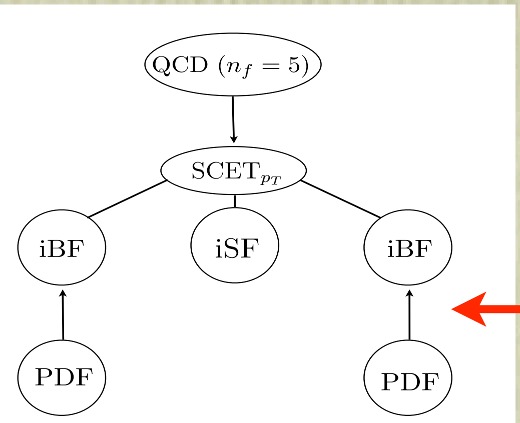
- The iBFs are matched onto PDFs to separate the perturbative and non-perturbative scales:

$$B_n = \mathcal{I}_{n,i} \otimes f_i \qquad\qquad B_{\bar{n}} = \mathcal{I}_{\bar{n},i} \otimes f_i$$

iBF

Matching
coefficient

PDF



We are here

iBFs to PDFs

- iBF is matched onto the PDF with matching coefficient defined as:

$$\tilde{B}_n^{\alpha\beta}(z, t_n^+, b_\perp, \mu) = -\frac{1}{z} \sum_{i=g,q,\bar{q}} \int_z^1 \frac{dz'}{z'} \mathcal{I}_{n;g,i}^{\alpha\beta}\left(\frac{z}{z'}, t_n^+, b_\perp, \mu\right) f_{i/P}(z', \mu)$$

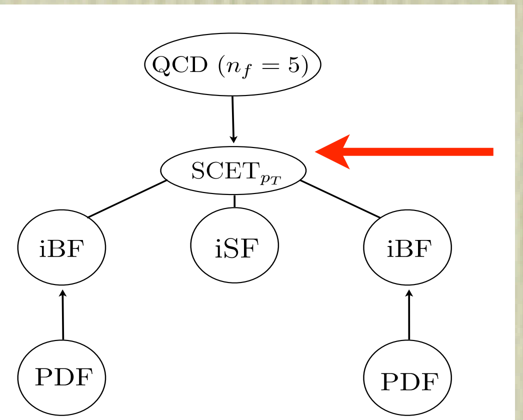
- The PDF is defined as:

Scaleless

$$f_{g/P}(z, \mu) = \frac{-z\bar{n} \cdot p_1}{2} \sum_{\text{spins}} \langle p_1 | \left[\text{Tr}\{ B_\perp^\mu(0) \delta(\bar{\mathcal{P}} - z\, \bar{n} \cdot p_1) B_{\perp\mu}(0) \} \right] | p_1 \rangle$$

- The matching coefficient is given by:

$$\mathcal{I}_{n;g,i}^{\beta\alpha}\left(\frac{z}{z'}, t_n^+, b_\perp, \mu\right) = -z \left[\tilde{B}_n^{\alpha\beta}\left(\frac{z}{z'}, z' t_n^+, b_\perp, \mu\right) \right]_{\text{finite part in dim-reg}}$$



We are here

Factorization in SCET

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes B_n \otimes B_{\bar{n}} \otimes S$$

- After matching the iBFs to the PDFs we get:

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes [\mathcal{I}_{n,i} \otimes f_i] \otimes [\mathcal{I}_{\bar{n},j} \otimes f_j] \otimes S$$

- Group the perturbative pT scale functions into transverse momentum dependent function:

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes [\mathcal{I}_{n,i} \otimes \mathcal{I}_{\bar{n},j} \otimes S] \otimes f_i \otimes f_j$$

$\mathcal{G}^{ij}$

Hard function

Transverse momentum dependent function

PDFs

Factorization Formula

- Factorization formula in full detail:

$$\frac{d^2\sigma}{d\mathrm{p}_T^2\,dY} = \frac{\pi^2}{4(N_c^2-1)^2Q^2}\int_0^1\frac{dx_1}{x_1}\int_0^1\frac{dx_2}{x_2}\int_{x_1}^1\frac{dx_1'}{x_1'}\int_{x_2}^1\frac{dx_2'}{x_2'}$$
$$\times \underbrace{H(x_1,x_2,\mu_Q;\mu_T)}_{\text{Hard function}}\,\underbrace{\mathcal{G}^{ij}(x_1,x_1',x_2,x_2',p_T,Y,\mu_T)}_{\text{Transverse momentum function}}\,\underbrace{f_{i/P}(x_1',\mu_T)\,f_{j/P}(x_2',\mu_T)}_{\text{PDFs}}$$

- The transverse momentum function is a convolution of the iBF matching coefficients and the soft function:

$$\mathcal{G}^{ij}(x_1,x_1',x_2,x_2',p_T,Y,\mu_T) = \int dt_n^+\int dt_{\bar n}^-\int\frac{d^2b_\perp}{(2\pi)^2}J_0(|\vec b_\perp|p_T)$$
$$\times\,\mathcal{I}^{\beta\alpha}_{n;g,i}\Big(\frac{x_1}{x_1'},t_n^+,b_\perp,\mu_T\Big)\,\mathcal{I}^{\beta\alpha}_{\bar n;g,j}\Big(\frac{x_2}{x_2'},t_{\bar n}^-,b_\perp,\mu_T\Big)$$
$$\times\,\mathcal{S}^{-1}\Big(x_1Q-e^{Y}\sqrt{\mathrm{p}_T^2+m_h^2}-\frac{t_{\bar n}^-}{Q},\,x_2Q-e^{-Y}\sqrt{\mathrm{p}_T^2+m_h^2}-\frac{t_n^+}{Q},\,b_\perp,\mu_T\Big)$$

Factorization Formula

- Factorization formula in full detail:

$$\frac{d^2\sigma}{dp_T^2\, dY} = \frac{\pi^2}{4(N_c^2-1)^2Q^2}\int_0^1 \frac{dx_1}{x_1}\int_0^1 \frac{dx_2}{x_2}\int_{x_1}^1 \frac{dx_1'}{x_1'}\int_{x_2}^1 \frac{dx_2'}{x_2'}$$
$$\times\, H(x_1,x_2,\mu_Q,\mu_T)\,\mathcal{G}^{ij}(x_1,x_1',x_2,x_2',p_T,Y,\mu_T)\,f_{i/P}(x_1',\mu_T)\,f_{j/P}(x_2',\mu_T)$$

RG evolution cut off at $\mu_T \sim p_T$, the matching scale from QCD $\to$ SCET_{p_T}, not $1/b_\perp$

Factorization Formula

Impact parameter appears, but only to simplify iBF→PDF matching; can transform this formula to be completely in momentum space

$$\mathcal{G}^{ij}(x_1, x_1', x_2, x_2', p_T, Y, \mu_T) = \int dt_n^+ \int dt_{\bar{n}}^- \int \frac{d^2 b_\perp}{(2\pi)^2} J_0(|\vec{b}_\perp| p_T)$$
$$\times\, \mathcal{I}_{n;g,i}^{\beta\alpha}\Big(\frac{x_1}{x_1'}, t_n^+, b_\perp, \mu_T\Big)\, \mathcal{I}_{\bar{n};g,j}^{\beta\alpha}\Big(\frac{x_2}{x_2'}, t_{\bar{n}}^-, b_\perp, \mu_T\Big)$$
$$\times\, \mathcal{S}^{-1}\Big(x_1 Q - e^{Y}\sqrt{\mathrm{p}_T^2 + m_h^2} - \frac{t_{\bar{n}}^-}{Q}, x_2 Q - e^{-Y}\sqrt{\mathrm{p}_T^2 + m_h^2} - \frac{t_n^+}{Q}, b_\perp, \mu_T\Big)$$

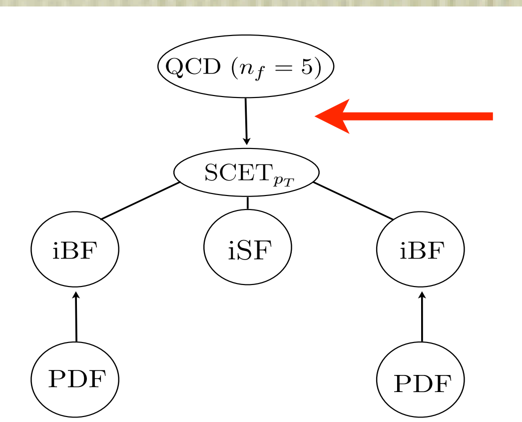
Factorization Formula

Impact parameter appears, but only to simplify iBF→PDF matching; can transform this formula to be completely in momentum space

$$\begin{aligned}\mathcal{G}^{ij}(x_1,x_1',x_2,x_2',p_T,Y,\mu_T) &= \frac{1}{2\pi}\int dt_n^+\int dt_{\bar{n}}^-\int d^2k_n^\perp\int d^2k_{\bar{n}}^\perp\int d^2k_{us}^\perp\frac{\delta(p_T-|\vec{k}_n^\perp+\vec{k}_{\bar{n}}^\perp+\vec{k}_{us}^\perp|)}{p_T}\\ &\times\mathcal{I}_{n;g,i}^{\beta\alpha}(\frac{x_1}{x_1'},t_n^+,k_n^\perp,\mu_T)\,\mathcal{I}_{\bar{n};g,j}^{\beta\alpha}(\frac{x_2}{x_2'},t_{\bar{n}}^-,k_{\bar{n}}^\perp,\mu_T)\\ &\times\mathcal{S}^{-1}(x_1Q-e^Y\sqrt{\mathrm{p}_T^2+m_h^2}-\frac{t_{\bar{n}}^-}{Q},x_2Q-e^{-Y}\sqrt{\mathrm{p}_T^2+m_h^2}-\frac{t_n^+}{Q},k_{us}^\perp,\mu_T)\end{aligned} \tag{54}$$

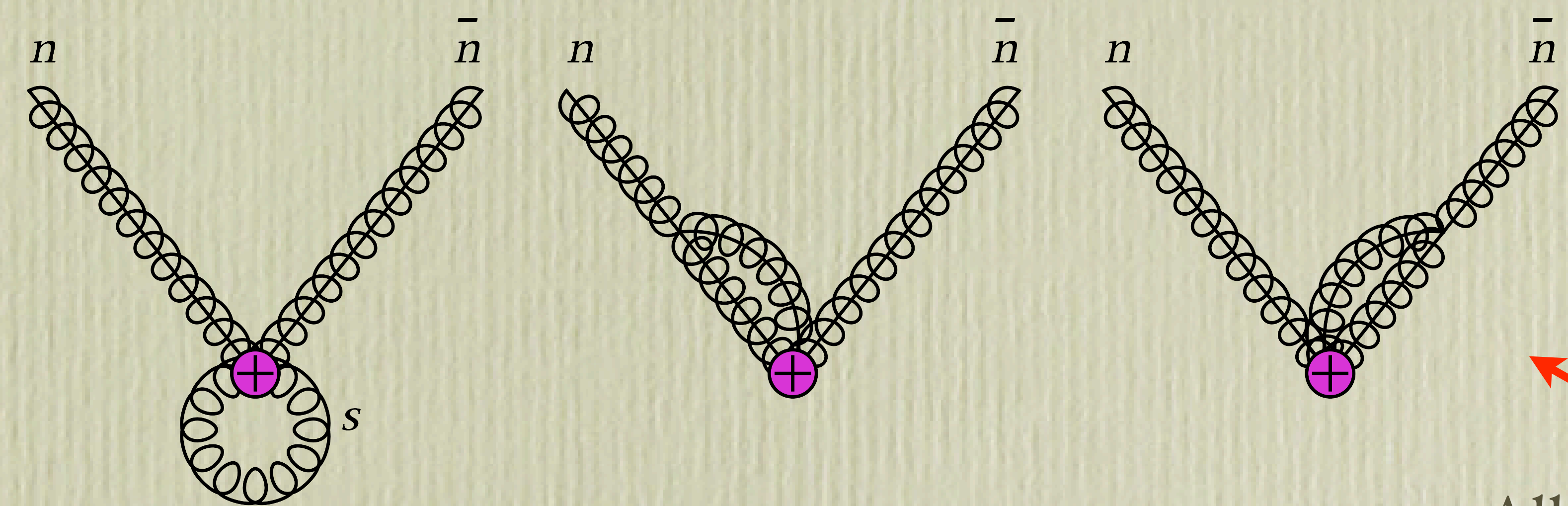
Fixed order and Matching Calculations

One loop Matching onto SCET



We are here

$$O_{QCD} = \int d\omega_1 \int d\omega_2 \, C(\omega_1, \omega_2) \, \mathcal{O}(\omega_1, \omega_2)$$



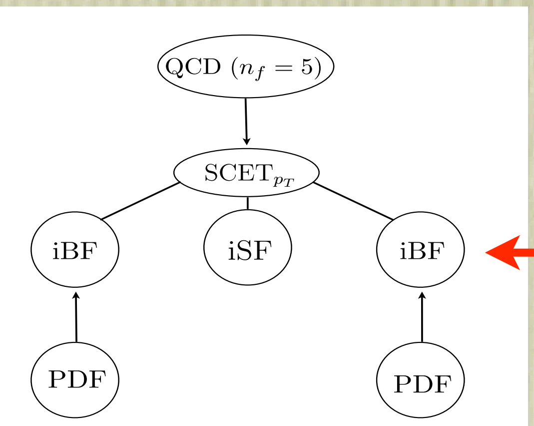
One loop SCET graphs

All graphs scaleless and vanish in dimensional regularization.

- Wilson Coefficient obtained from finite part in dimensional regularization of the QCD result for gg→h. At one loop we have:

$$C(\bar{n}\cdot\hat{p}_1 n\cdot\hat{p}_2, \mu) = \frac{c\,\bar{n}\cdot\hat{p}_1 n\cdot\hat{p}_2}{v}\left\{1 + \frac{\alpha_s}{4\pi}C_A\left[\frac{11}{2} + \frac{\pi^2}{6} - \ln^2\left(-\frac{\bar{n}\cdot\hat{p}_1 n\cdot\hat{p}_2}{\mu^2}\right)\right]\right\}$$

iBFs

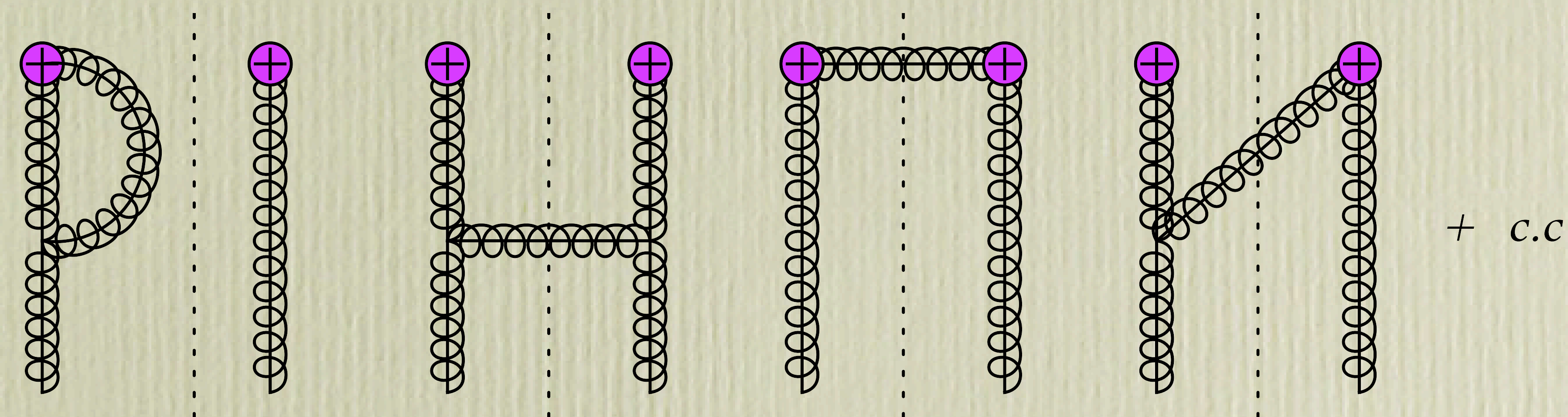


We are here

- Definition of the iBF:

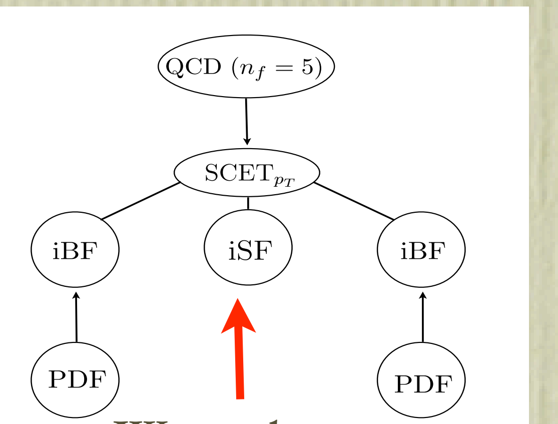
$$\tilde{B}_n^{\alpha\beta}(x_1, t_n^+, b_\perp, \mu) = \int \frac{db^-}{4\pi} e^{\frac{i}{2}\frac{t_n^+ b^-}{Q}} \sum_{\text{initial pols.}} \sum_{X_n} \langle p_1 | \left[g B^A_{1n\perp\beta}(b^-, b_\perp) | X_n \rangle \right.$$

$$\times \left. \langle X_n | \delta(\bar{\mathcal{P}} - x_1 \bar{n} \cdot p_1) g B^A_{1n\perp\alpha}(0) \right] | p_1 \rangle,$$



One loop graphs

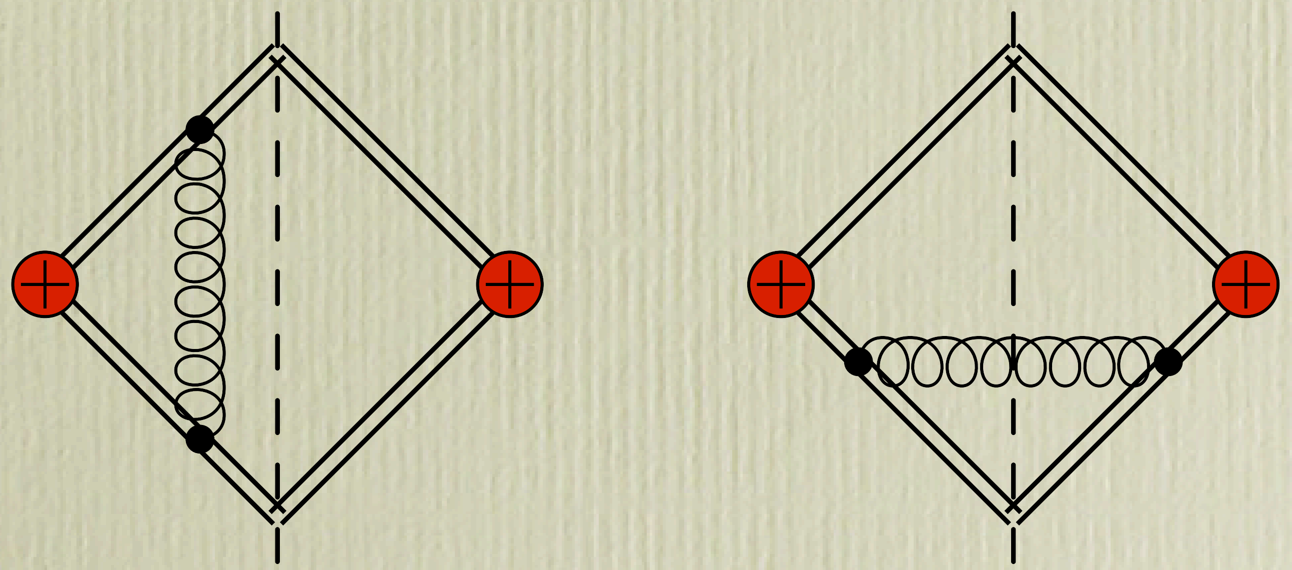
Soft function



We are here

- Soft function definition:

$$S(z) \;=\; \langle 0|\mathrm{Tr}\big(\bar{T}\{S_{\bar{n}}T^D S^\dagger_{\bar{n}} S_n T^C S^\dagger_n\}\big)(z)\mathrm{Tr}\big(T\{S_n T^C S^\dagger_n S_{\bar{n}} T^D S^\dagger_{\bar{n}}\}\big)(0)|0\rangle$$



One loop graphs

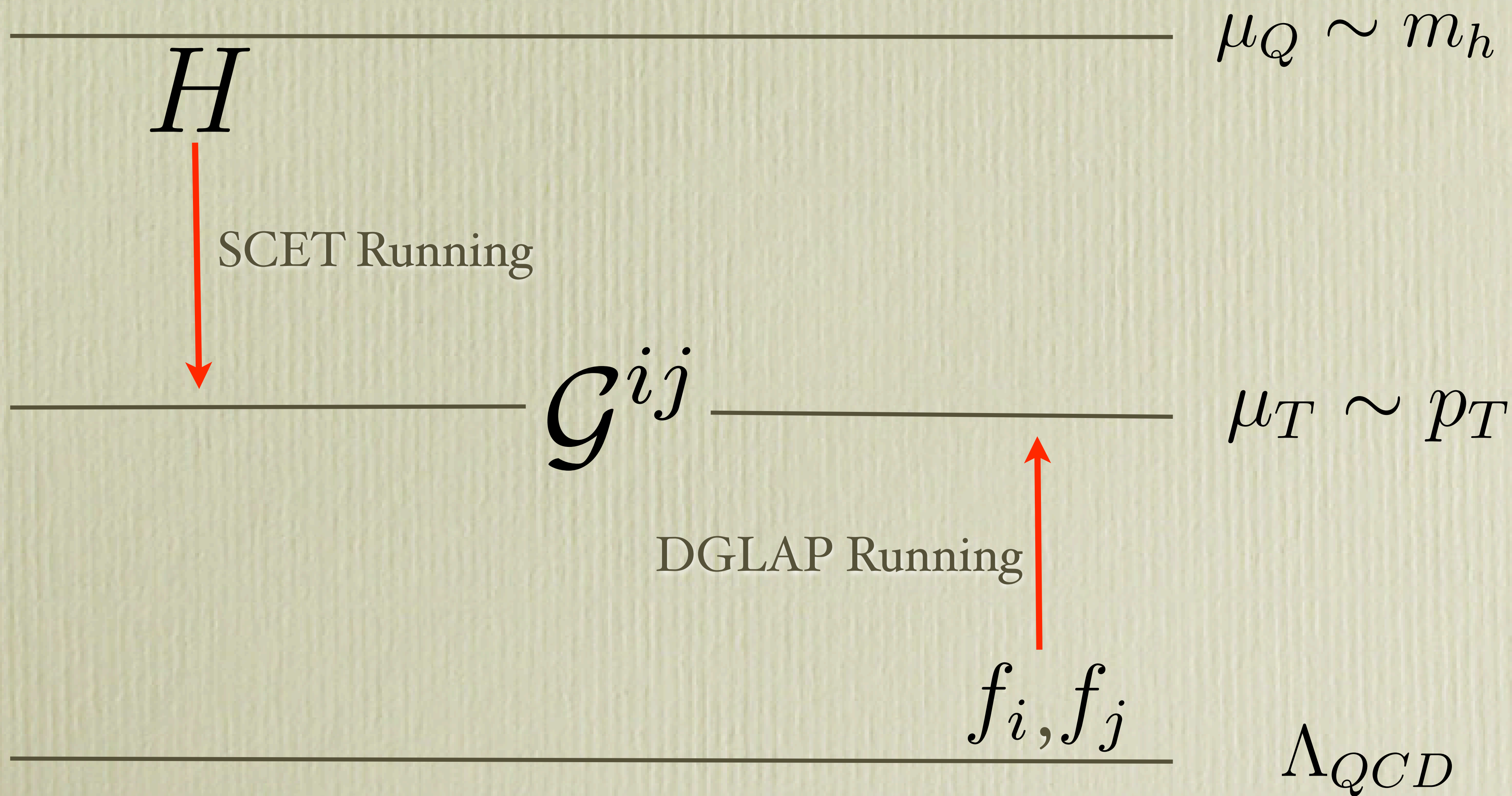
Running

Running

- Factorization formula:

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

- Schematic picture of running:



Running

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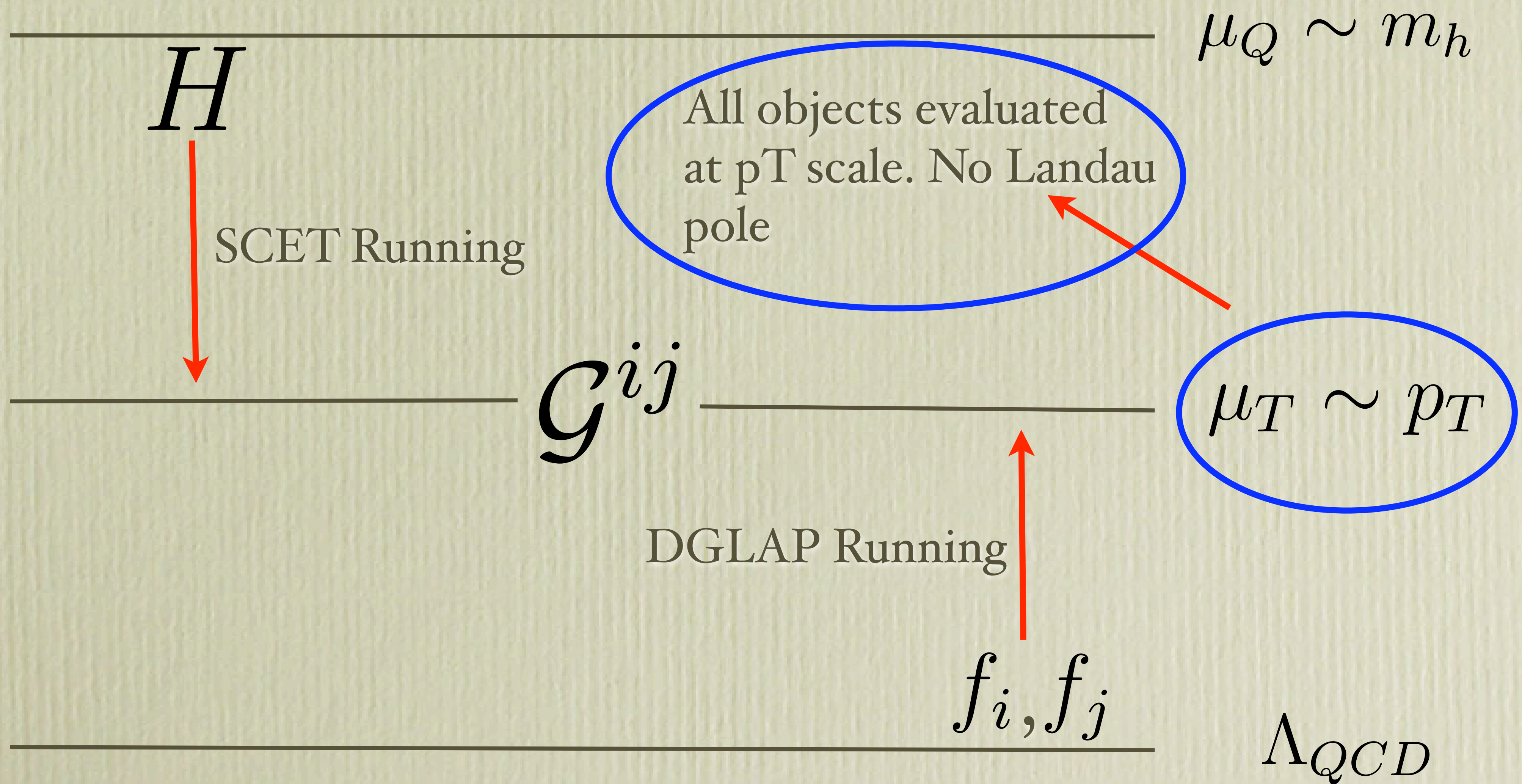

Limit of very small pT

- We derived a factorization formula in the limit:

$$m_h \gg p_T \gg \Lambda_{QCD}$$

- For smaller values of pT, one can introduce a non-perturbative model for the transverse momentum function:

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

Hard function.

Transverse momentum function.

PDFs.

Can make non-perturbative model at low p_T

Field theoretically defined object

Scale dependence and running known

Numerics

Preliminary plot

Conclusions

- Derived factorization formula for the Higgs transverse momentum distribution in an EFT approach:

$$\frac{d^2\sigma}{dp_T^2\,dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

- Resummation via RG equations in EFTs

- Formulation is free of Landau poles; easy matching to fixed-order

- Next steps: W/Z production at the Tevatron, higher-order calculations of iBF, iSF to enable NNLL+NLO result