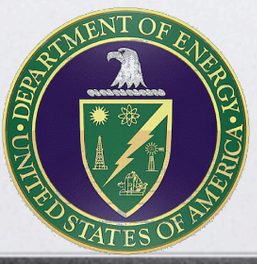
Three Years of Lattice Strong Dynamics:

Have we accomplished anything yet?

George T. Fleming (Yale U.)

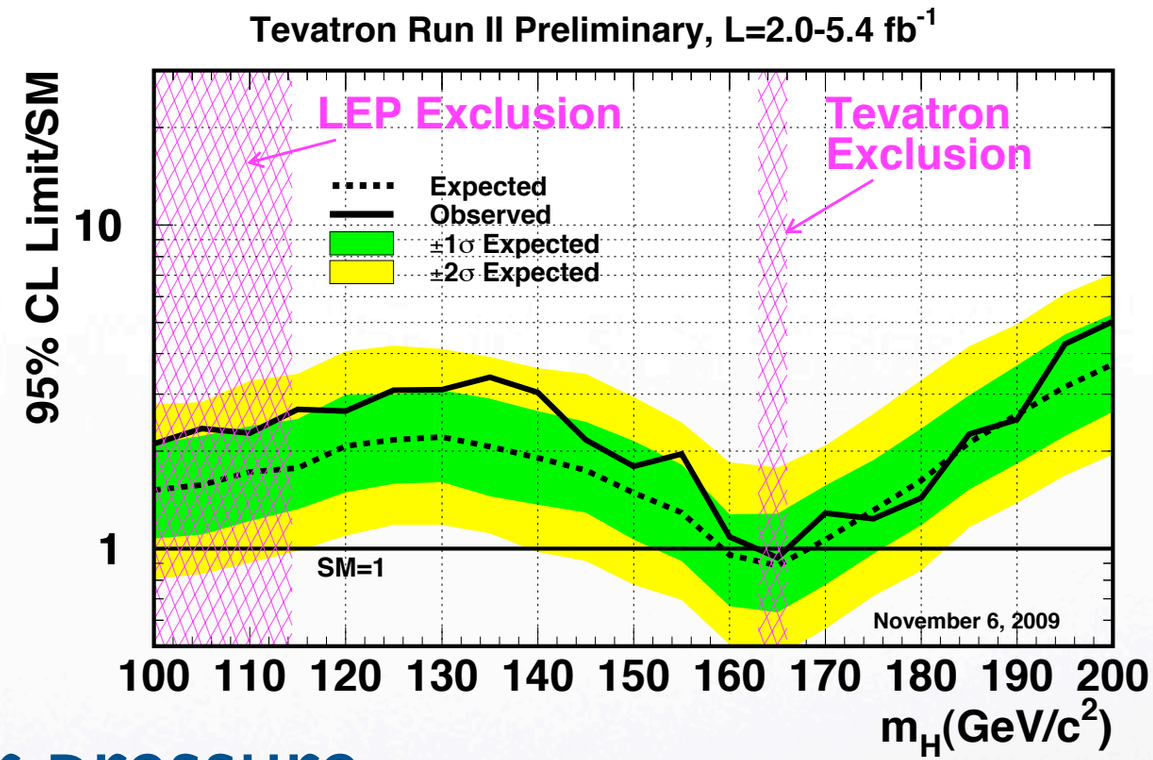
Fermilab Theory Seminar

13 May 2010



Waiting for (no?) Higgs boson...

- CDF: 7.0 fb^{-1}, D0: 7.6 fb^{-1}.
- By end of FY11, more than 10 fb^{-1} per experiment.
- In less than 1.5 years, SM Higgs boson could be excluded at 95% CL.



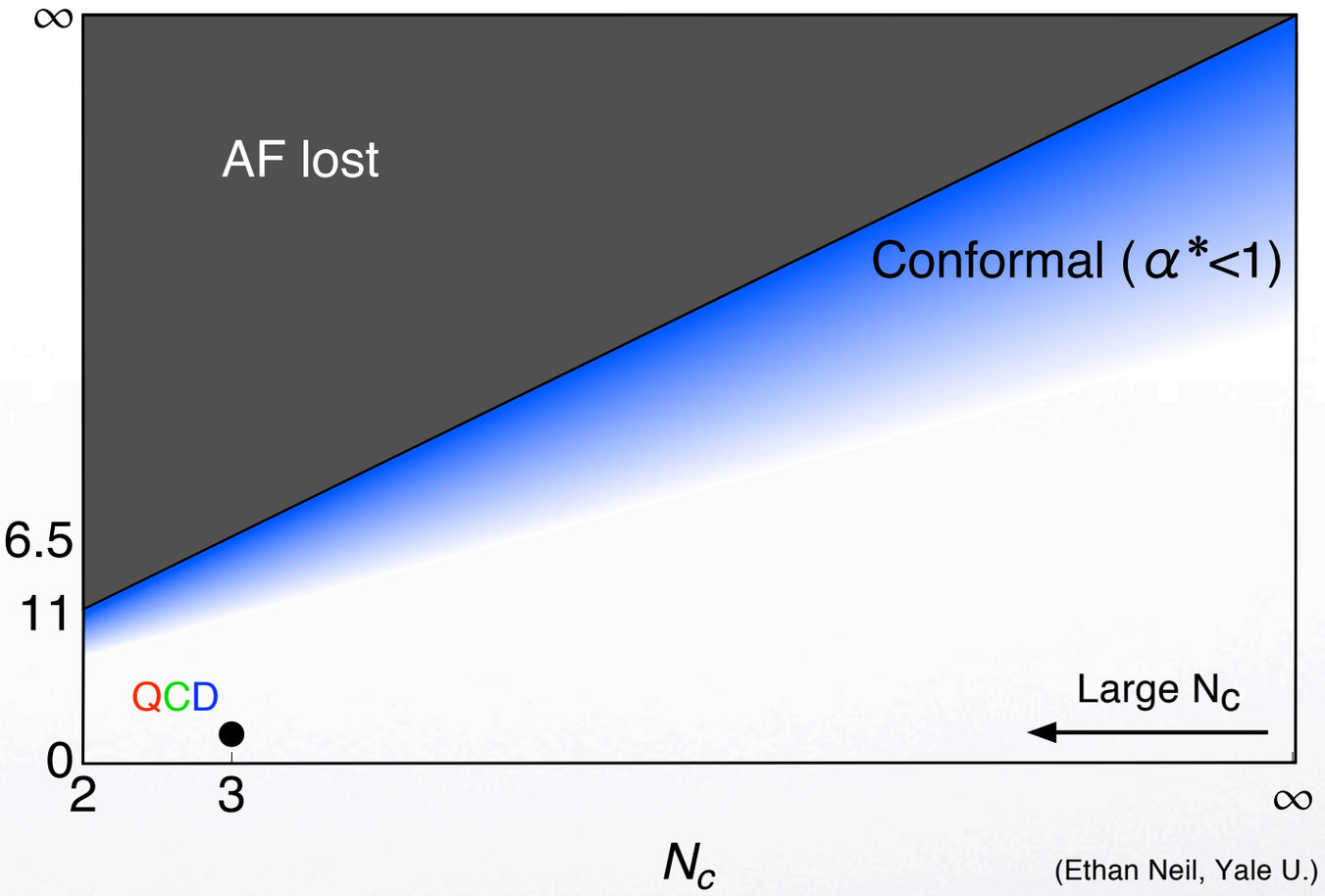
- Most MSSM models under similar pressure.
- Can strong interactions even produce light, narrow scalar resonances (pseudo-dilatons, ...)? Not seen in QCD or lattice.
- Overall, not finding a light SM Higgs boson strengthens the case for new strong interactions at TeV energies.

The Many Scales of the Standard Model

- The SM has many scales: quark and lepton masses, hadronic masses, Higgs VEV, ...
- How are these scales (ultimately) related to the Planck scale? Can they be generated dynamically?
- QCD naturally generates the hadronic scale far below M_{Pl}.
- Many DEWSB scenarios rely on QCD-like mechanism to generate another scale: the Higgs VeV.
- Can QCD-like mechanisms generate all the scales of the SM?
- Do SCGT's exist which can dynamically generate more than one scale? Are they common or rare?

How generic is QCD?

- For $N_f = 0–1$, confinement but no NG bosons.
- For $N_c = 2$, enhanced chiral symmetry means special case: Pattern of symmetry breaking yet to be determined.
- Pert. theory indicates IRFP for $N_f \lesssim 5.5 \cdot N_c$.



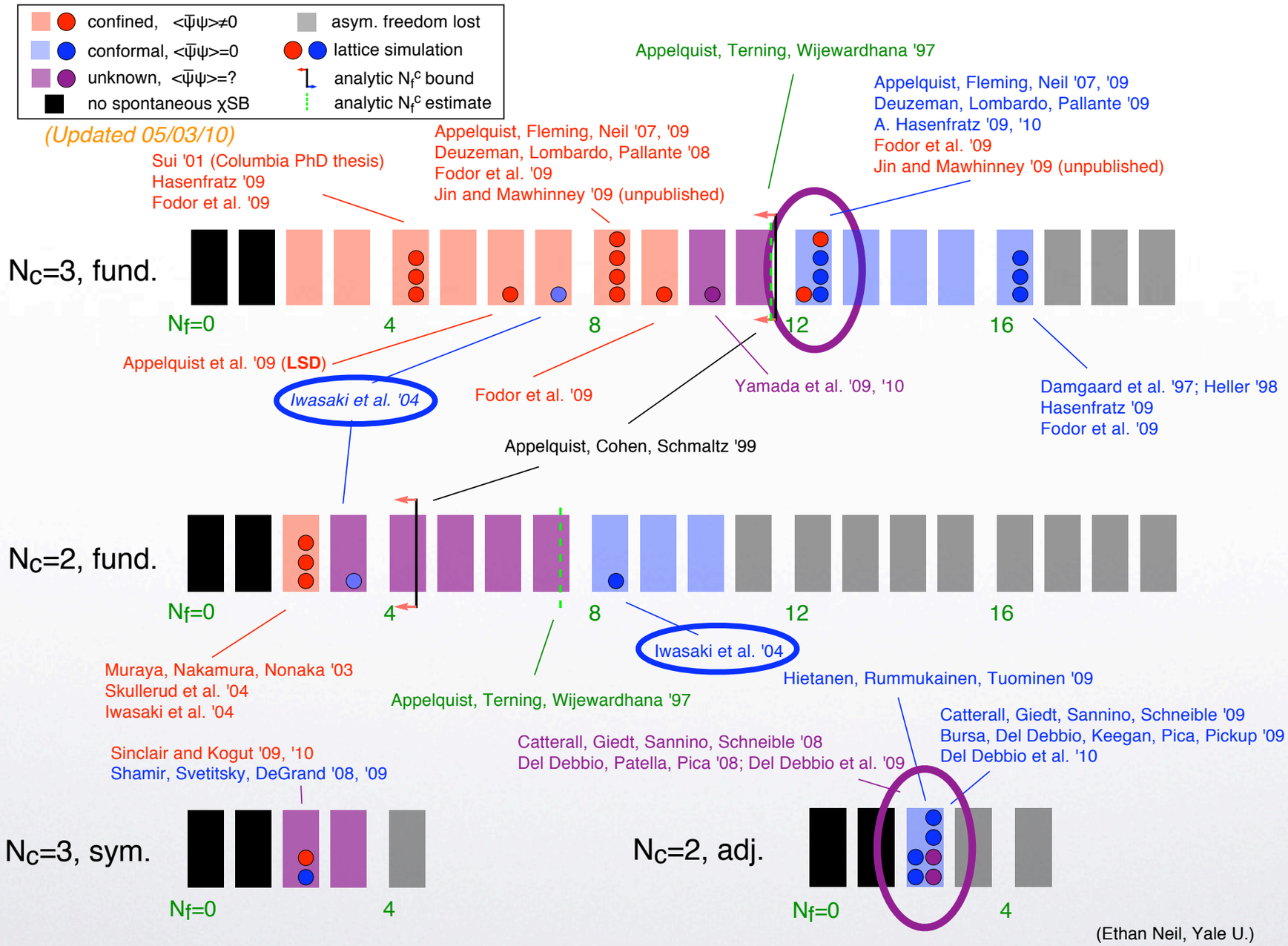
(Ethan Neil, Yale U.)

- Phenomenological success of large N_c calculations suggest QCD-like theories for N_f=2–3 and $N_c \geq 3$.
- Surprises may lurk in the larger space of little-known theories.

Tools of the Trade

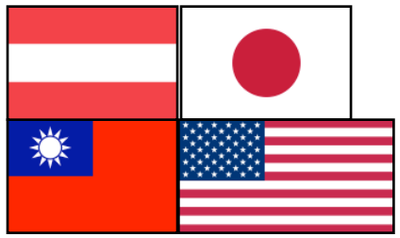
- Tools developed for study of Lattice QCD:
  - Non-perturbative Running Coupling
  - Non-perturbative Renormalization of Operators
  - Light Hadron and Glueball Spectrum
  - Chiral Observables (condensate, Dirac eigenvalues)
  - Thermodynamic Observables (T_c , EoS)
- Are tools optimized for QCD useful for non-QCD studies?
  - Exception: Monte Carlo methods using Wilsonian RG.
  - Can finite-size scaling methods be adapted from stat. mech.?

After Hiatus, Searching in Earnest



(Ethan Neil, Yale U.)

A Dozen Lattice BSM Efforts Worldwide



Aoyama et al.

DeGrand et al.

Del Debbio et al.

Deuzeman et al.

Catteral et al.

LSD

Hietanen et al.

A. Hasenfratz

LHC

Jin-Mawhinney

Yamada et al.

Kogut-Sinclair

Traveling Road Show

1. Lattice Gauge Theory for LHC Physics, Livermore, CA, May 2-3, 2008. http://www.yale.edu/LSD/workshop08/

2. XXVI International Symposium on Lattice Field Theory, College of William and Mary, Williamsburg, VA, July 14-19, 2008. http://conferences.jlab.org/lattice2008/

3. Workshop on Dynamical Electroweak Symmetry Breaking, University of Southern Denmark, Odense, Denmark, September 9-13, 2008. http://hep.sdu.dk/dewsb/

4. New frontiers in large N gauge theories, University of Washington, Seattle, WA, February 3-6, 2009. http://www.int.washington.edu/PROGRAMS/09-41w.html

5. Large N@Swansea, Swansea University, Swansea, Wales UK, July 7-10, 2009. http://www.ippp.dur.ac.uk/Workshops/09/largeN/

6. XXVII International Symposium on Lattice Field Theory, Peking University, Beijing, China, July 25-31, 2009. http://rchep.pku.edu.cn/workshop/lattice09/index.xml

7. Les Houches Summer School, Session XCIII: Modern perspectives in lattice QCD: Quantum field theory and high performance computing, August 3-28, 2009. http://giulio.tau.ac.il/~bqs/Houches2009/Houches0809.html

8. Universe in a Box: LHC, Cosmology and Lattice Field Theory, Leiden University, Leiden, The Netherlands, August 24-28, 2009. http://www.lorentzcenter.nl/lc/web/2009/366/info.php3?wsid=366

9. 2nd Workshop on Lattice Gauge Theory for LHC Physics, November 6-7, 2009, Boston University, Boston, MA. http://www.yale.edu/LSD/workshop/

10. Origin of Mass 2010, CP^3-Origins, Odense, Denmark, May 3-7, 2010. http://cp3-origins.dk/events/meetings/mass-2010

Lattice Strong Dynamics (LSD) Collaboration

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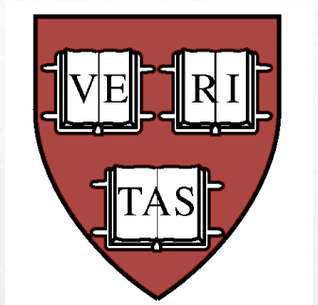

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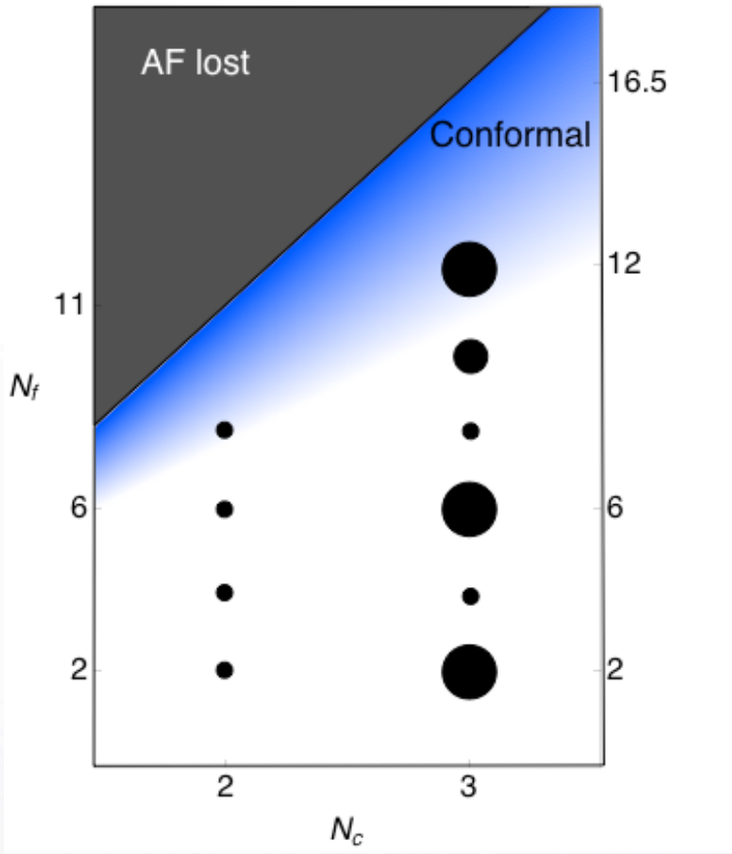
LSD Program Overview

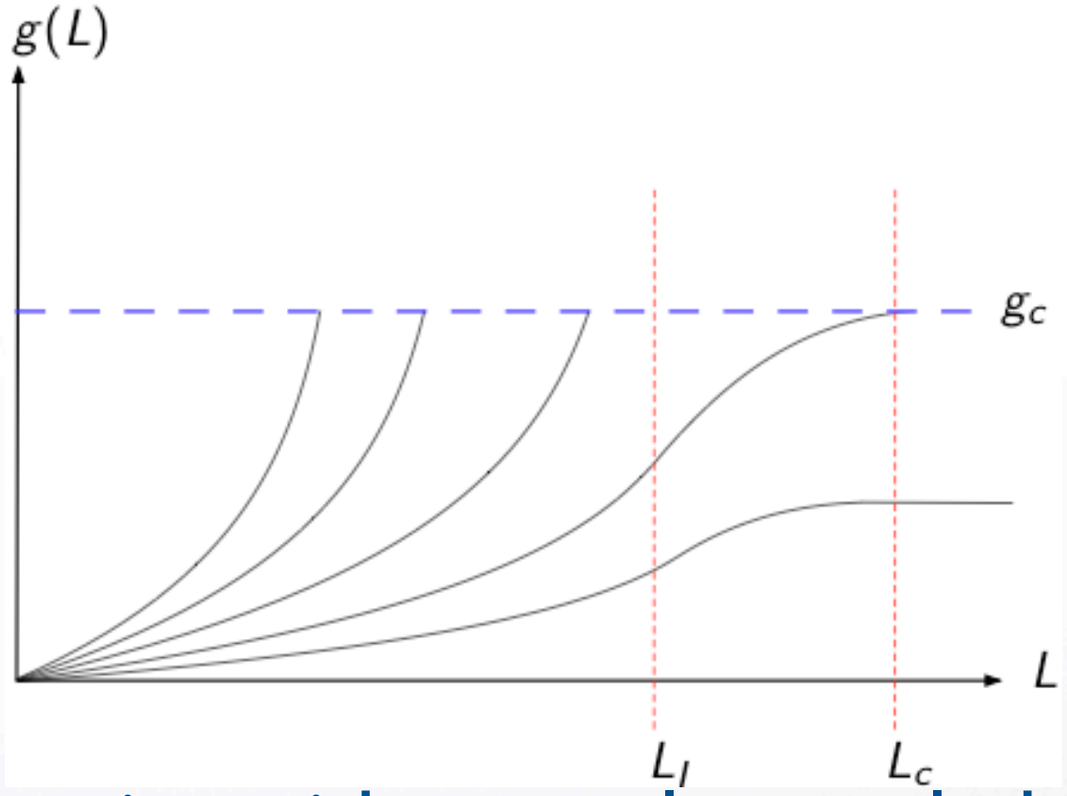
- SU(2) and SU(3) gauge theories with N_f domain wall fundamental fermions.
- Initial focus on SU(3): code readiness and QCD experience.
- Preparing SU(2) code for production.
- Majority of flops so far spent on SU(3) with N_f=2,6,10.



- Exploration of IR: QCD-like, conformal or "walking".
- Phenomenology: S parameter and condensate enhancement.
- Recent PRL with PRD in draft.

Can the running coupling be our guide?

- Typically in Lattice QCD, we must satisfy L^{-1}, $m \ll M_\pi \ll \Lambda_c \ll a^{-1}$ to have a reliable calculation. Surprisingly, this is possible for $L/a \sim 32$.
- $g(\Lambda_c) \sim g_c$ and $g(a^{-1})$ can be perturbative for $a \cdot \Lambda_c \geq 1/4$.



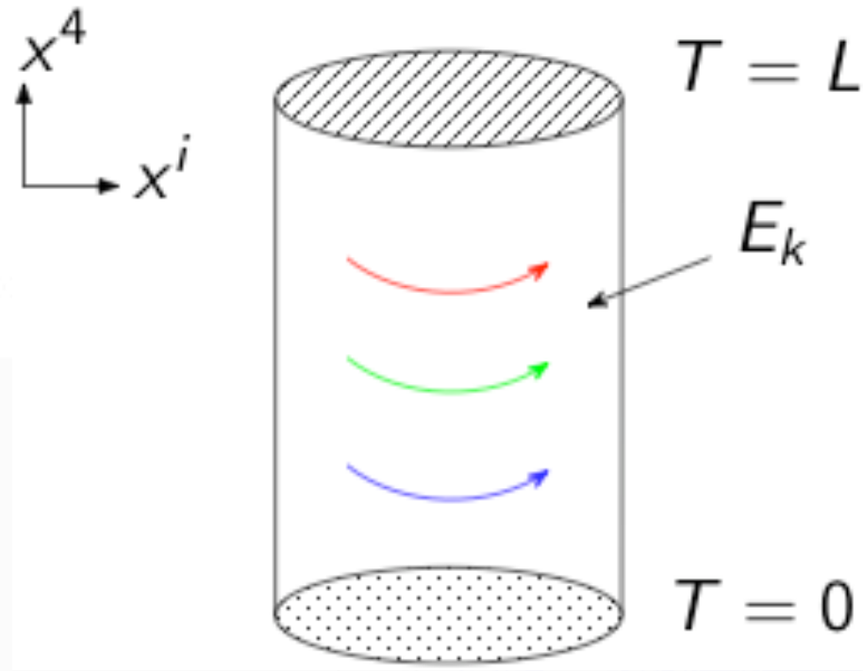
- For large N_f, $g(\mu)$ flows to g_* at IRFP.
- Walking theories may exist nearby theories with strongly-coupled IRFP: $g_* \lesssim g_c$. Walking theories should have two dynamically generated scales: the inflection point and confinement.
- **Caution**: Slower running/walking means $L/a \sim 32$ likely not a big enough box.

The Schrödinger Functional Scheme

- Dirichlet boundary conditions in Euclidean time.
- Non-trivial fixed gauge fields on boundary.
- Classical solution is constant chromo-electric background field.
- BC lift Dirac zero modes.
- SF running coupling inversely proportional to response of system to variation of strength of background field, controlled by η,

$$\frac{dS}{d\eta} = \frac{k}{\overline{g}^2(L)}\bigg|_{\eta=0}$$

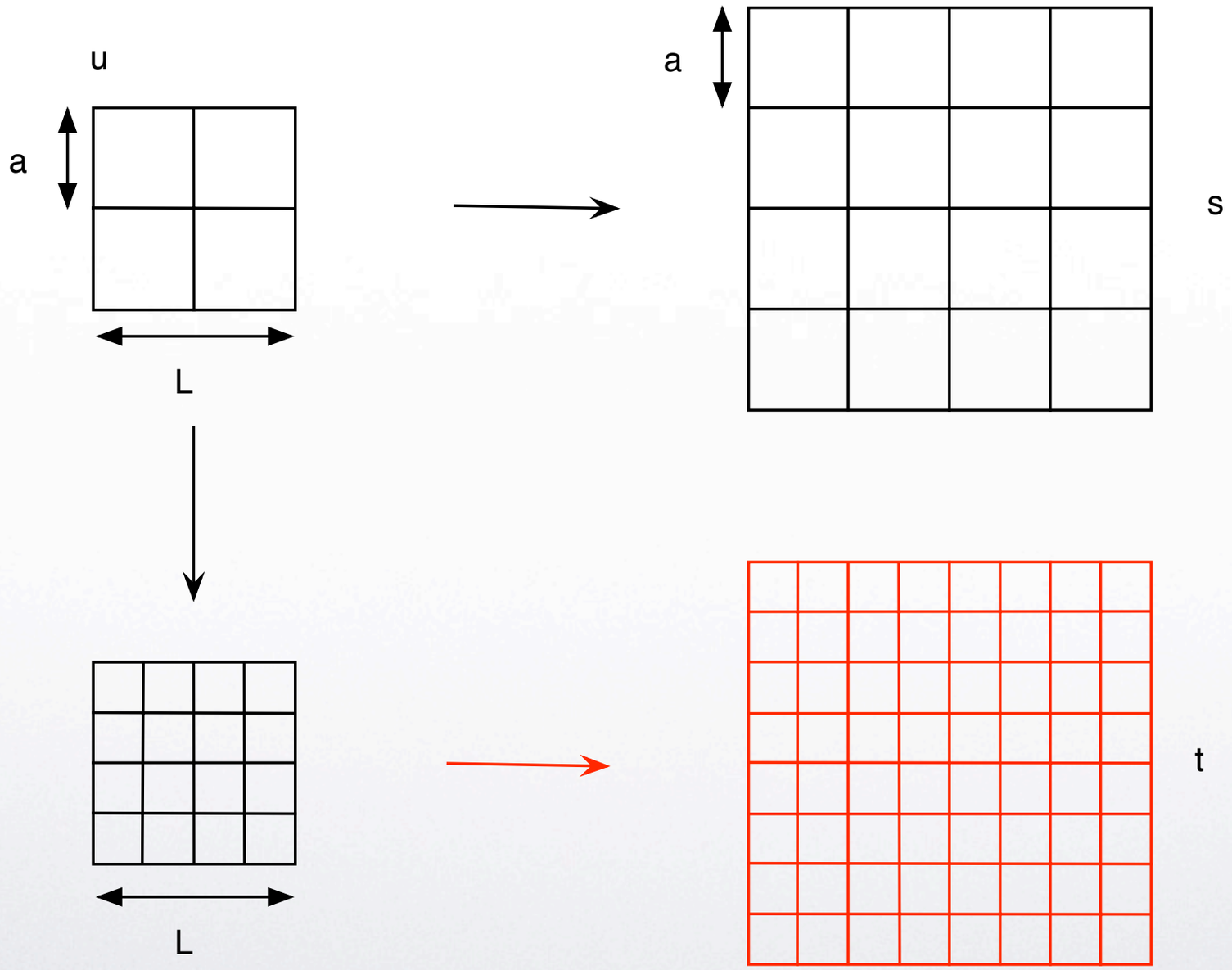
- Comprehensive review by R. Sommer, hep-lat/0611020



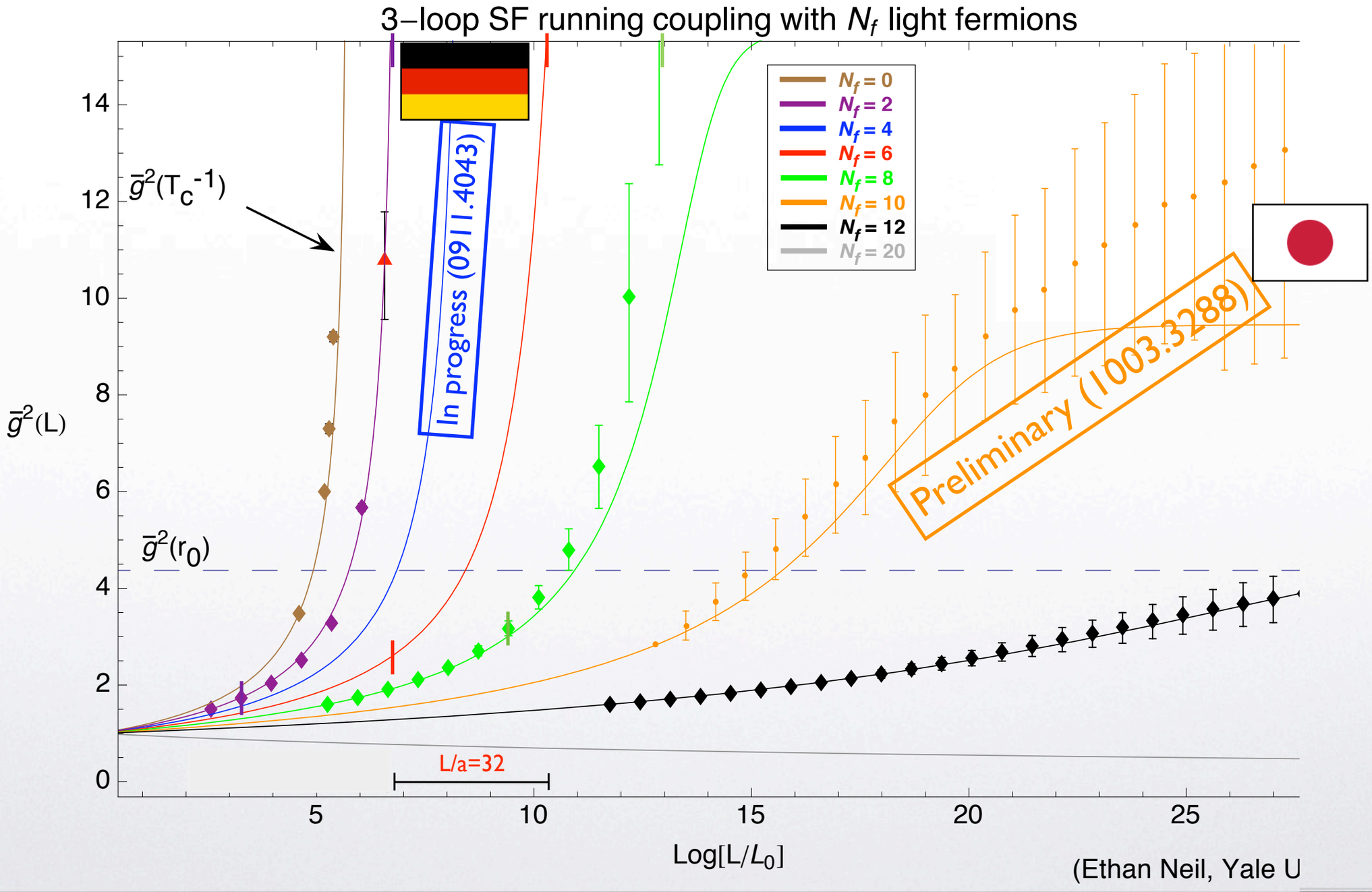
Continuum Running Coupling via Step Scaling

- $g^2(L)$, defined directly at scale L, avoids $L \to \infty$ extrapolation.
- Evolution of $g^2(L)$ *vs.* L determined in discrete steps $g^2(L_0) \to g^2(2L_0) \to \ldots$ relative to L_0 by ***step scaling***.
- Conformal invariance means $g^2(L) = g^2(2L)$ in continuum limit.
- Step scaling function: $\Sigma(2, g^2(L), a/L) \equiv g^2(2L) + O(a/L)$.
- The continuum limit $\sigma(2,u) \equiv \lim_{a \to 0} \Sigma(2, u, a/L)$ is discrete analog of continuum beta function.
- $g^2(L)$ will not be conformal at any finite lattice spacing due to $O(a/L)$ terms.

Visualizing Step Scaling



Non-perturbative SF running coupling



Running Couplings in Other NP Schemes

- Twisted gauge field on torus ('t Hooft 1979) lifts zero modes:

$$U_\mu(x+\hat{\nu}L) = \Omega_\nu U_\mu(x)\Omega_\nu^\dagger \ , \ \nu = 1,2 \ , \ \Omega_1\Omega_2 = e^{i2\pi/3}\Omega_2\Omega_1$$

$$\Omega_\mu\Omega_\mu^\dagger = 1 \ , \ (\Omega_\mu)^3 = 1 \ , \ \mathrm{Tr}(\Omega_\mu) = 0$$

- Twisted fermions have new "smell" dof, N_s=N_c: (Parisi 1983)

$$\psi(x+\hat{\nu}L+\hat{\rho}L) = \Omega_\rho\Omega_\nu\psi(x) \neq \Omega_\nu\Omega_\rho\psi(x)$$

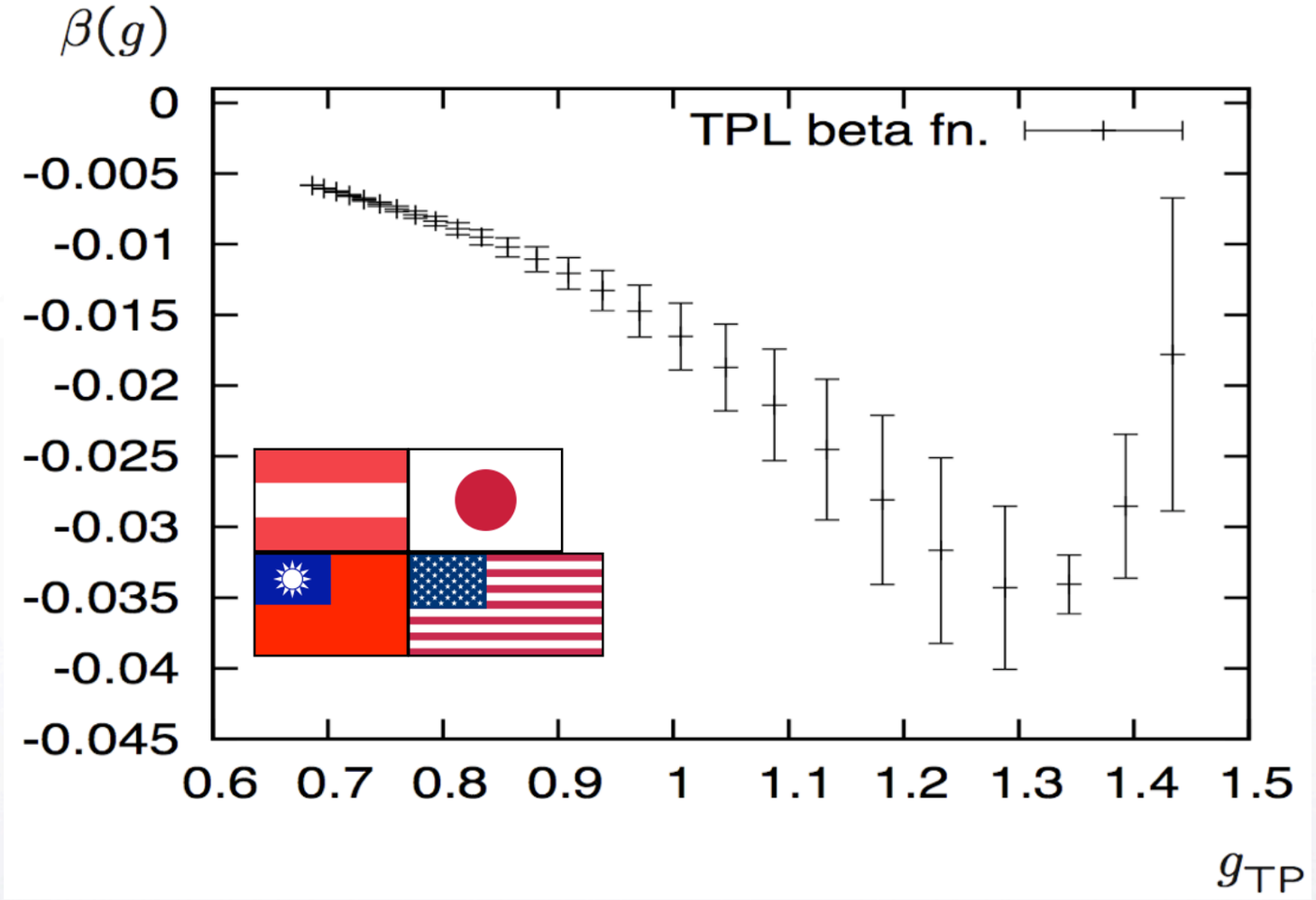
$$\Longrightarrow \quad \psi_\alpha^a(x+\hat{\nu}L) = e^{i\pi/3}\Omega_\nu^{ab}\psi_\beta^b(\Omega_\nu^\dagger)_{\beta\alpha}$$

- Twisted Polyakov Loop scheme (de Divitiis *et al.* 1994)

$$P_1(y,z,t) = \mathrm{Tr}\left\langle \Pi_j U_1(j,y,z,t)\Omega_1 e^{i2\pi y/3L}\right\rangle$$

$$g^2_{\mathrm{TP}}(L) \equiv \frac{1}{k}\frac{\left\langle \sum_{y,z} P_1(y,z,L/2)P_1^*(0,0,0)\right\rangle}{\left\langle \sum_{x,y} P_3(x,y,L/2)P_3^*(0,0,0)\right\rangle}$$

TPL Running coupling for SU(3), N_f=12

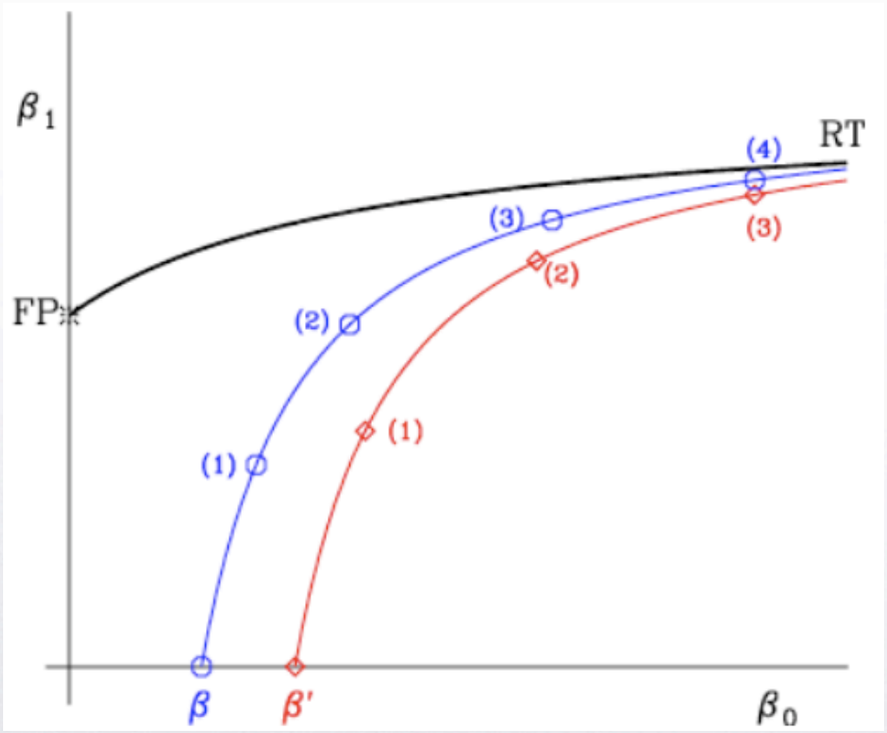


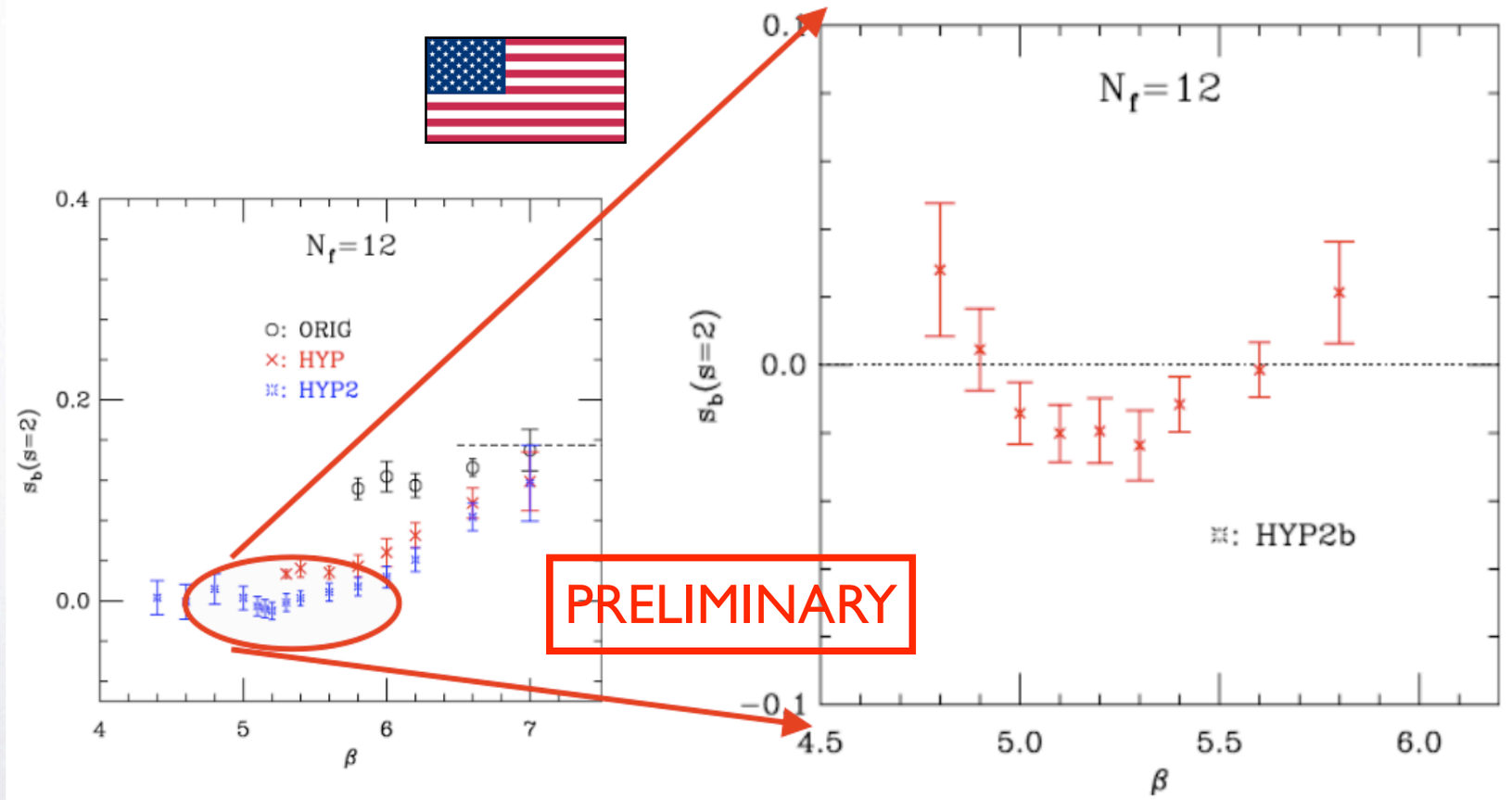
- One twisted staggered fermion: three "smells" times four "tastes"
- PRELIMINARY: Aoyama *et al.*, see also arXiv:0910.4196

Running Couplings in Other NP Schemes

- Monte Carlo Renormalization Group (MCRG) 2-Lattice Method
  Anna Hasenfratz

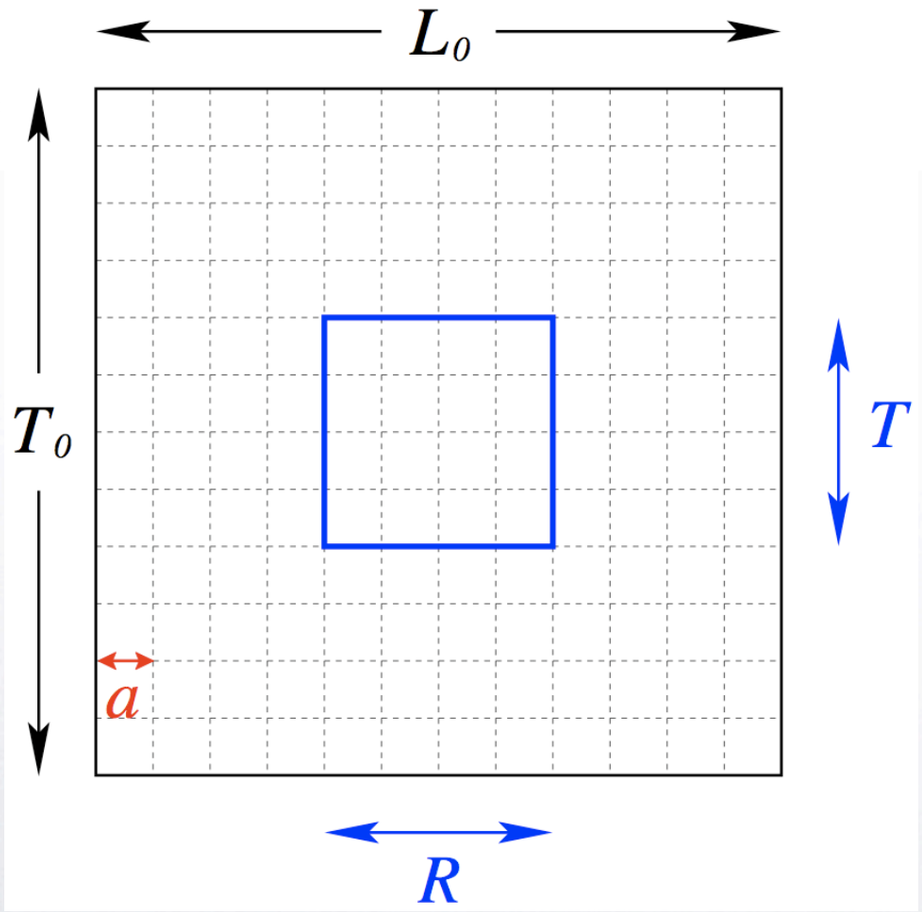
$$s_b(\beta) = \beta - \beta' \ , \ s_b(\beta^*) = 0$$

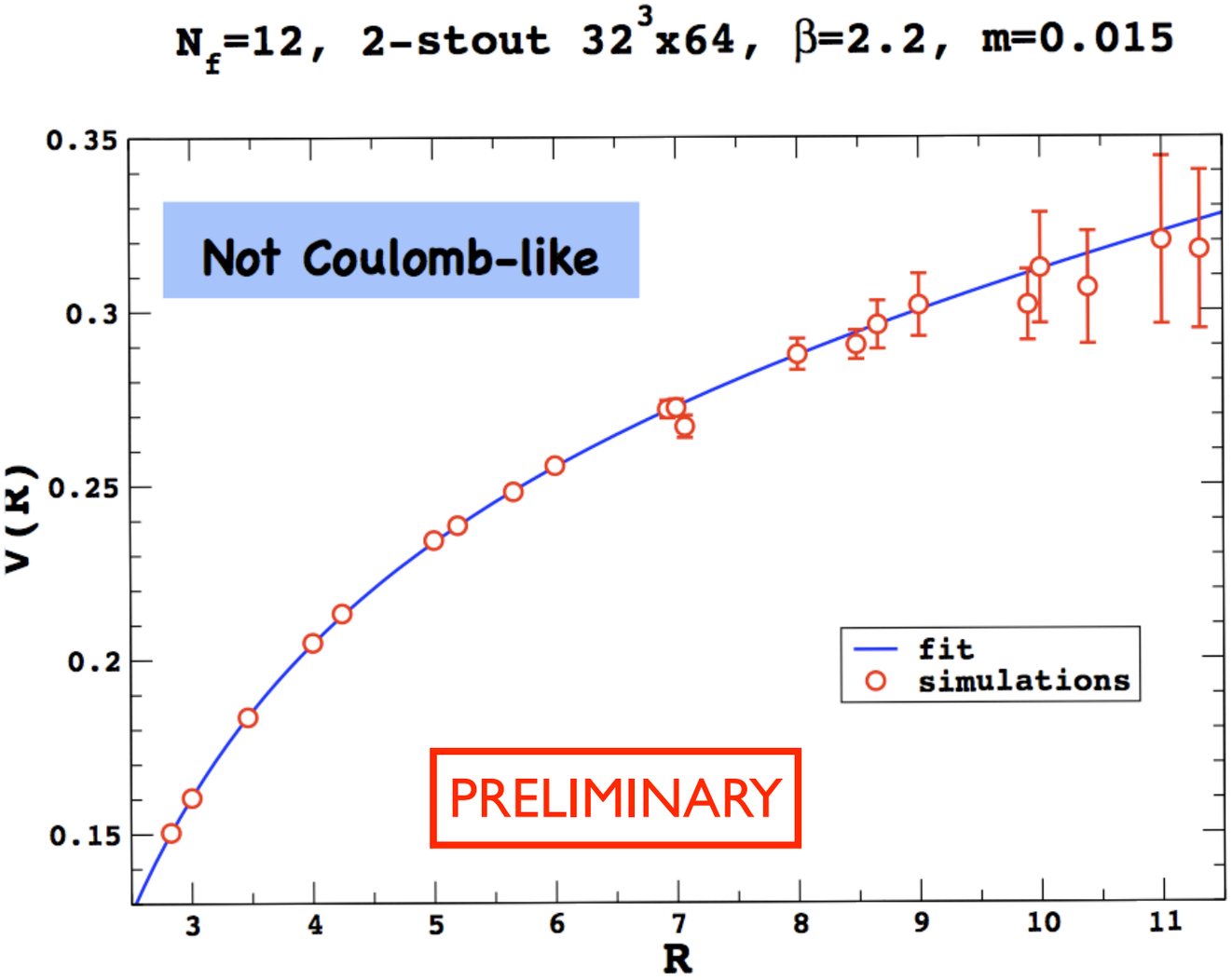




Running Couplings in Other NP Schemes

- Methods based on Wilson Loops (***LHC***: Fodor *et al.*)



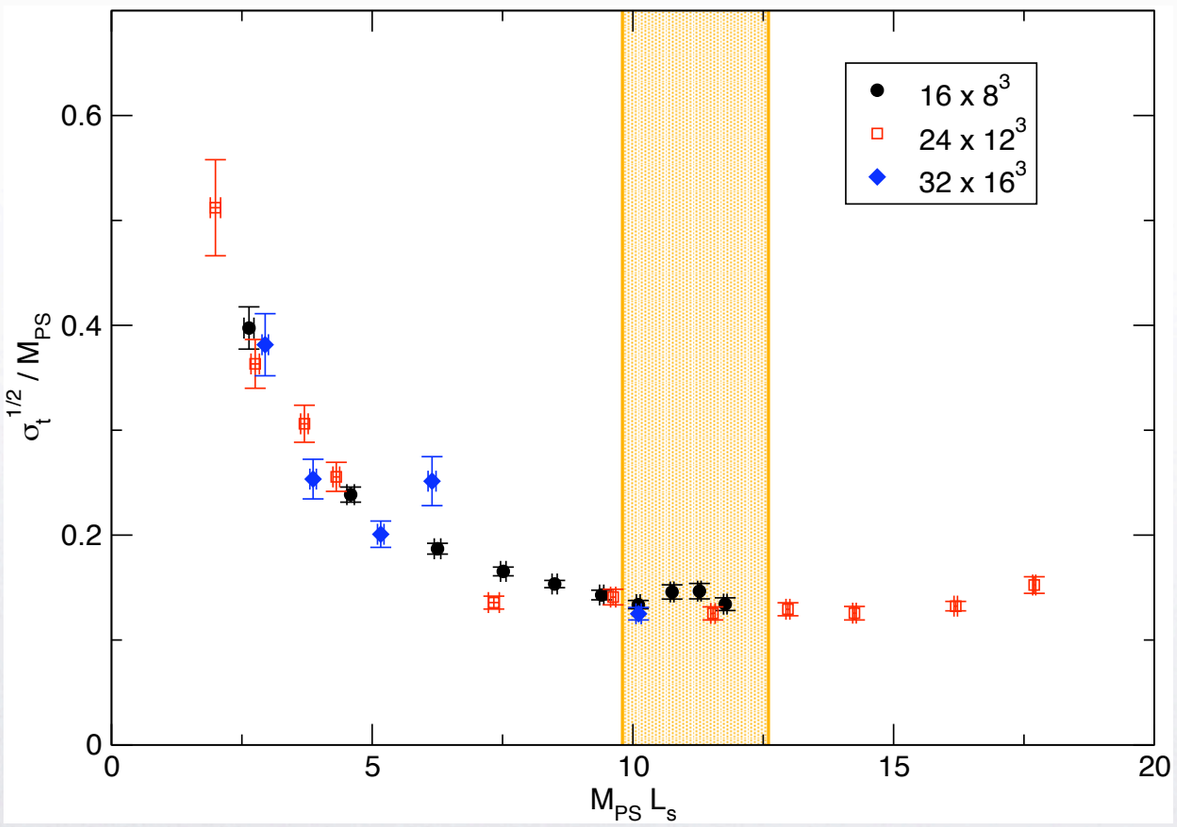


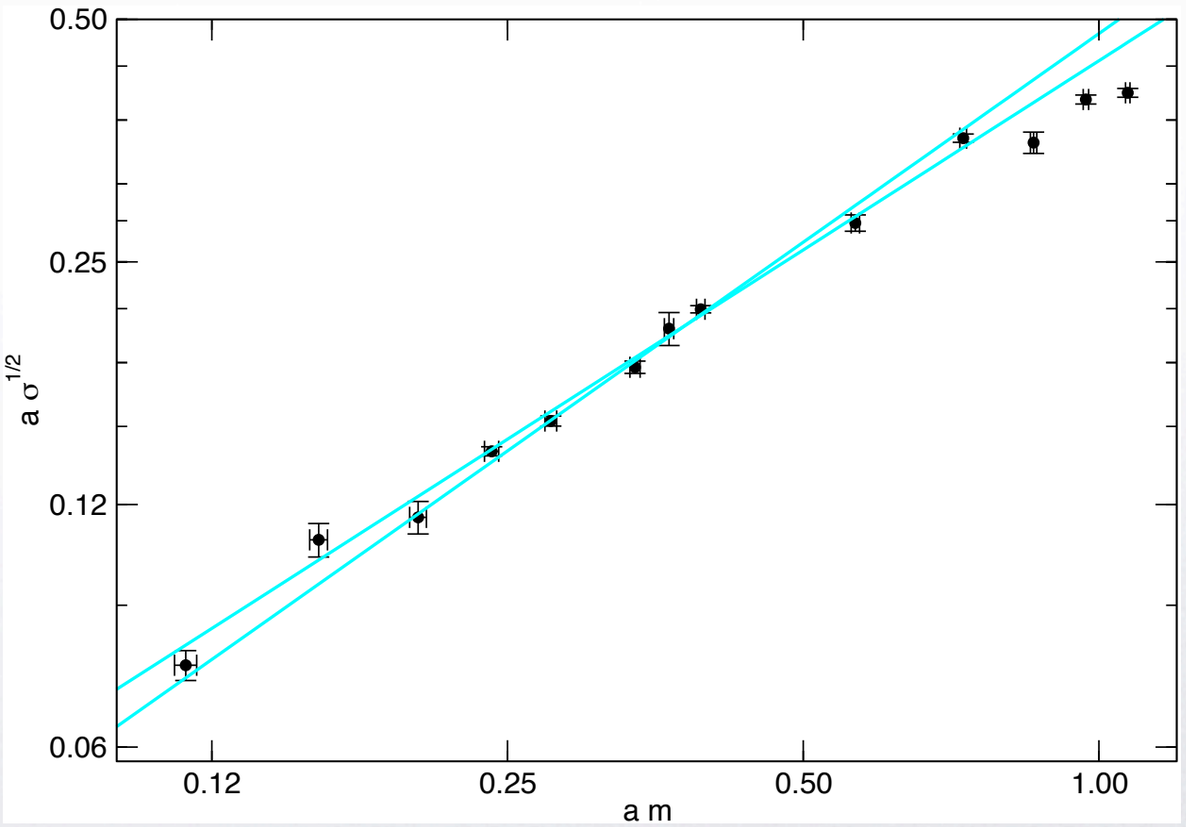
Anomalous Dimension from FSS

- Finite Size Scaling (FSS) can be used to extract the anom. dim. of the relevant mass if the scaling region is found.

$$M_X L = f(x) \ , \quad x = (L/a)(am)^{\rho} \ , \quad \rho = 1/(1+\gamma_*)$$

- In scaling region, $\gamma_* = 0.16 - 0.28$.





Flavor dependence of NLO ChiPT

$$M_\pi^2 = 2mB\left\{1+\frac{2mB}{(4\pi F)^2}\left[2\alpha_8-\alpha_5+N_f\left(2\alpha_6-\alpha_4\right)+\frac{1}{N_f}\log\frac{2mB}{(4\pi F)^2}\right]\right\}$$

$$F_\pi = F\left\{1+\frac{2mB}{(4\pi F)^2}\left[\frac{1}{2}\left(\alpha_5+N_f\alpha_4\right)-\frac{N_f}{2}\log\frac{2mB}{(4\pi F)^2}\right]\right\}$$

$$\langle\bar{q}q\rangle = F^2B\left\{1+\frac{2mB}{(4\pi F)^2}\left[\frac{1}{2}\left(2\alpha_8+\eta_2\right)+2N_f\alpha_6-\frac{N_f^2-1}{N_f}\log\frac{2mB}{(4\pi F)^2}\right]\right\}$$

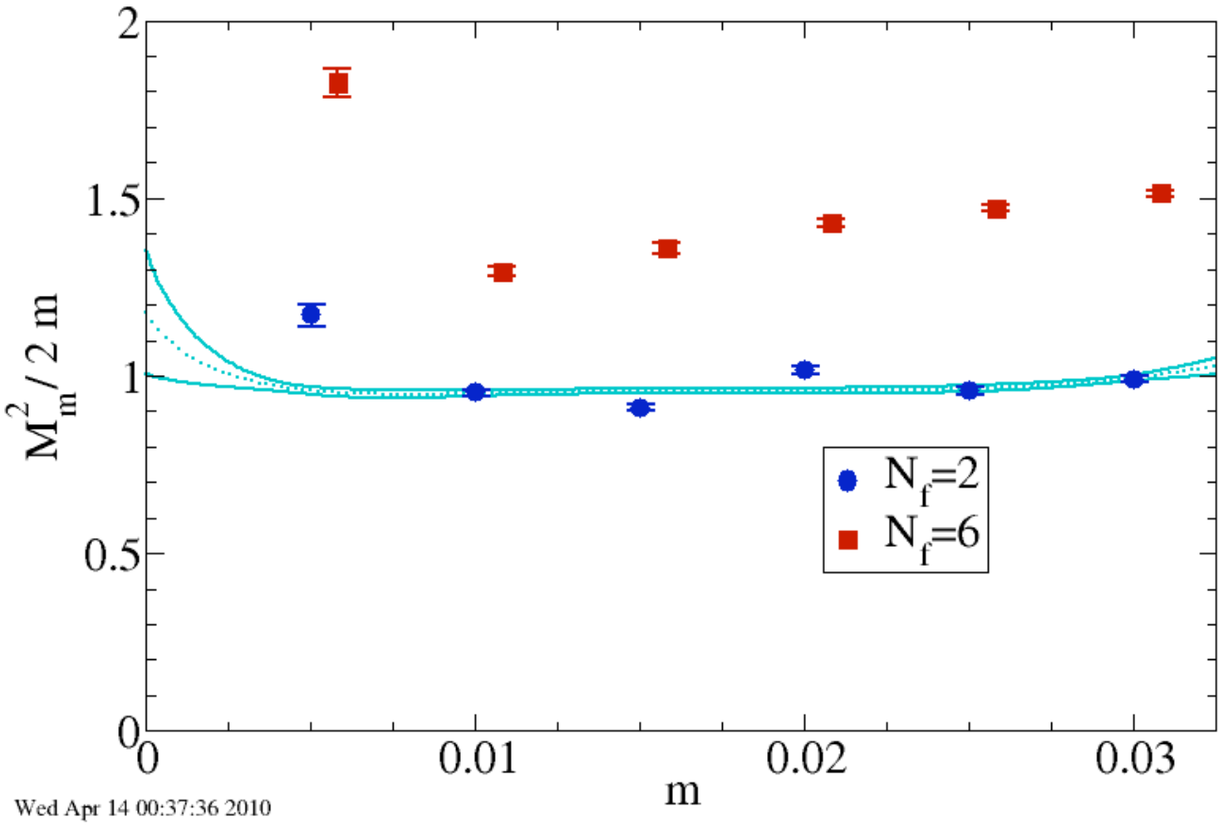
- The leading non-analytic terms are enhanced in the condensate and f_π but suppressed in $(M_\pi)^2$.
- The $\alpha_i \sim O(1)$ low energy constants.
- $\eta_2 \sim O(a^{-2})$ contact term: UV-sensitive slope for condensate.

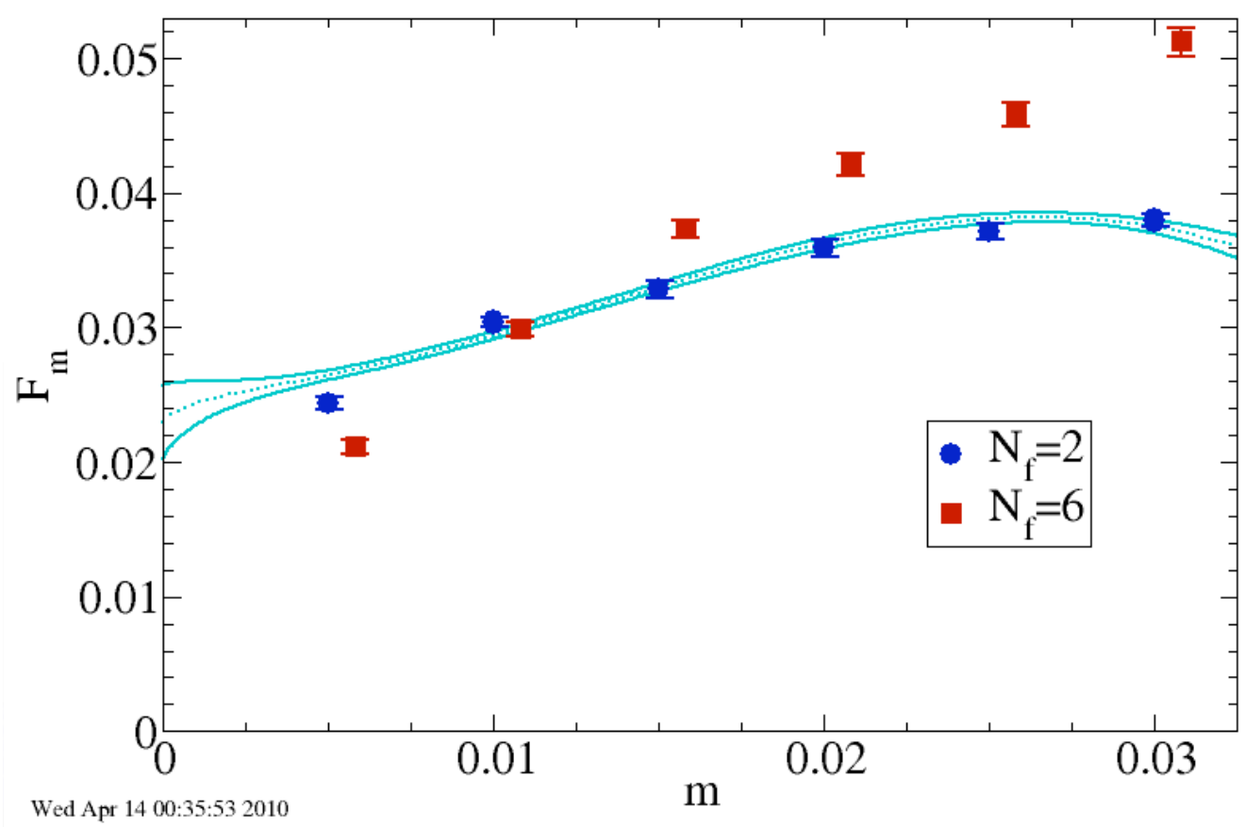
Non-analytic flavor factors in NNLO ChiPT

	$m \log(m)$	$m^2 \log^2(m)$
M_π^2	N_f^{-1}	$-3/8\ N_f^2 + 1/2 - 9/2\ N_f^{-2}$
F_π	$-1/2\ N_f$	$3/16\ N_f^2 + 1/2$
$\langle qq \rangle$	$-N_f + N_f^{-1}$	$3/2 - 3/2\ N_f^{-2}$

- J. Bijnens and J. Lu, JHEP11(2009)116 [arXiv:0910.5424]
- Small NLO coeff for M_π^2 is not generic and doesn't persist to higher orders.
- Can NNLO formulae help us extrapolate $N_f \gg 2$ results?

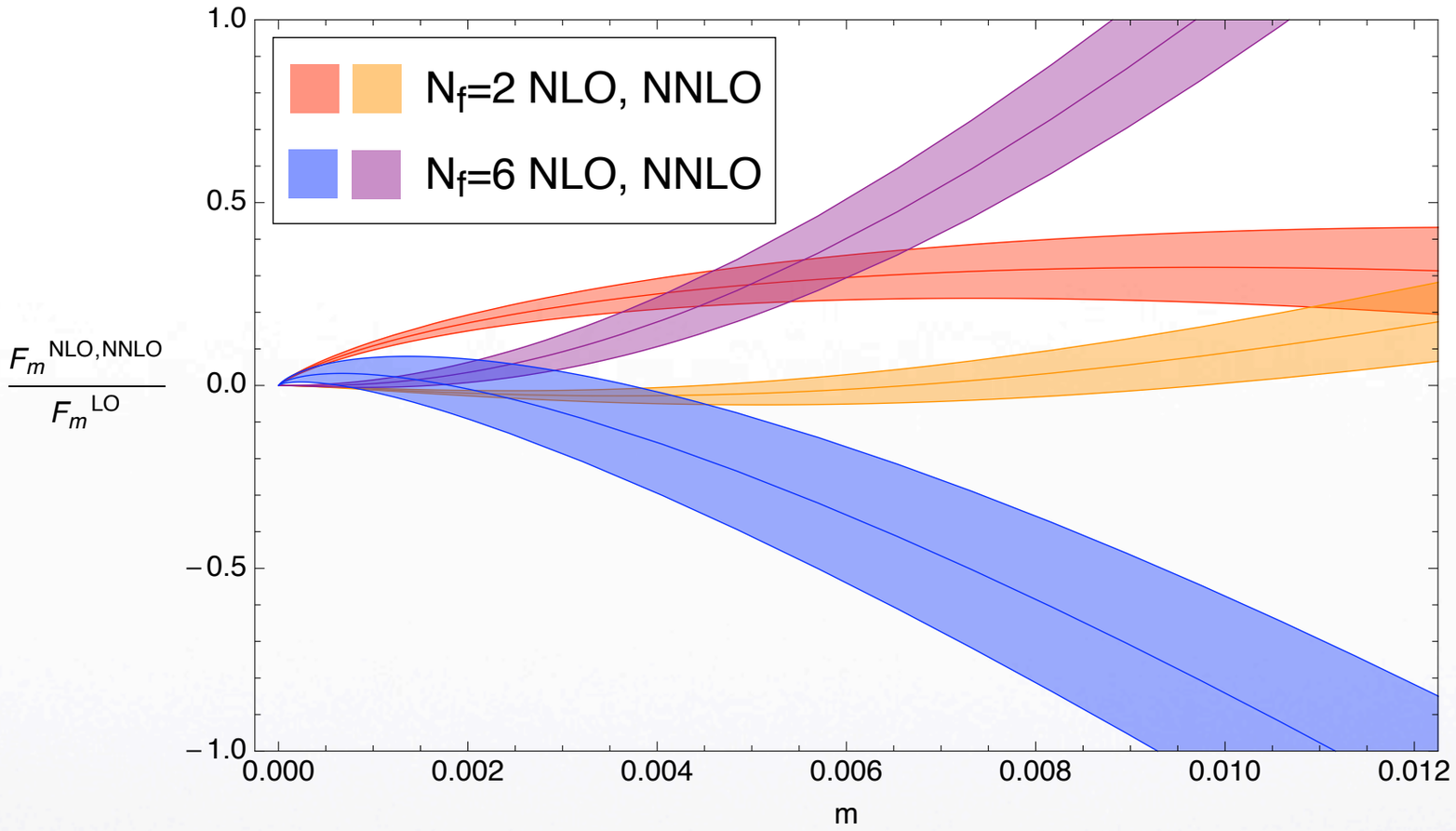
Preliminary: Basic Chiral Observables





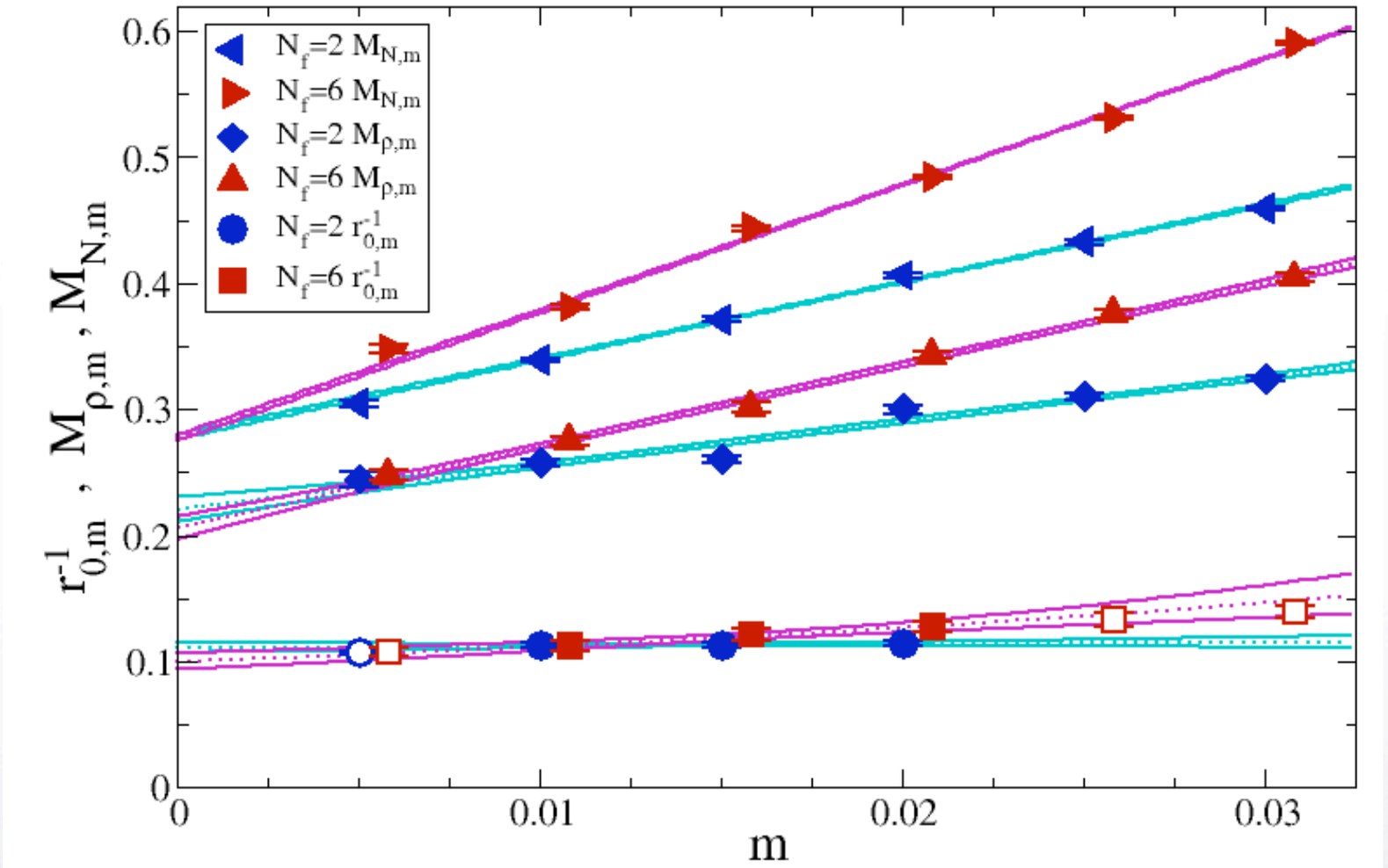
- NNLO ChiPT fits work fine for Nf=2.
- NNLO expression for general Nf recently derived by Bijnens and Lu [JHEP11(2009)116].

Preliminary: χPT Radius of Convergence



- Smaller quark masses needed for reliable NNLO extrapolation for $N_f>2$ [E.T. Neil *et al.*, PoS(CD09)088].
- On $32^3\times64$, $m\cong0.01$: $M_\pi \cdot L\sim4$ and $F_\pi \cdot L\sim1$. $48^3\times64$ lattices needed to reach smaller quark masses [requires LC resources].

Preliminary: LSD Nf=2 and 6 scale setting



- Lattice scale from M_N, M_ρ, r_0 all matched at 10% level with more masses and increased statistics.

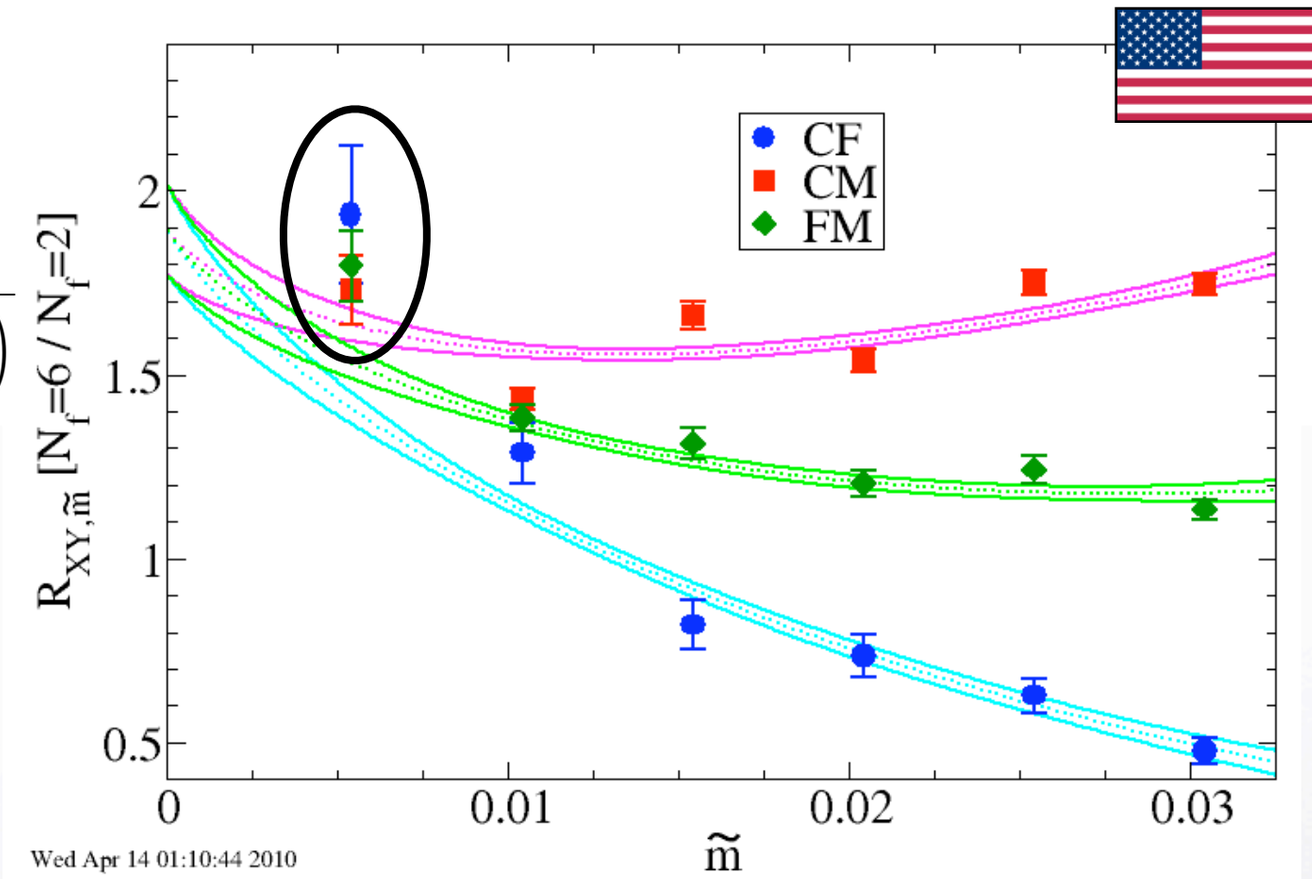
LSD Preliminary: Condensate Enhancement

- Definition of Enhancement

$$\left.\frac{\langle\overline{\psi}\psi\rangle^{(N_f)}}{\langle\overline{\psi}\psi\rangle^{(2)}}\right|_{5M_\rho} \equiv \mathcal{R}(5M_\rho) \approx \frac{\exp\left(\int_{\alpha(5M_\rho)}^{\alpha(M_\rho)} \left.\frac{\gamma(\alpha)}{\pi\beta(\alpha)}\right|_{N_f} d\alpha\right)}{\exp\left(\int_{\alpha(5M_\rho)}^{\alpha(M_\rho)} \left.\frac{\gamma(\alpha)}{\pi\beta(\alpha)}\right|_{N_f=2} d\alpha\right)}$$

- GMOR Ratios

$$R = \overbrace{\frac{\langle\overline{\psi}\psi\rangle}{F_\pi^3}}^{\text{CF}} = \overbrace{\frac{M_\pi^3}{\sqrt{(2m)^3\langle\overline{\psi}\psi\rangle}}}^{\text{CM}} = \overbrace{\frac{M_\pi^2}{2mF_\pi}}^{\text{FM}} \quad \text{as} \quad m \to 0$$



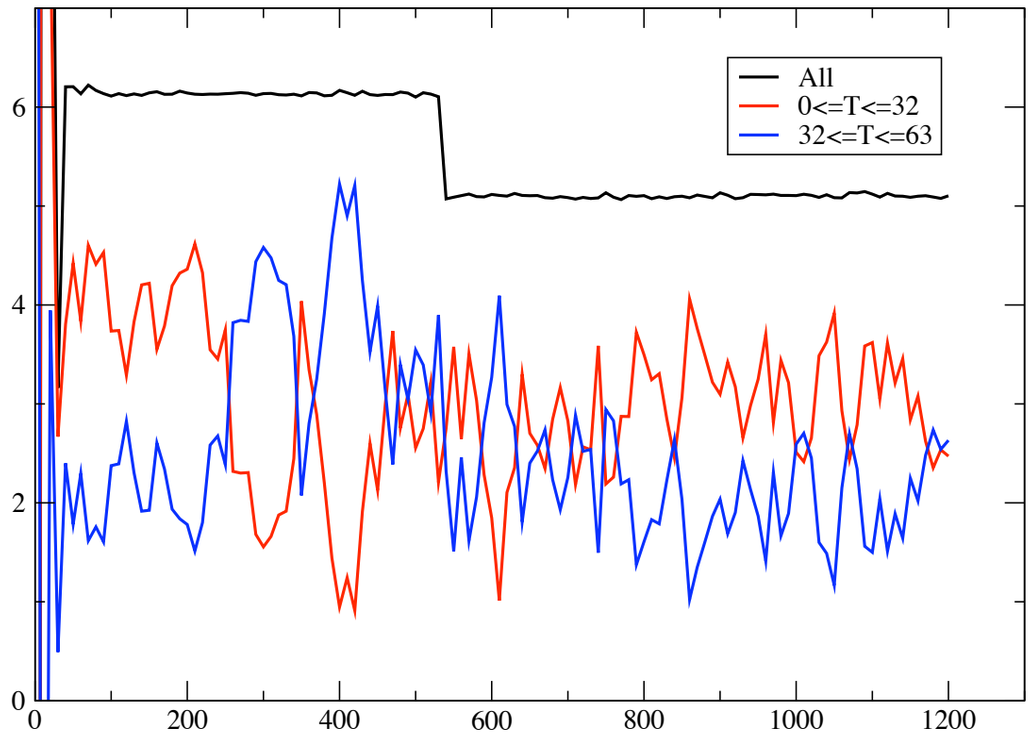
- Chiral extrapolation

$$\mathcal{R}_{XY,\widetilde{m}} = \frac{R^{(N_f)}}{R^{(2)}}\left[1 + \widetilde{m}\left(\alpha_{XY10} + \alpha_{11}\log\widetilde{m}\right)\right] \ , \quad \widetilde{m} = \sqrt{m^{(N_f)}m^{(2)}}$$

- Perturbative estimates of enhancement $\mathcal{R}(5M_\rho) \sim 1.2$–$1.3$ (lat scheme)
- NLO extrapolation of ratio reliable (?) due to cancellations.

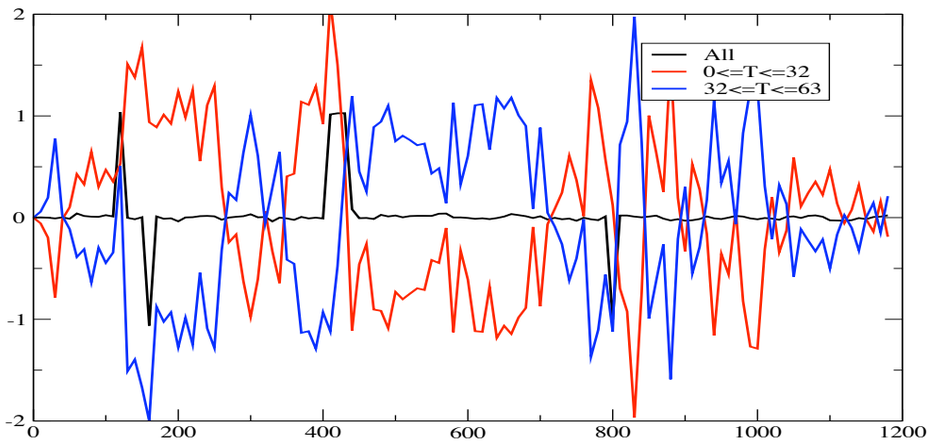
Evolution of Q_{top} for N_f=6, m_f=0.005

6 flavor, beta = 2.10, mf=0.05, Disordered start, topological charge

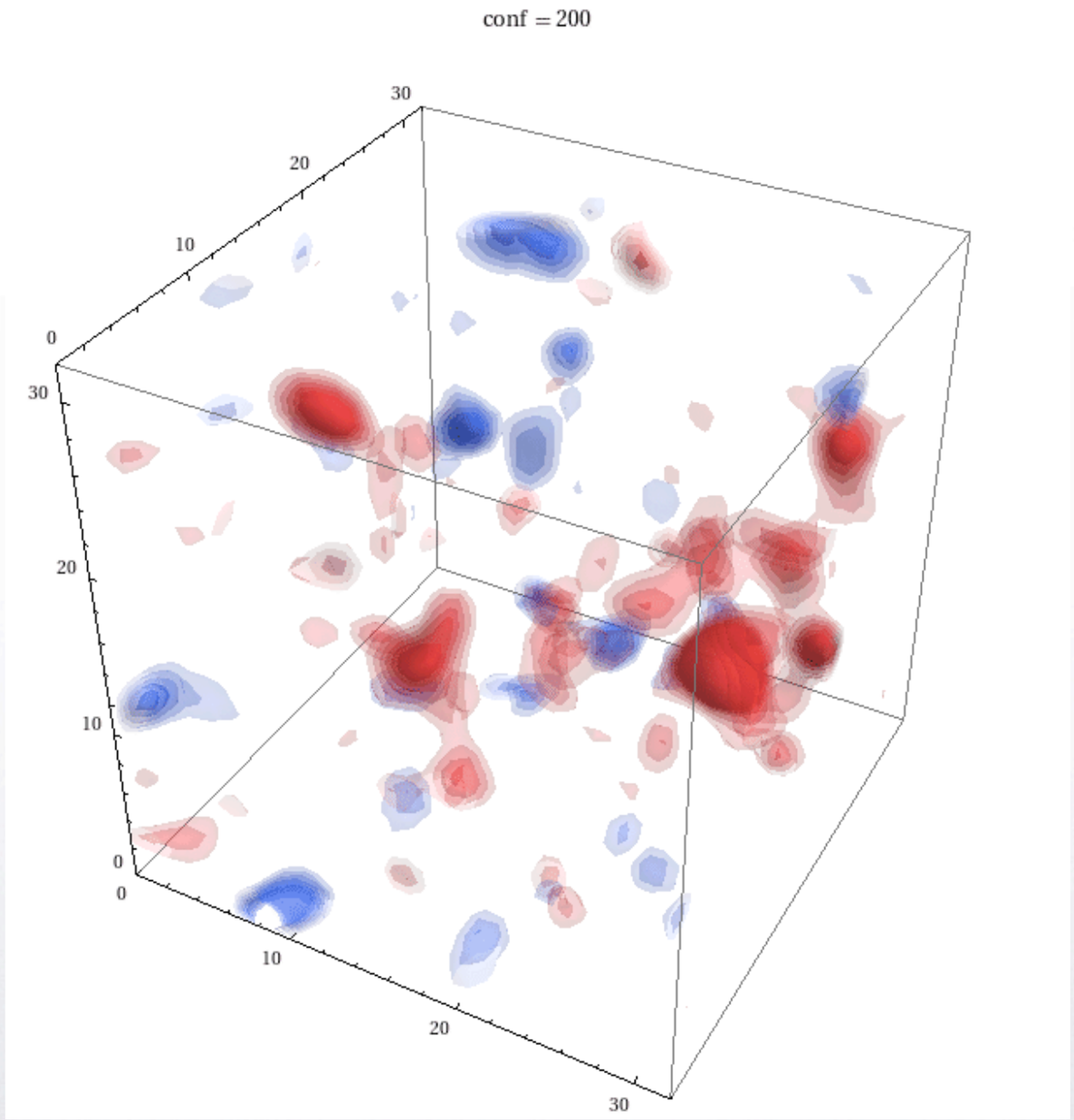


- Non-normal distribution for Q_{top} expected to enhance finite volume effects.

6 flavor, beta = 2.10, mf=0.05, Ordered start, topological charge

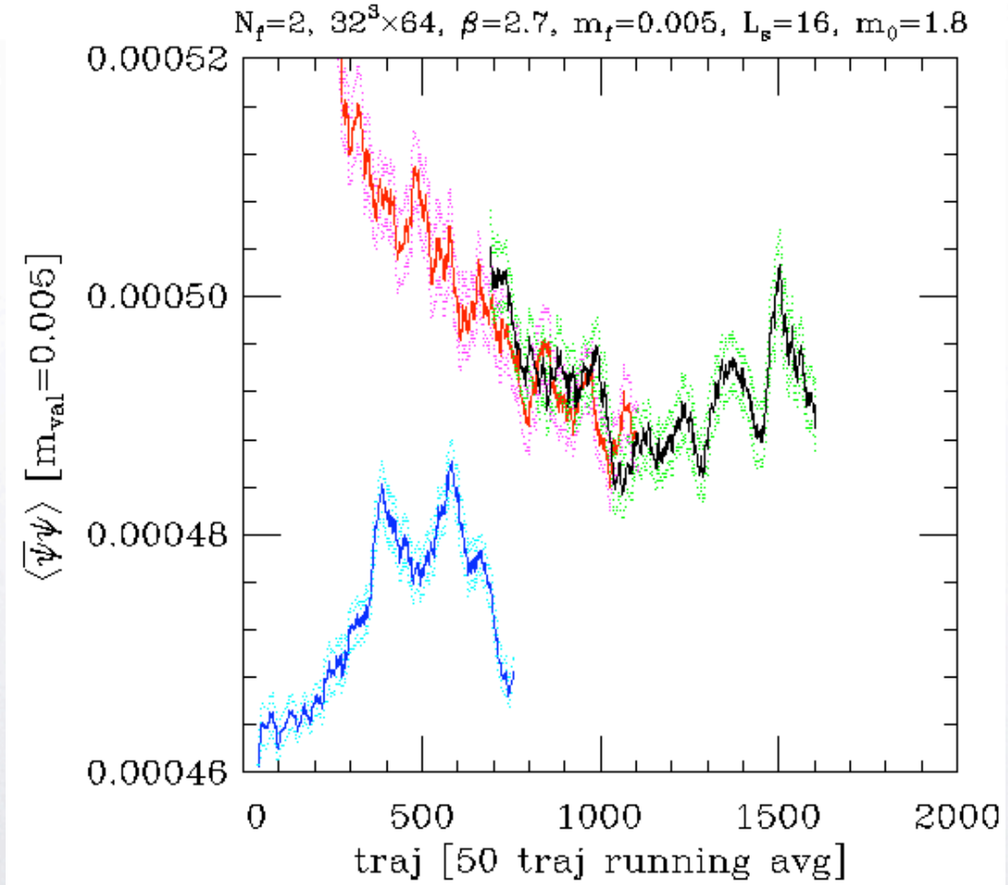


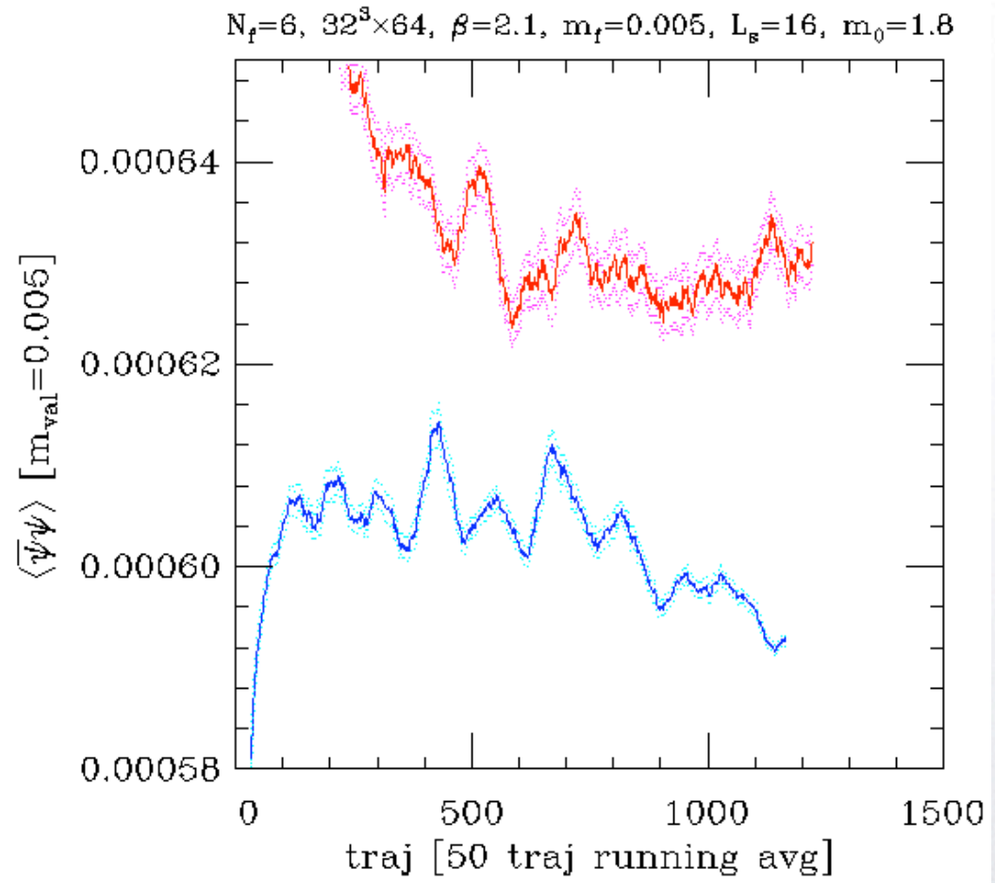
Evolution of Q_{top} for N_f=6, m_f=0.005



Effect of Q_{top} on N_f=2,6, m_f=0.005 condensates

- For N_f=6, condensate and f_π sensitive to Q_{top}.
- For N_f=2, runs thermalizing on O(1000) traj.





Weinberg's Sum Rules

Zeroth Weinberg sum rule:

$$S = \frac{1}{3\pi}\int_0^\infty \frac{ds}{s}\left[R_V(s) - R_A(s)\right]$$

This is the S parameter before the SM contribution is subtracted.

First Weinberg sum rule:

$$\frac{1}{3\pi}\int_0^\infty ds\left[R_V(s) - R_A(s)\right] = 4\pi F_\pi^2$$

Second Weinberg sum rule:

$$\frac{1}{3\pi}\int_0^\infty dss\left[R_V(s) - R_A(s)\right] = 0$$

- Peskin-Takeuchi (1992) used vector-pole dominance and Weinberg's sum rules to estimate the S parameter.
- How well do the WSRs with vector-pole dominance work?

Weinberg's Sum Rules

Vector-pole dominance:

$$\begin{aligned} R_V(s) &= 12\pi^2 F_\rho^2 \delta(s - m_\rho^2) \\ R_A(s) &= 12\pi^2 F_{a_1}^2 \delta(s - m_{a_1}^2) \end{aligned}$$

With this assumption, WSRs read:

$$\begin{aligned} 4\pi \left[\frac{F_\rho^2}{M_\rho^2} - \frac{F_{a_1}^2}{M_{a_1}^2} \right] &= S \\ F_\rho^2 - F_{a_1}^2 &= F_\pi^2 \\ F_\rho^2 M_\rho^2 - F_{a_1}^2 M_{a_1}^2 &= 0 \end{aligned}$$

- These sum rules hold only in the chiral limit.
- Lacking reliable chiral extrapolations, how well do they hold at finite quark masses?

Weinberg's Sum Rules

Vector-pole dominance:

$$\begin{aligned} R_V(s) &= 12\pi^2 F_\rho^2 \delta(s - m_\rho^2) \\ R_A(s) &= 12\pi^2 F_{a_1}^2 \delta(s - m_{a_1}^2) \end{aligned}$$

With this assumption, WSRs read:

$$\begin{aligned} 4\pi \left[\frac{F_\rho^2}{M_\rho^2} - \frac{F_{a_1}^2}{M_{a_1}^2} \right] &= S \\ F_\rho^2 - F_{a_1}^2 &= F_\pi^2 \\ F_\rho^2 M_\rho^2 - F_{a_1}^2 M_{a_1}^2 &= 0 \end{aligned}$$

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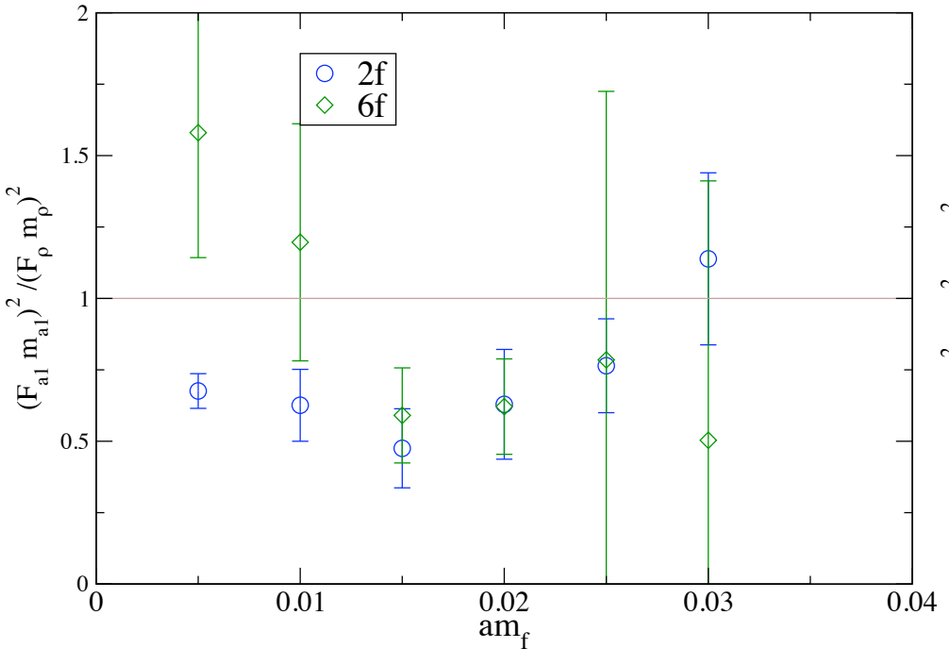
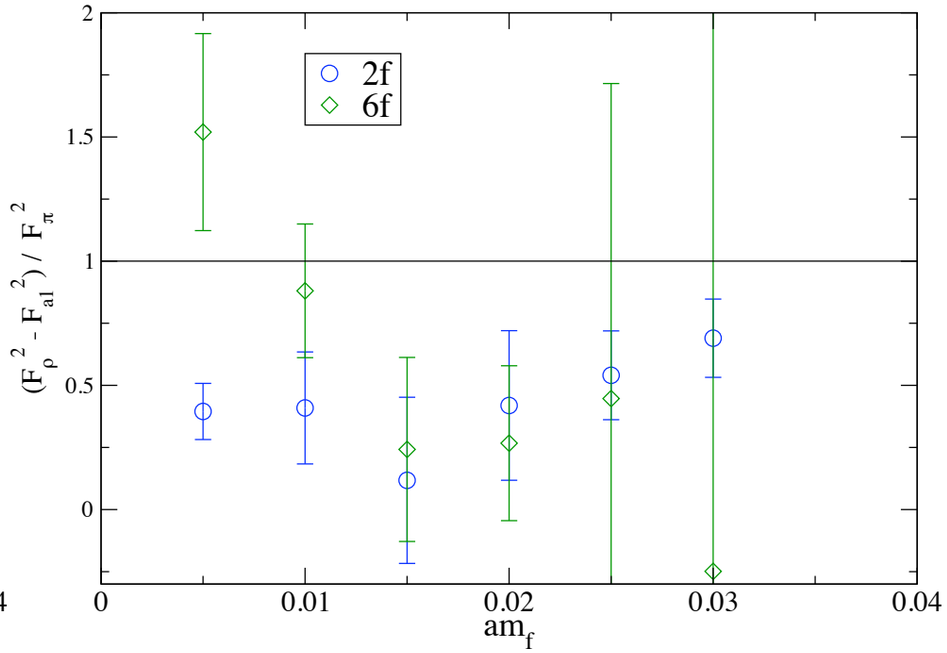
Weinberg's First and Second Sum Rules at Finite Quark Masses

PRELIMINARY



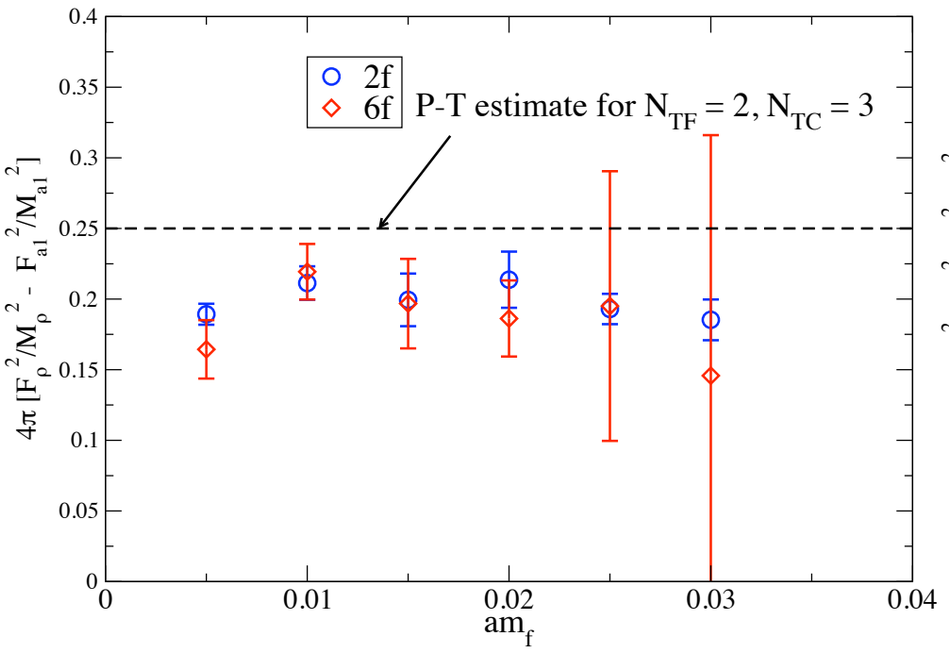


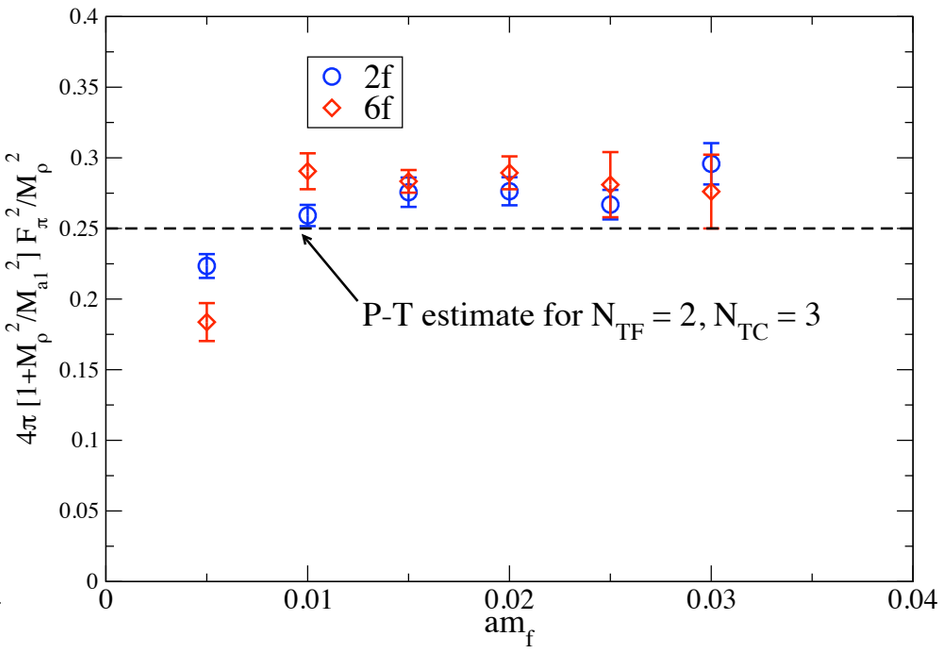
- Deviations from Weinberg's sum rules are seen.
- However, vector and axial-vector decay constants are very poorly determined, and may change significantly with fit ranges.
- Need reliable chiral extrapolations to check the sum rules in the chiral limit.

Zeroth Weinberg Sum Rule at Finite Quark Masses

- Zeroth Weinberg sum rule at the chiral limit, combined with the vector-pole dominance assumption, gives an estimate of the S parameter.
- Making use of the Weinberg's first and second sum rules, Peskin and Takeuchi gave (before SM subtraction)

$$S \approx 0.25 \frac{N_{TC}}{3} \frac{N_{TF}}{2}$$





PRELIMINARY

- Our normalization eliminates the naive $N_{TF}/2$ scaling for S so that $N_f = 2$ and 6 results can be compared directly.

Preliminary: Polarization Tensor for S Parameter

- χPT for polarization tensor

$$\sigma = \sqrt{1 + \frac{4m_\pi^2}{q^2}}$$

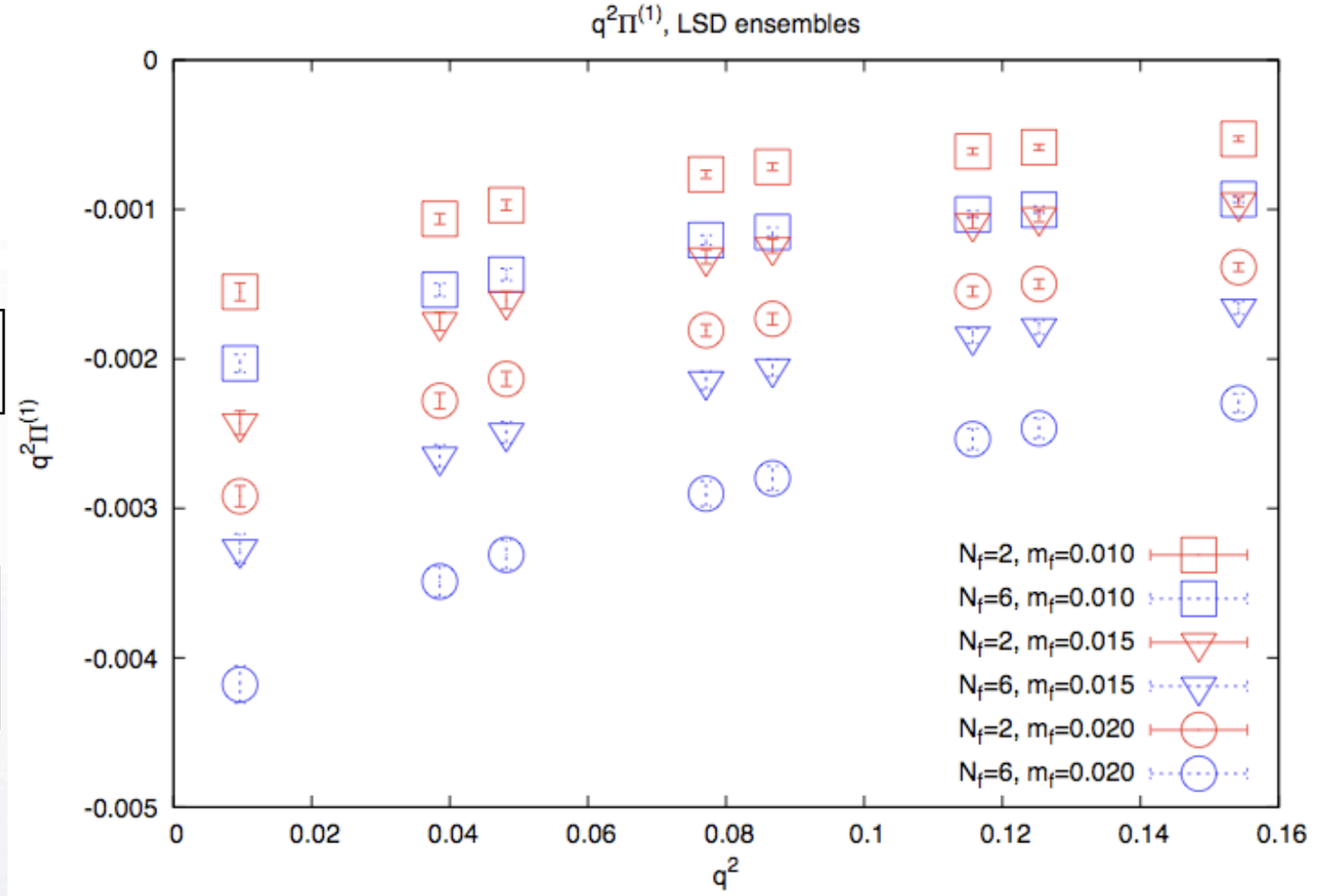
$$\overline{J}(\sigma) = \frac{1}{16\pi^2}\left(\sigma \log\left[\frac{\sigma-1}{\sigma+1}\right] + 2\right)$$

$$q^2\Pi^{(1)}_{V-A} = -f_\pi^2 + q^2\left[\frac{1}{24\pi^2}\left(L_{10} - \frac{1}{3}\right) + \frac{2}{3}\sigma^2\overline{J}(\sigma)\right]$$

- S parameter from $L_{10}(\mu)$

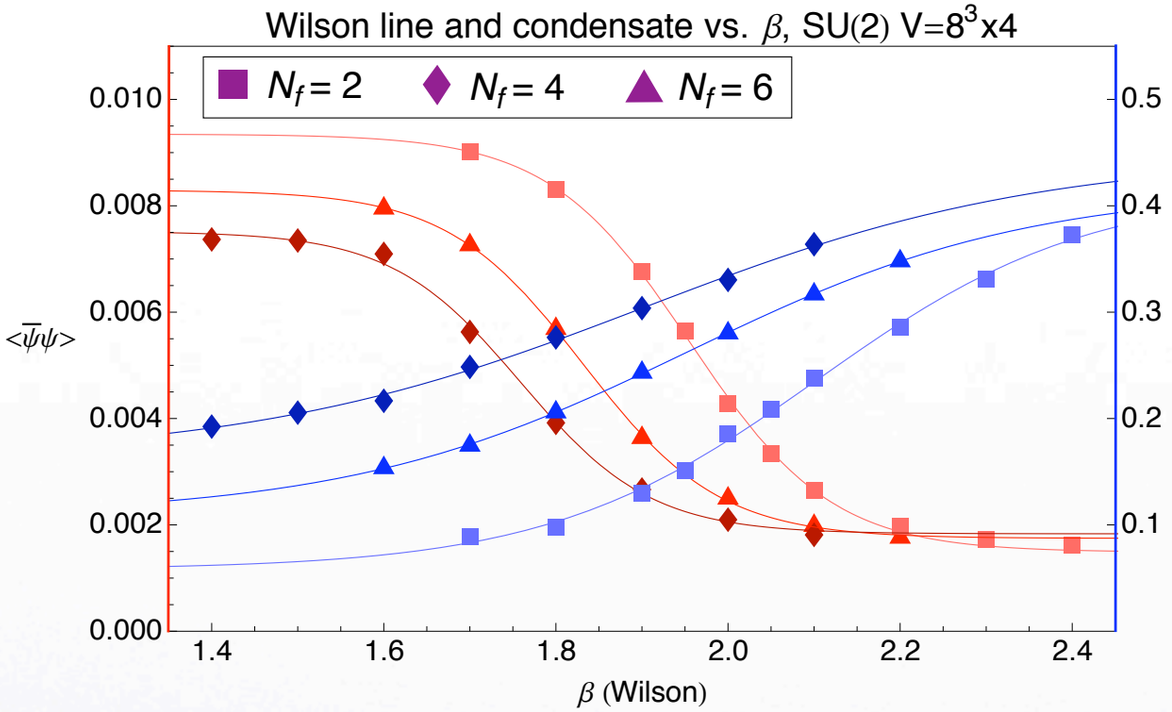
$$S = \frac{1}{12\pi}\left[L^r_{10}(\mu) + \log\frac{\mu^2}{m_H^2} - \frac{1}{6}\right]$$

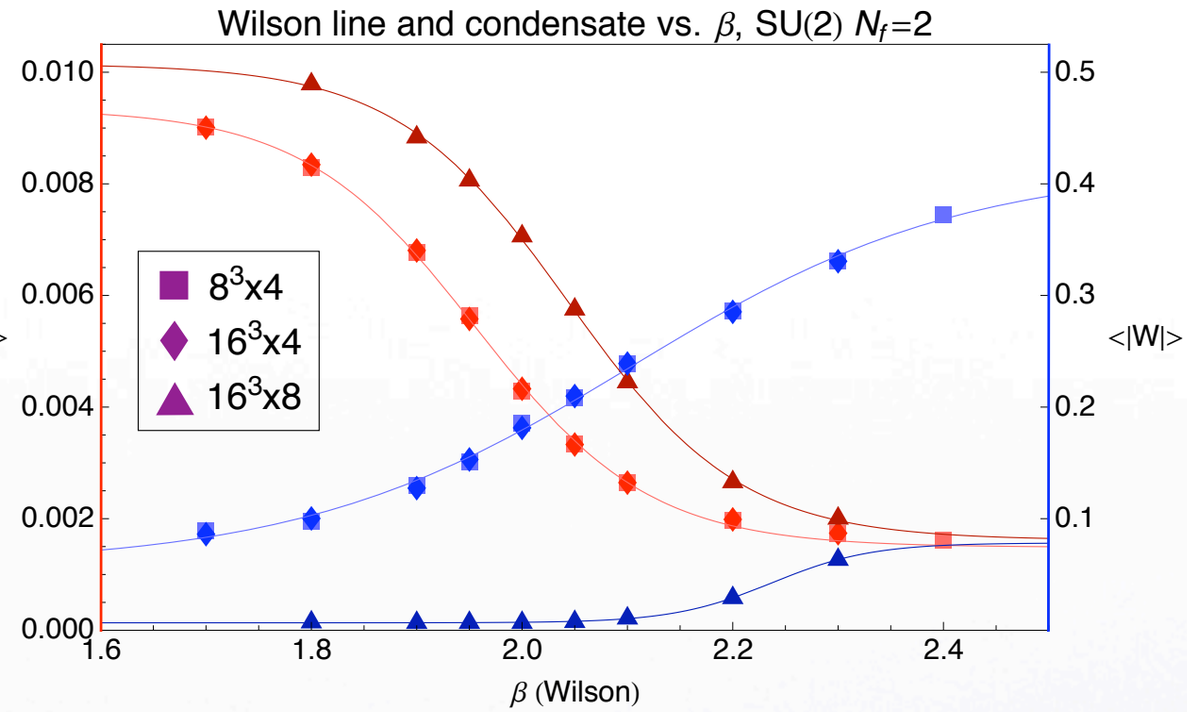
- LSD planning PRL soon.



(David Schaich)

LSD Preliminary: SU(2) Parameter Studies





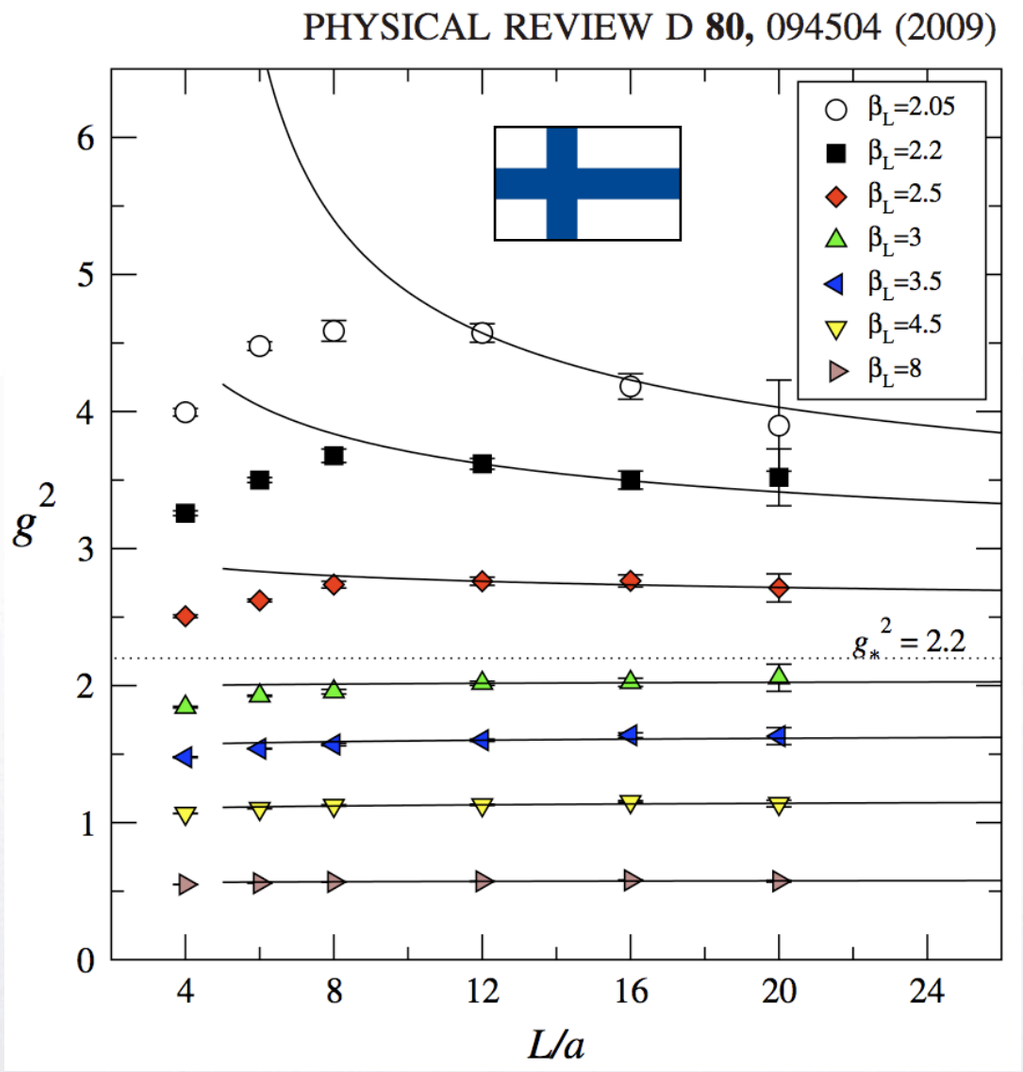
- Finite T studies ($L^3 \times L/2$) are low-cost ($4\times$) way to map out parameter space before starting zero T studies ($L^3 \times 2L$) using physical observables.
- Same approach used for SU(3) DWF in 1990's [my thesis].
- In progress: $m_{res}(\beta_c)$ on $L^3 \times 2L$ lattices at β_c.

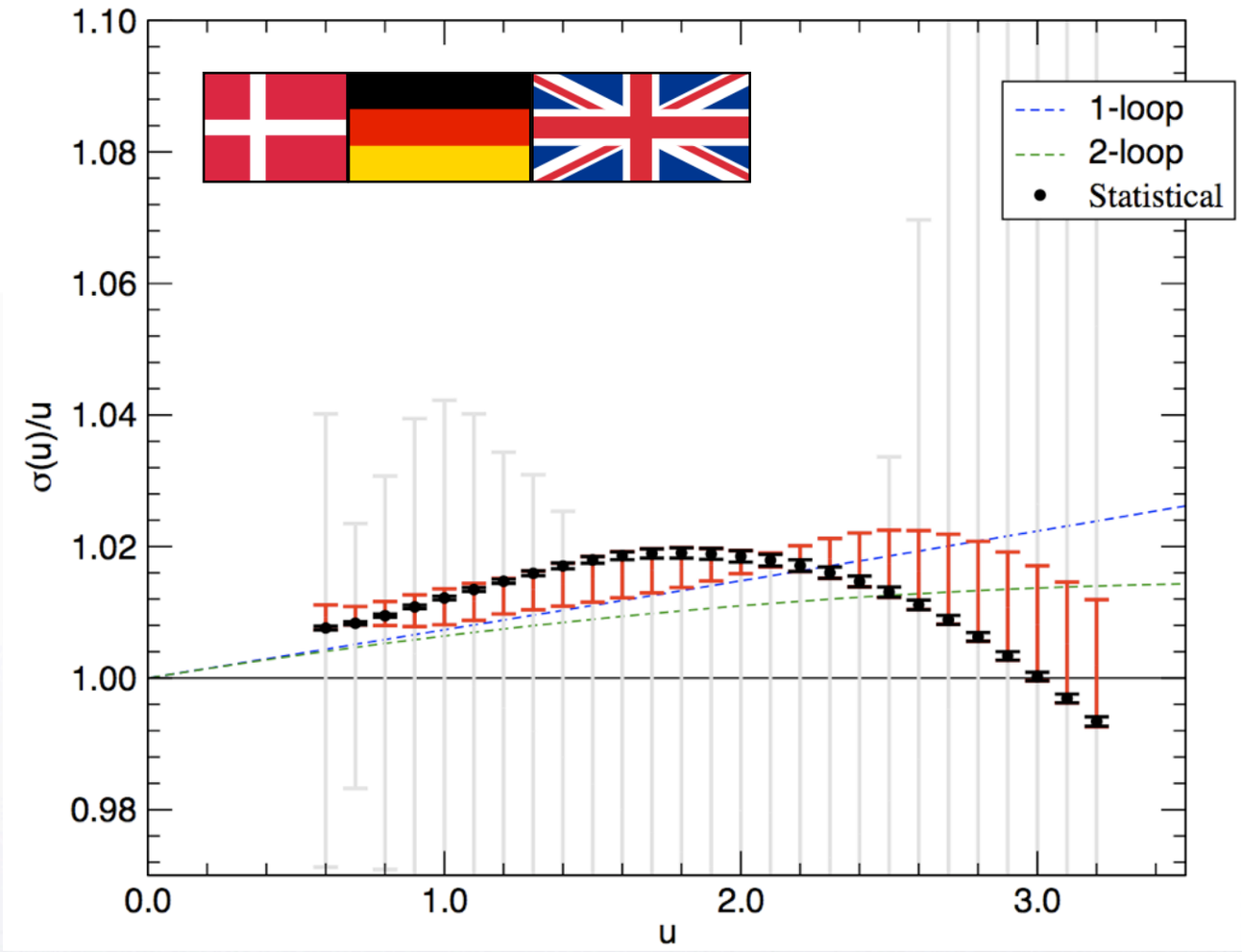
Have we accomplished anything yet?

- ***YES!!!***
- For SU(3), N_f=12 fund, most running coupling schemes show at least an inflection point, if not an IRFP. If it ultimately confines, then it should walk.
- For SU(2), N_f=2 adj, consensus is building around IRFP. Anomalous dimension is perhaps not as large as hoped for model building.
- For SU(3), N_f=6 fund, condensate enhancement suggests anomalous dimension is bigger than predicted by pert. theory. An encouraging trend toward the conformal window?

Backup Slides

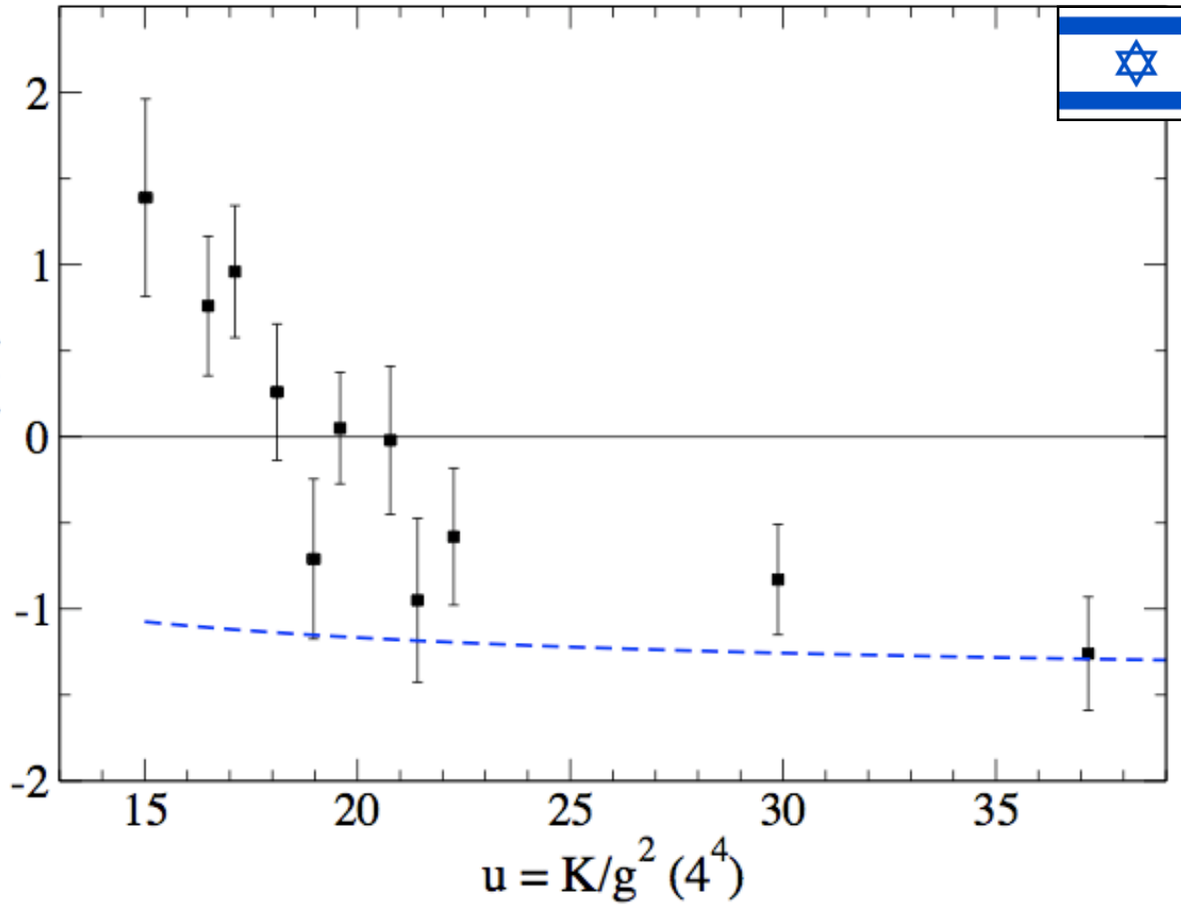
Schrödinger Functional for other representations

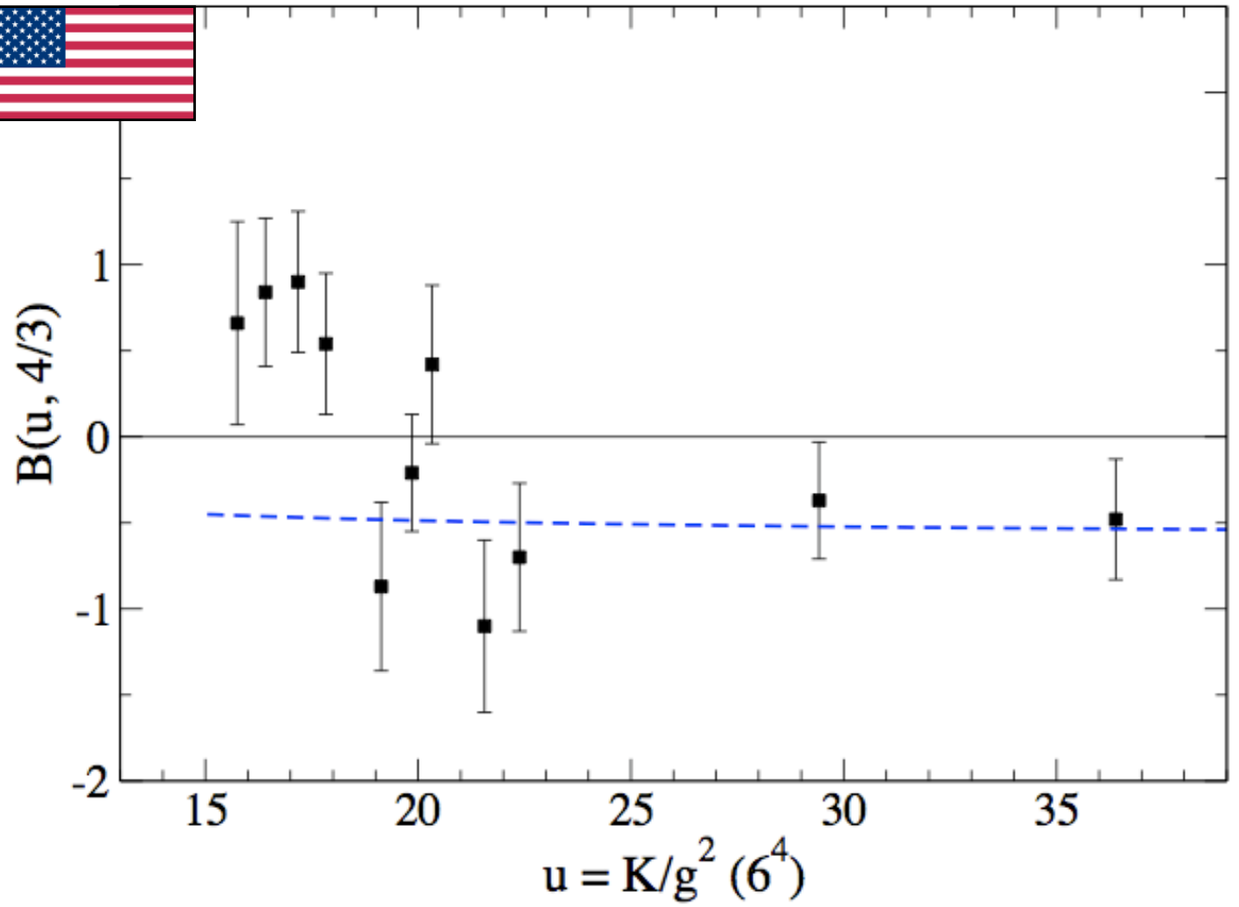




- SU(2) N_f=2 Adjoint: Hietanen *et al.*, Bursa *et al.*

Schrödinger Functional for other representations

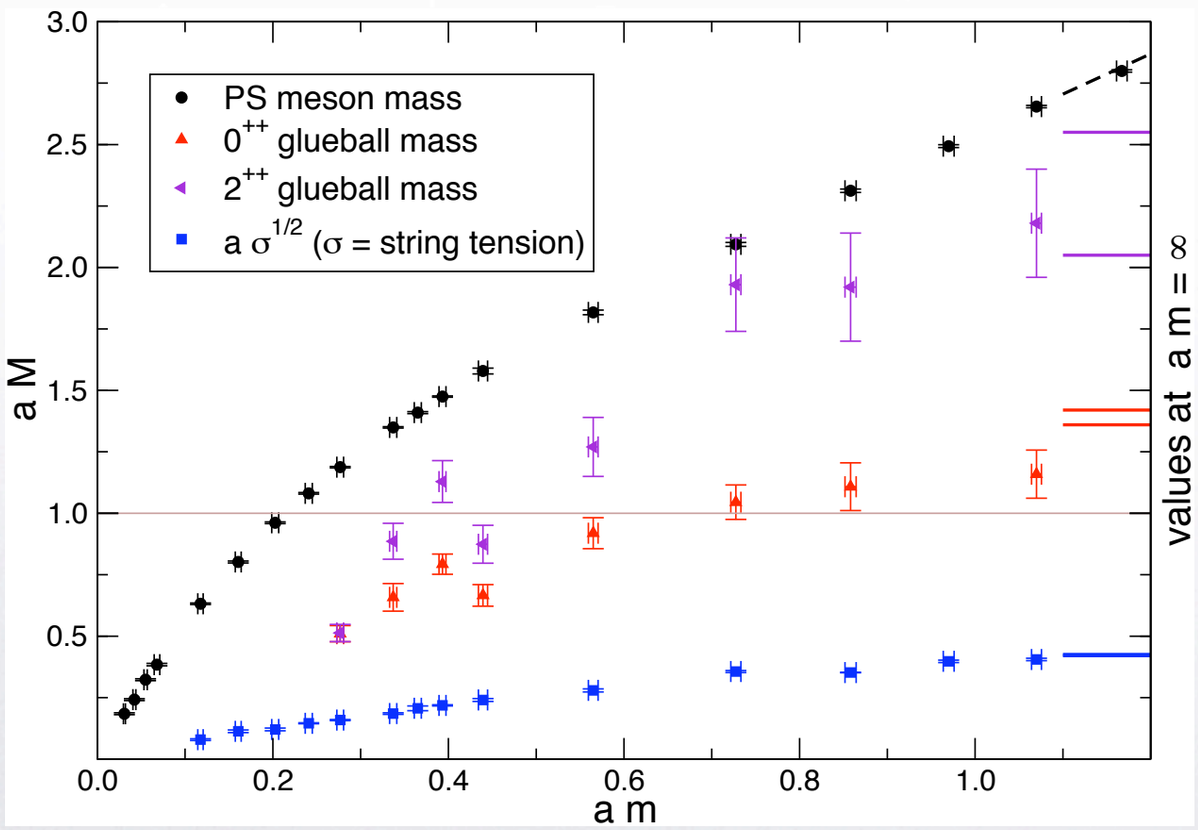


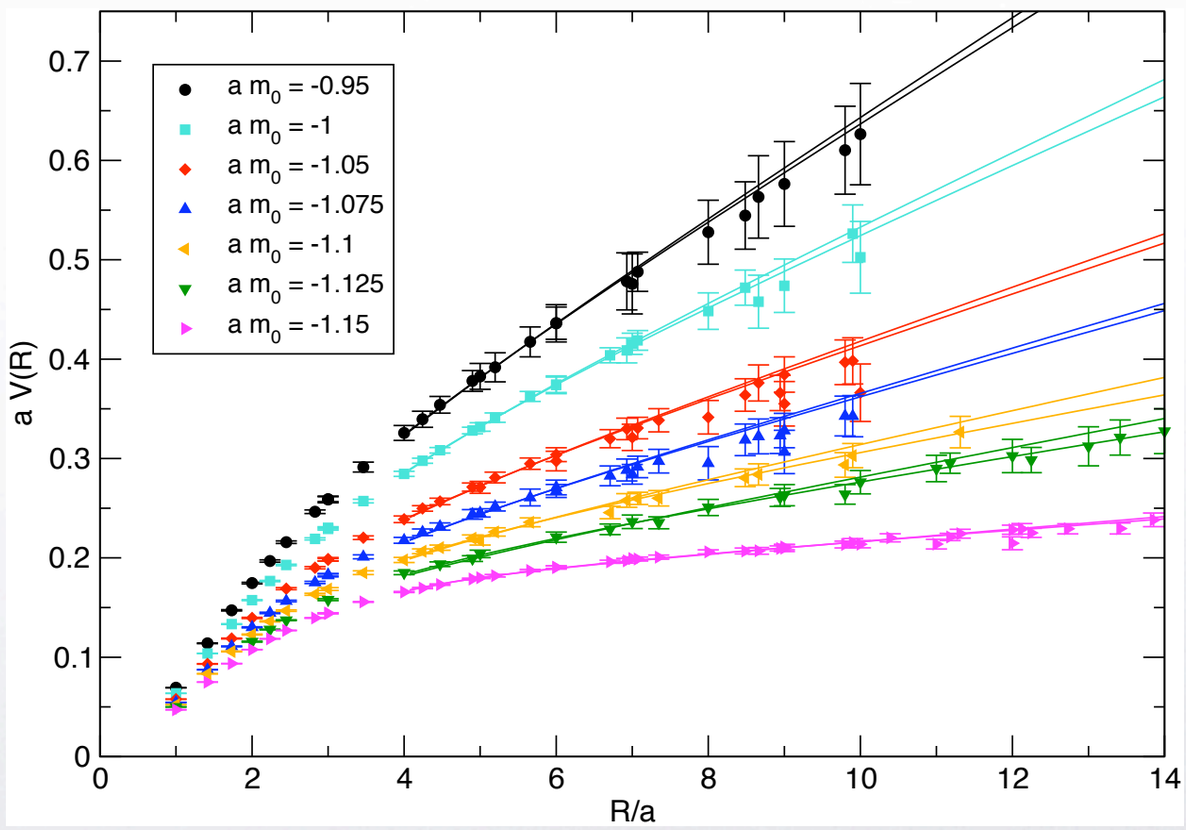


- Discrete beta function: $B(g^2(L),s) \equiv K/g^2(sL) - K/g^2(L)$.
- SU(3) N_f=2 Sextet: DeGrand *et al.*, Phys. Rev. D78, 031502 (2008)

Hyperscaling in SU(2), N_f=2 adj

- Substantial evidence for all scales collapsing together as m_{PCAC} is tuned to the critical value.
- Del Debbio *et al.*, arXiv:1004.3197 and arXiv:1004.3206





Why does LSD explore with DWF?

- In the early days of Lattice QCD, spontaneous symmetry breaking on the lattice was poorly understood and yet many interesting calculations were done which agreed with experiment.
- A long time later, Aoki and Sharpe-Singleton explained how Wilson fermions actually work (only for $N_c>3$ and $N_f=2$). Staggered fermions are still a topic of some debate.
- For SU(2) fund., staggered starts with SU(2) that can only break to SO(2), probably not useful for studying continuum vacuum alignment.
- Working out the structure of various possible Aoki phases and the number of NGBs seems a daunting task for Wilson fermions.
- Without knowing the continuum answer in advance (as in QCD), "cheaper" actions require very expensive continuum extrapolation.
- The safest choice is the one with the continuum symmetries.

SU(2) Vacuum Alignment and Lattice Fermions

- All continuum gauge theories with N_f Dirac fermions in real or pseudo-real representations have enhanced $SU(2N_f)$ global symmetry which mixes flavor and chirality.
- The larger symmetry has more options for spontaneous breaking leading to the dynamical question of vacuum alignment.
- A mass term aligns the vacuum along a specific direction. For SU(2) fund, m $\langle\psi\psi\rangle$ breaks $SU(2N_f) \rightarrow Sp(2N_f)$.
- Wilson fermions start with $Sp(2N_f)$ which may further break spontaneously *à la* Aoki-Sharpe-Singleton.
- DW fermions have full $SU(2N_f)$ in the $L_s \rightarrow \infty$ limit. At finite L_s, m_f+m_{res} breaks $SU(2N_f) \rightarrow Sp(2N_f)$.