Lattice Studies of the Nearly Conformal Composite Higgs Mechanism

Julius Kuti

Fermilab seminar

April 1, 2010

With Lattice Higgs Collaboration members:

Z. Fodor, K. Holland, D. Nogradi, C. Schroeder

Video Games in Technicolor

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The New York Times

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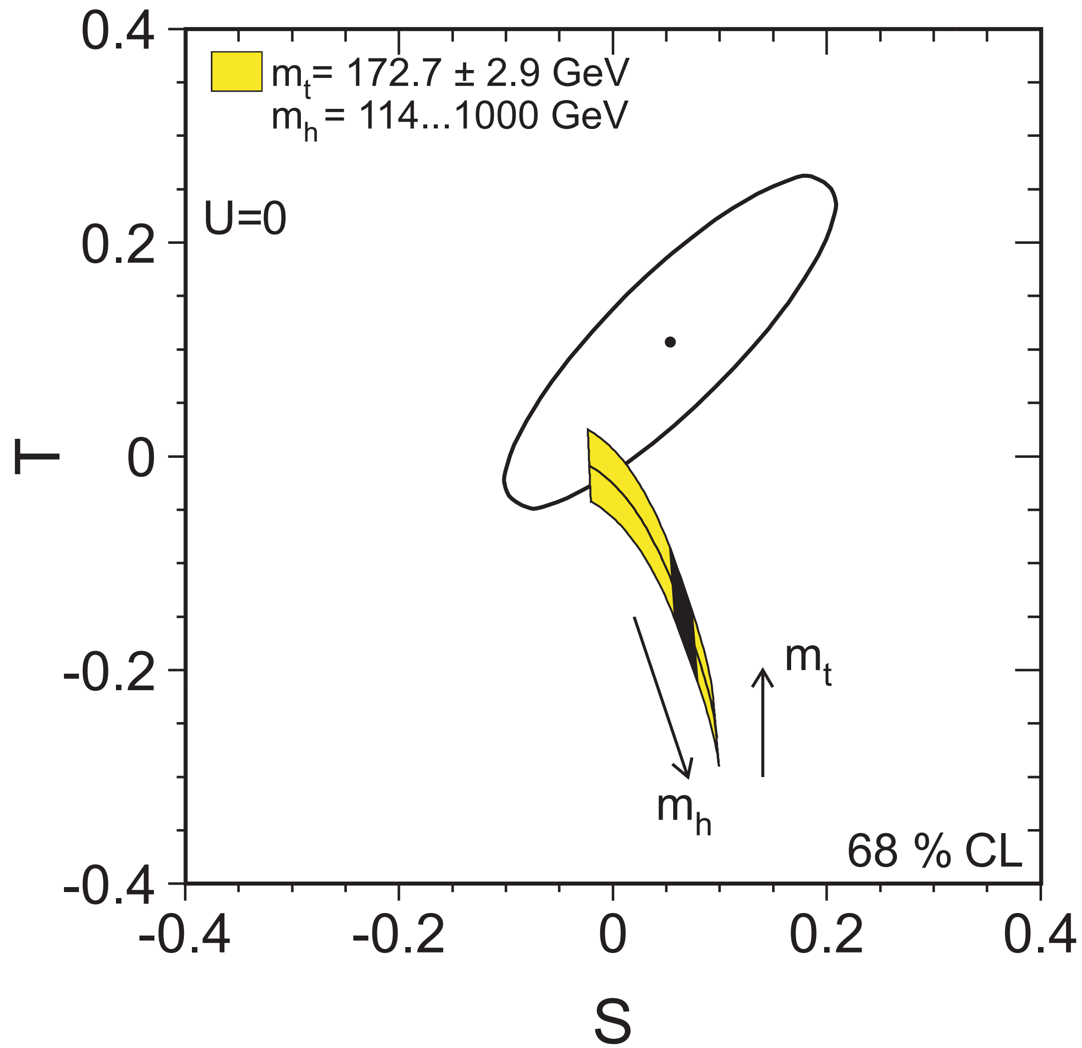
Physicists at CERN in Geneva find the Higgs particle with unexpected characteristics

By Jane Ellis

The properties of the newly found Higgs particle shook the foundations of modern particle physics. Although its decay properties are very similar to what was expected, the mass at 507 GeV is far too heavy and the width far too narrow to accommodate what is know to be the Standard Model of modern particle physics. Physicists are turning now to lattice gauge theorists who are trying to explain with nearly conformal gauge theories the experiments at the Large Hadron Collider. *Continued on page 11 ...*



How to pull out the heavy Higgs particle to 507 GeV from the allowed oval region without violating EW precision data?

Nearly conformal gauge theories might work?

Requires unusual nonperturbative properties which can be studied on the lattice with extreme computing resources

Can the lattice be transformational when we do not know the answer?

Outline

1. Overview of three coordinated projects in our program
   - SU(3) color, fundamental rep, staggered Nf=4-20
   - sextet representation with SU(3) color
   - Running coupling (new ideas, first results)
2. Chiral symmetry breaking
   - Finite volume p-regime, delta-regime, epsilon-regime
   - Goldstone spectra and staggered CHPT
   - New results at Nf=4,8,9,12 will be presented
3. Inside and above the conformal window
   - Zero momentum dynamics at Nf=16,20
4. Conclusions and Outlook
   - Prospects towards model building ?
   - Can lattice studies be transformational ?
   - Is peta-scale to exa scale power needed for definitive phenomenology ?

Talk is based on published results last year:

1. **Topology and higher dimensional representations.**
   Published in **JHEP 0908:084,2009**.
   e-Print: **arXiv:0905.3586** [hep-lat]

2. **Nearly conformal gauge theories in finite volume.**
   Phys.Lett.B681:353-361,2009.
   e-Print: **arXiv:0907.4562** [hep-lat]

3. **Chiral properties of SU(3) sextet fermions**
   e-Print: **arXiv:0908.2466** [hep-lat]

4. **Chiral symmetry breaking in nearly conformal gauge theories**
   e-Print: **arXiv:0911.2463** [hep-lat] **posted**

5. **Calculating the running coupling in strong electroweak models**
   e-Print: **arXiv:0911.2934** [hep-lat]

and some unpublished new analysis

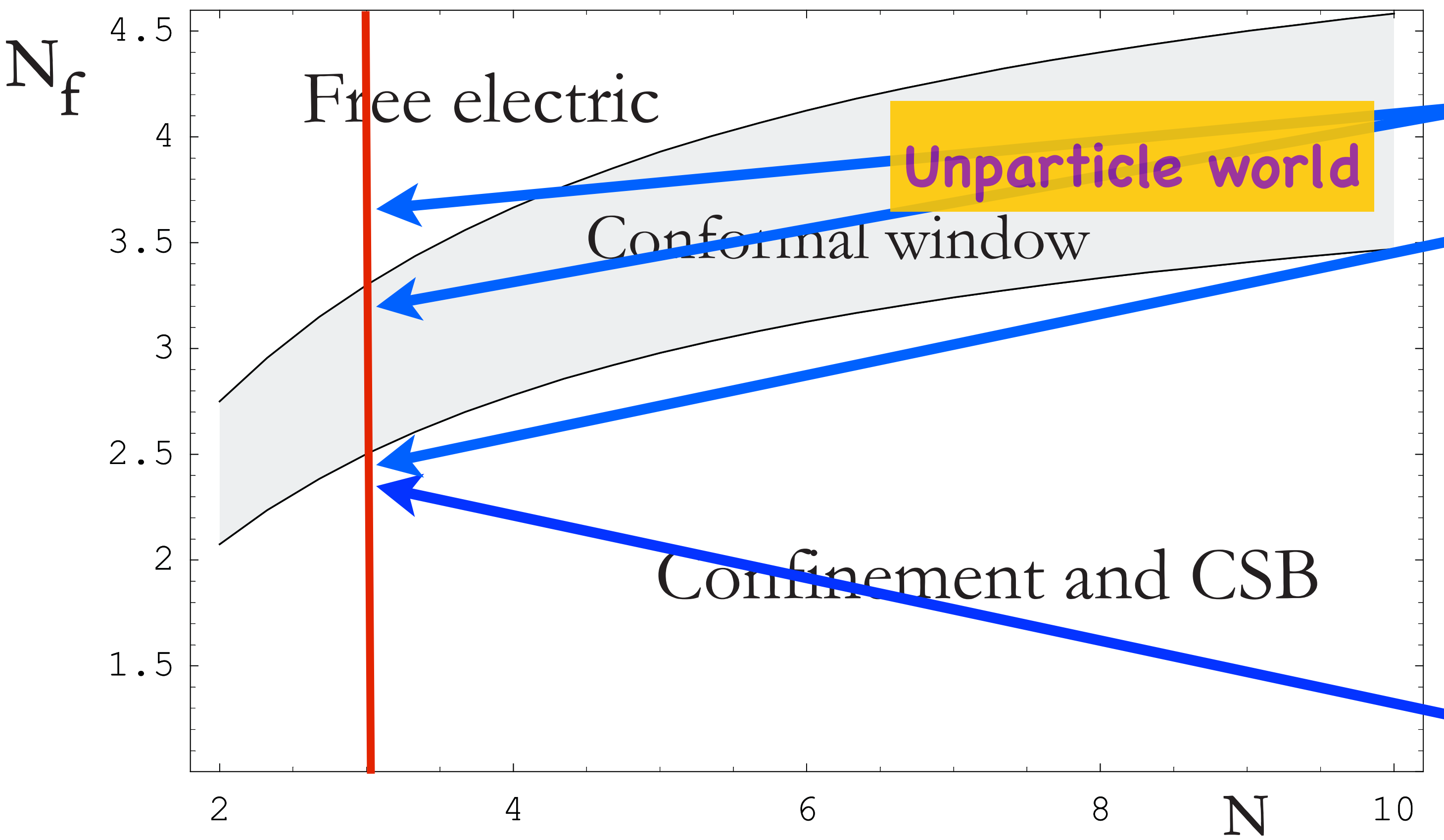
USQCD got into BSM studies ~3 yrs ago

(early work on supersymmetry, top-Higgs physics, ...)

Kudos to Yale group for stimulating lattice interest in conformality

Now the genie is out of the bottle

Phase diagram of TWO projects as nearly conformal gauge theories in flavor-color space ?



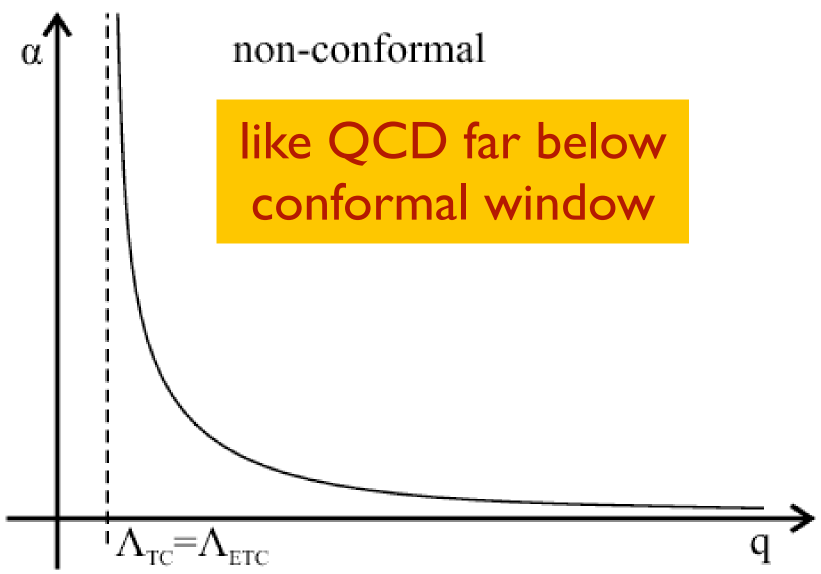
Project 1:
Fundamental rep Nf=4,8-12,14,16,20 flavors and three colors with staggered fermions.

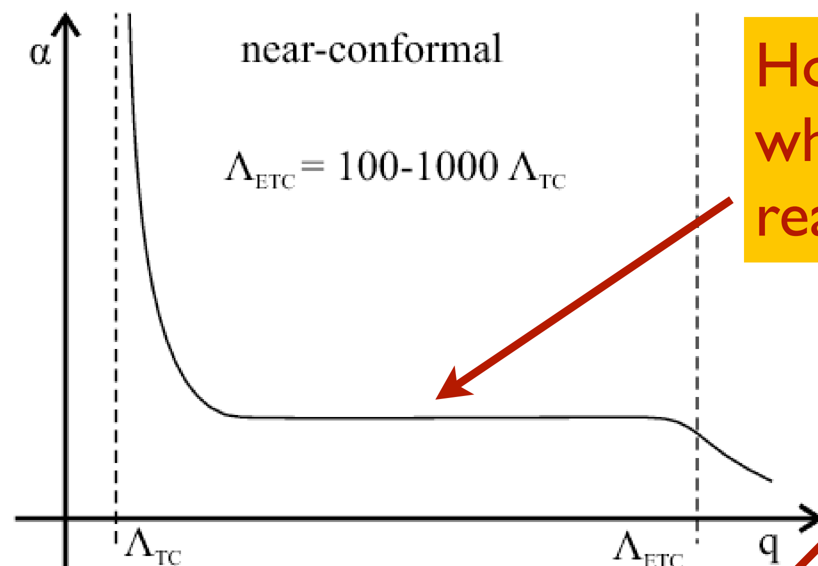
Project 2:
2-index symmetric rep with Nf=2 flavors and three colors with overlap chiral fermions → staggered
(will be briefly discussed here, but quenched results with interesting topology are published and full dynamical simulations are running)
Our unified GPU/MPI code ready near the conformal window (walking):
Higgs phenomenology with nearly vanishing beta function

Phenomenology goal: nearly conformal gauge theory with minimal realization of the composite Higgs mechanism
Consistent with ElectroWeak Precision Data ?

They are fun lattice field theories anyway!

Project 3: Important to complement the test of chirality with running coupling and beta function

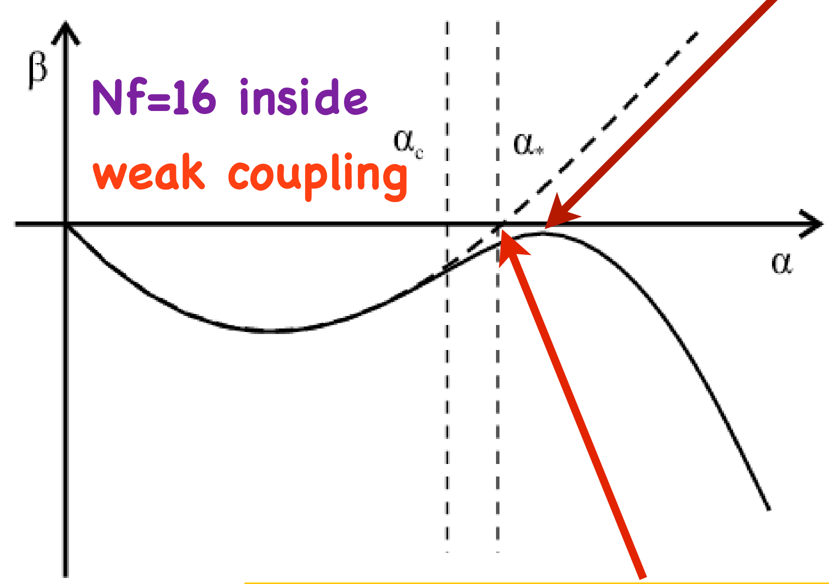




How to reach walking scale which is wanted for several reasons in BSM?

Is 2-index symmetric rep nearly conformal?

Fundamental rep with Nf=4,8,9 should be similar



would be Banks-Zaks FP

Nf=10,11,12,14,16,20 under continued study

Nf=12 controversial

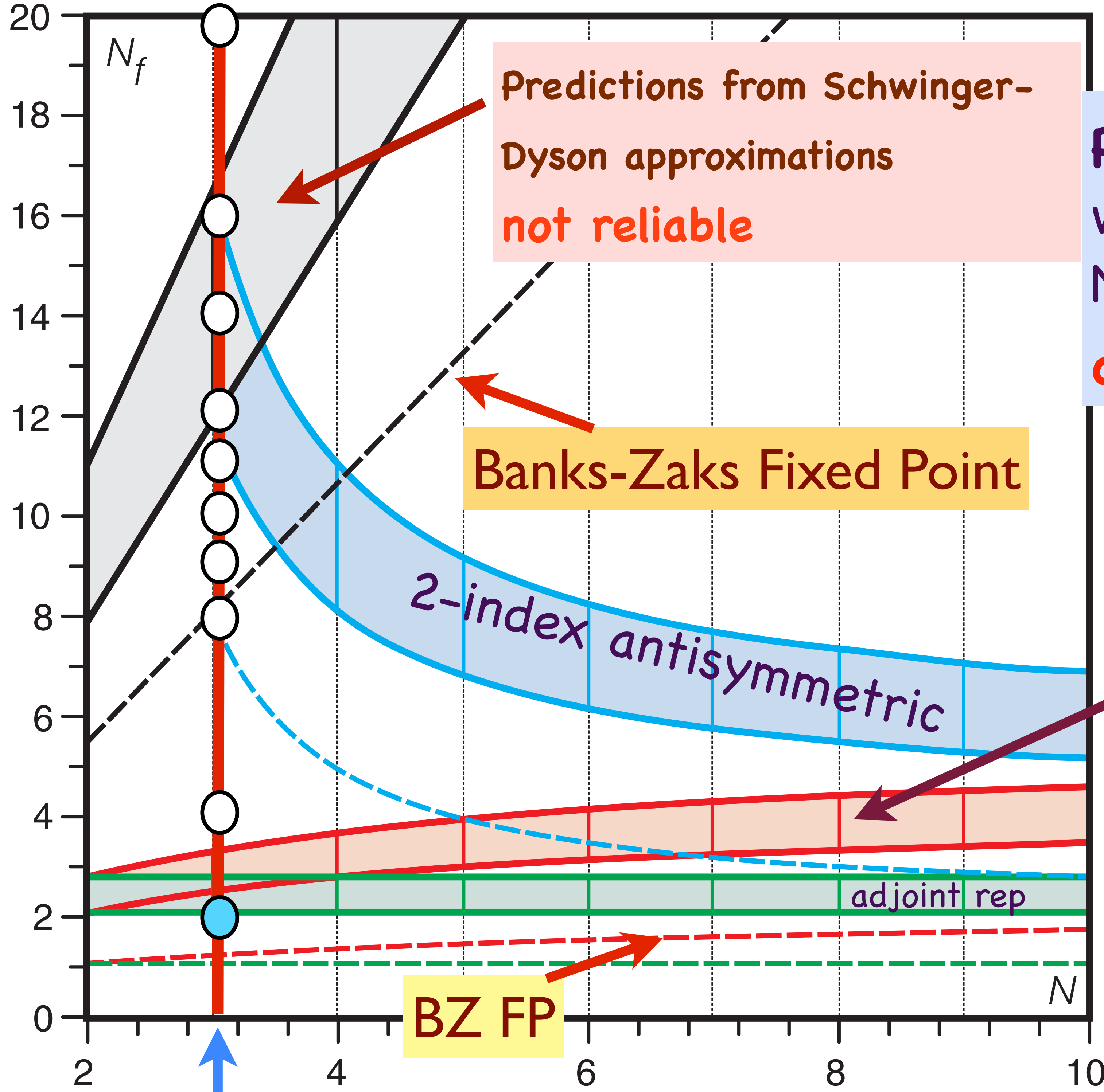
DeGrand et al. (conformal?)

our staggered simulations disagree with conformal phase

important in model building

Theory space and conformal windows

Important early work by Bardeen, Leung, Love on Schwinger-Dyson



We only run with N=3 colors

Project 1: in fundamental rep with N=3 colors with Nf=4,8,9,10,11,12,14,16,20 flavors

dynamical staggered

Project 2: 2-index symmetric rep (sextet)

N=3 colors and Nf=2 flavors

dynamical overlap

Running on CPU clusters and GPU clusters

Very demanding

Unified code

We are supported by the Wuppertal hardware/software infrastructure

Zoltan Fodor
Kalman Szabo
Sandor Katz

GPU Hardware

CUDA code:
Kalman Szabo
Sandor Katz

GTX 280
Flops: single 1 Tflop, double 80 Gflops
Memory 1GB, Bandwidth 141 GBs^{-1}
230 Watts, $350

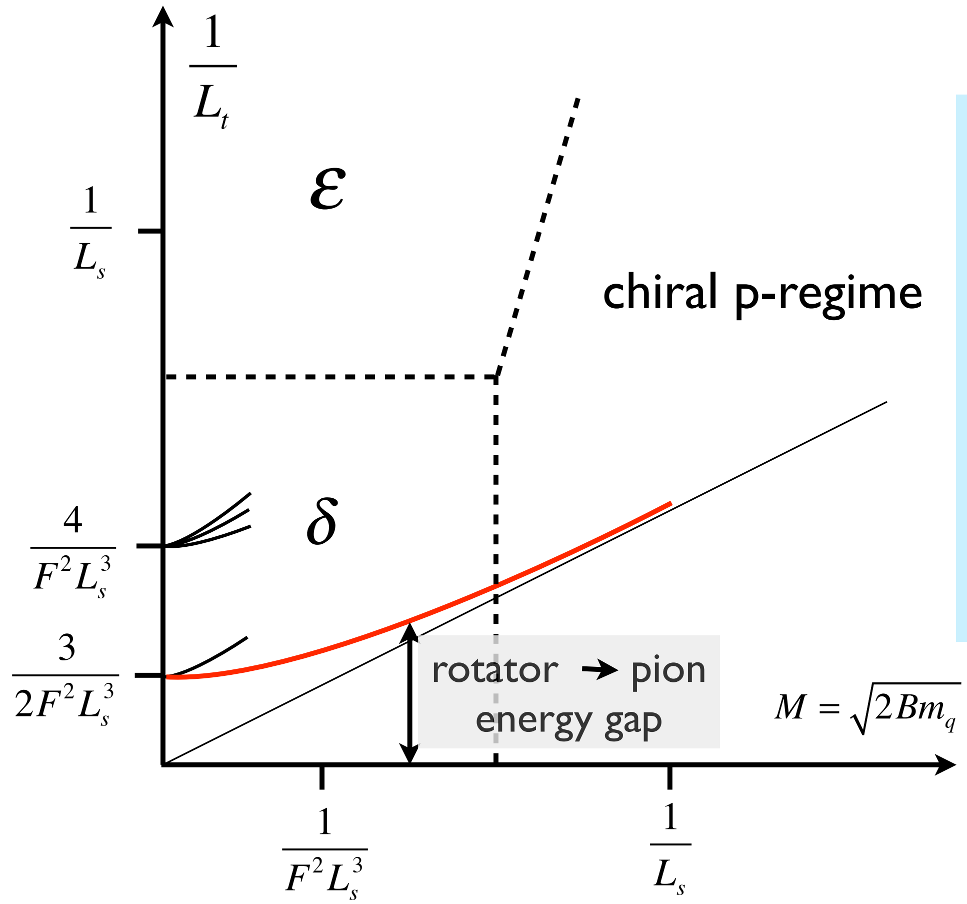
also USQCD CPU
cluster support

UCSD Tesla cluster
ARRA funded by DOE
waiting for Fermi cards

Tesla 1060
Flops: single 1 Tflop, double 80 Gflops
Memory 4GB, Bandwidth 102 GBs^{-1}
230 Watts, $1200

Tesla 1070
Flops: single 4 Tflops, double 320 Gflops
Memory 16GB, Bandwidth 408 GBs^{-1}
900 Watts, $8000

Chiral regimes to identify in theory space:



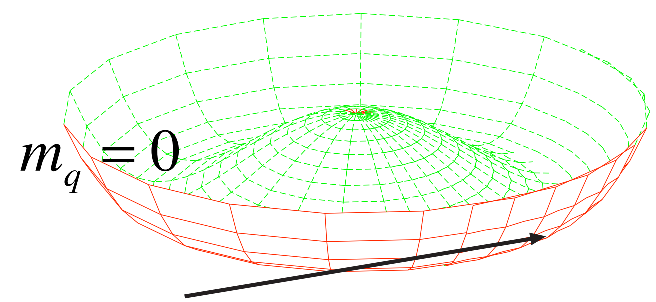
Goldstone dynamics is different in each regime

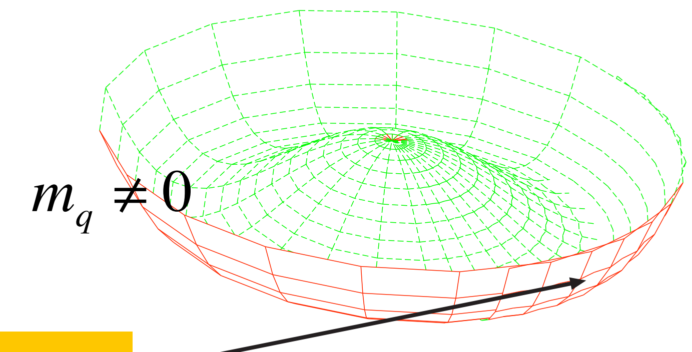
We study δ and ε -regimes (RMT)

and p-regime (probing chiral loops)

complement each other

interpretation of rotator levels in $m_q \to 0$ limit:

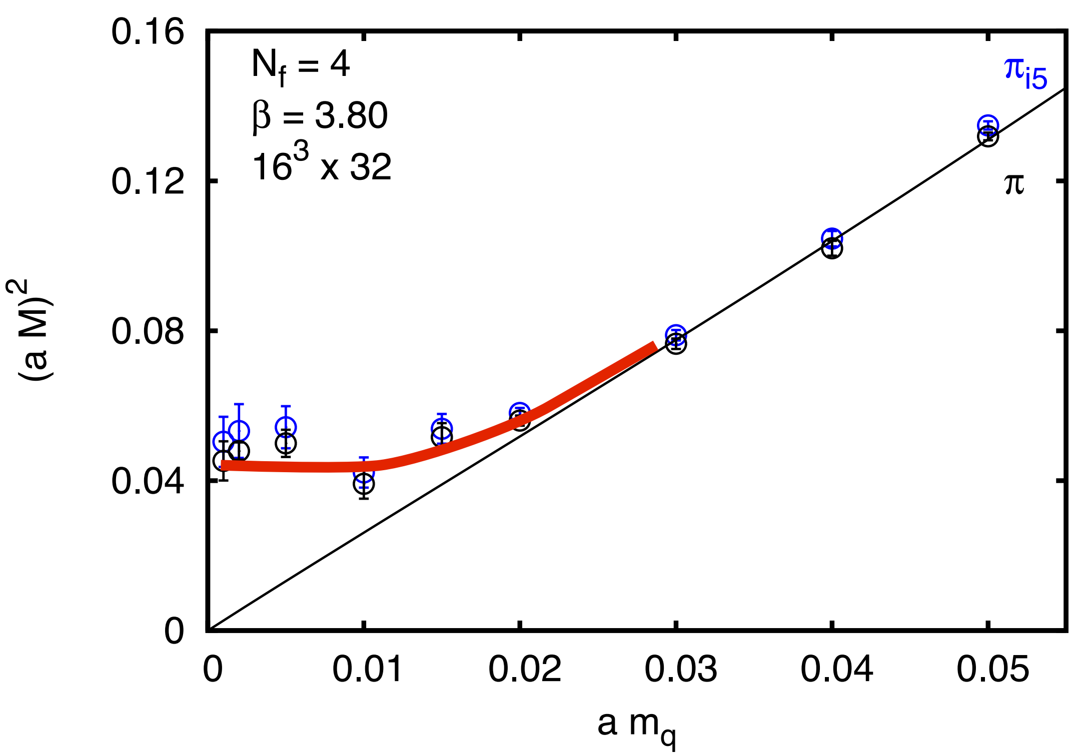


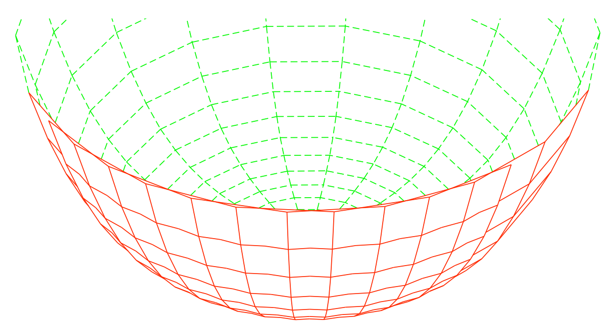


Veff: chiral condensate in flavor space

arbitrary orientation of condensate

tilted condensate



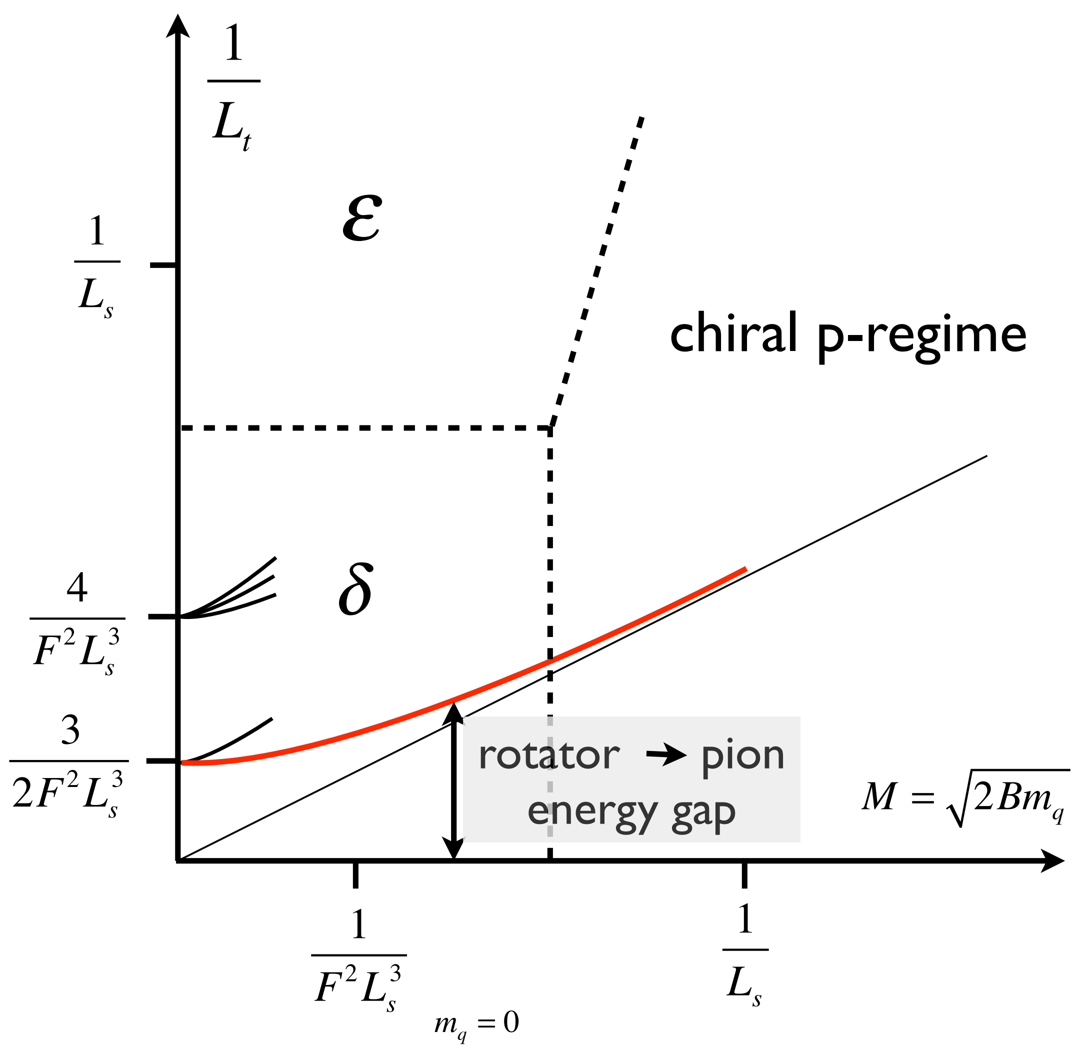


Not to misidentify rotator gaps as evidence of chirally symmetric phase !!

One-loop expansion in our analysis of p-regime:

Leutwyer, Gasser, P. Hasenfratz,

Niedermayer, Hansen, Neuberger, ...



$$M_\pi^2 = M^2\left[1 - \frac{M^2}{8\pi^2 N_f F^2} ln\left(\frac{\Lambda_3}{M}\right)\right],$$

$$F_\pi = F\left[1 + \frac{N_f M^2}{16\pi^2 F^2} ln\left(\frac{\Lambda_4}{M}\right)\right],$$

Note Nf scaling of pion mass!

warning: 2-loop ~Nf^2 (Bijnens)

$$M_\pi(L_s,\eta) = M_\pi\left[1 + \frac{1}{2N_f}\frac{M^2}{16\pi^2 F^2}\cdot\widetilde{g}_1(\lambda,\eta)\right], \quad \lambda = ML_s$$

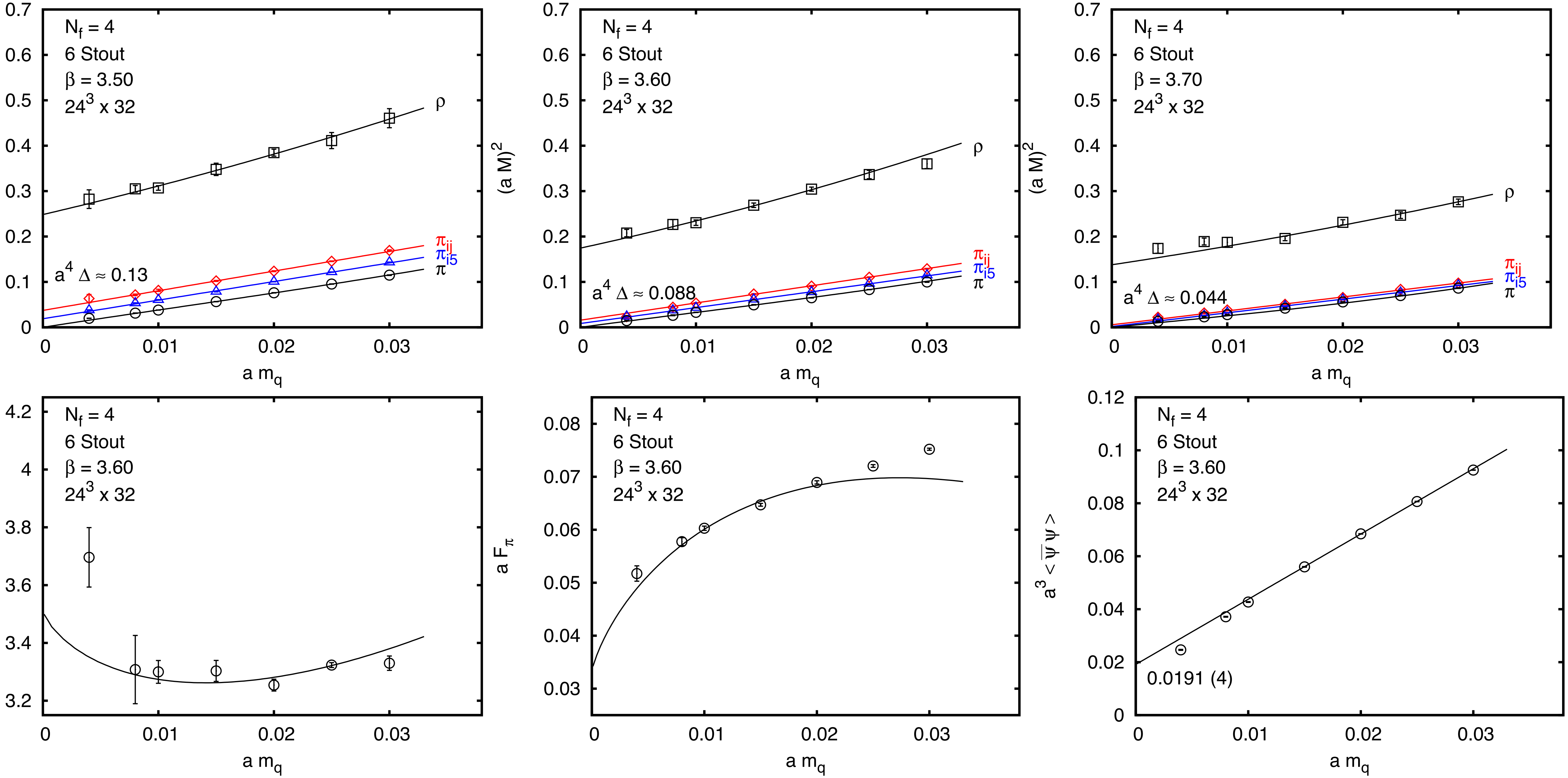
$$F_\pi(L_s,\eta) = F_\pi\left[1 - \frac{N_f}{2}\frac{M^2}{16\pi^2 F^2}\cdot\widetilde{g}_1(\lambda,\eta)\right],$$

We use staggered action with stout smearing

Taste breaking included in staggered perturbation theory!

structure changing as Nf grows

Nf=4 NLO chiral analysis in p-regime:

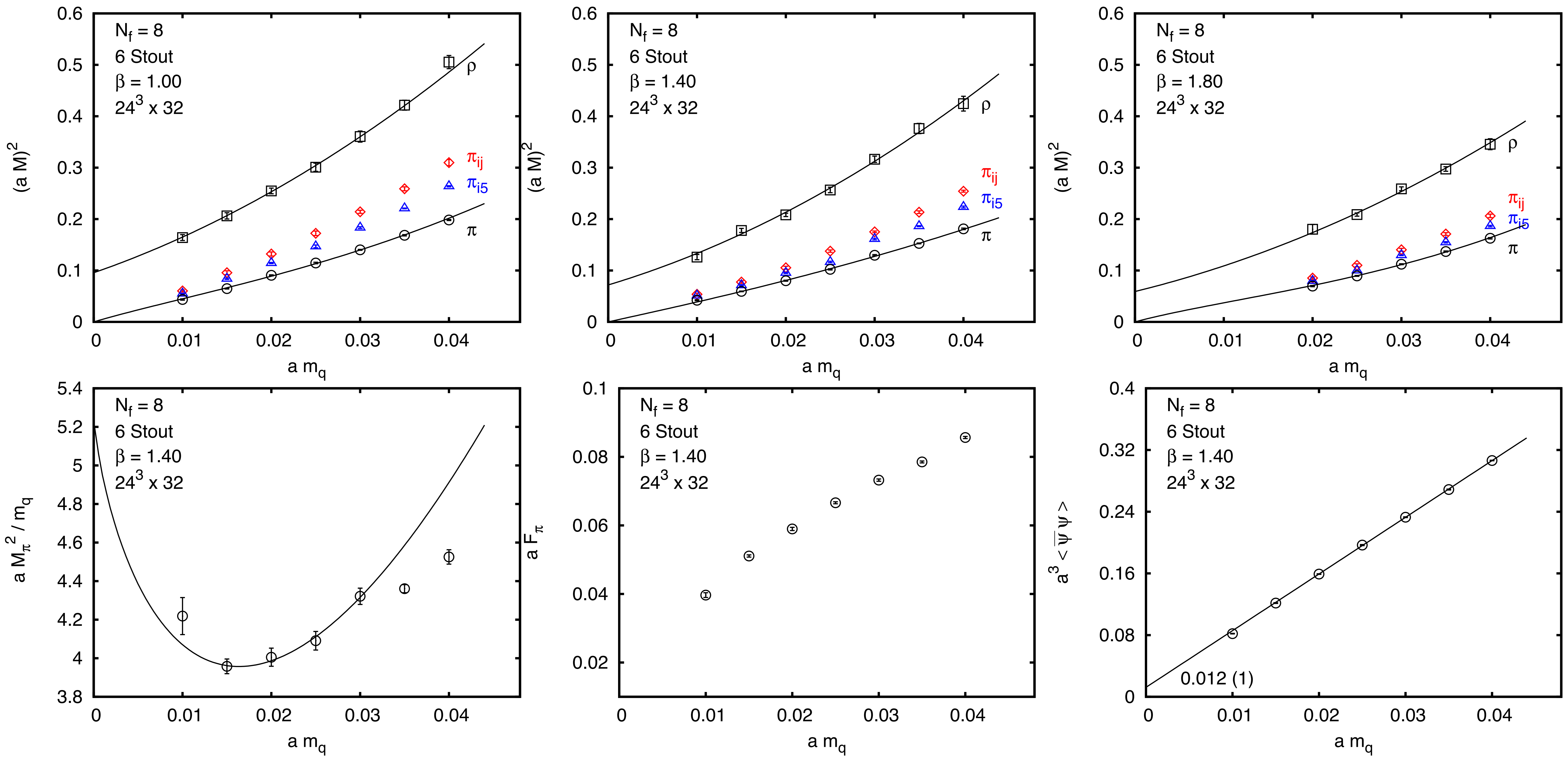


$$\mathcal{L}_\chi^{(4)} = \frac{F^2}{4}\mathrm{Tr}(\partial_\mu\Sigma\partial_\mu\Sigma^\dagger) - \frac{1}{2}B\,m_q\,F^2\mathrm{Tr}(\Sigma + \Sigma^\dagger) \;+V_\chi^{(6)}$$

$$M_\pi(T_a)^2 = 2Bm_q + a^2\Delta(T_a) + O(a^2 m_q) + O(a^4)$$

$$\begin{aligned}
\Delta(\xi_5) &= 0\,, \\
\Delta(\xi_\mu) &= \frac{8}{F^2}(C_1 + C_2 + C_3 + 3C_4 + C_5 + 3C_6)\,, \\
\Delta(\xi_{\mu 5}) &= \frac{8}{F^2}(C_1 + C_2 + 3C_3 + C_4 - C_5 + 3C_6)\,, \\
\Delta(\xi_{\mu\nu}) &= \frac{8}{F^2}(2C_3 + 2C_4 + 4C_6)\,.
\end{aligned}$$

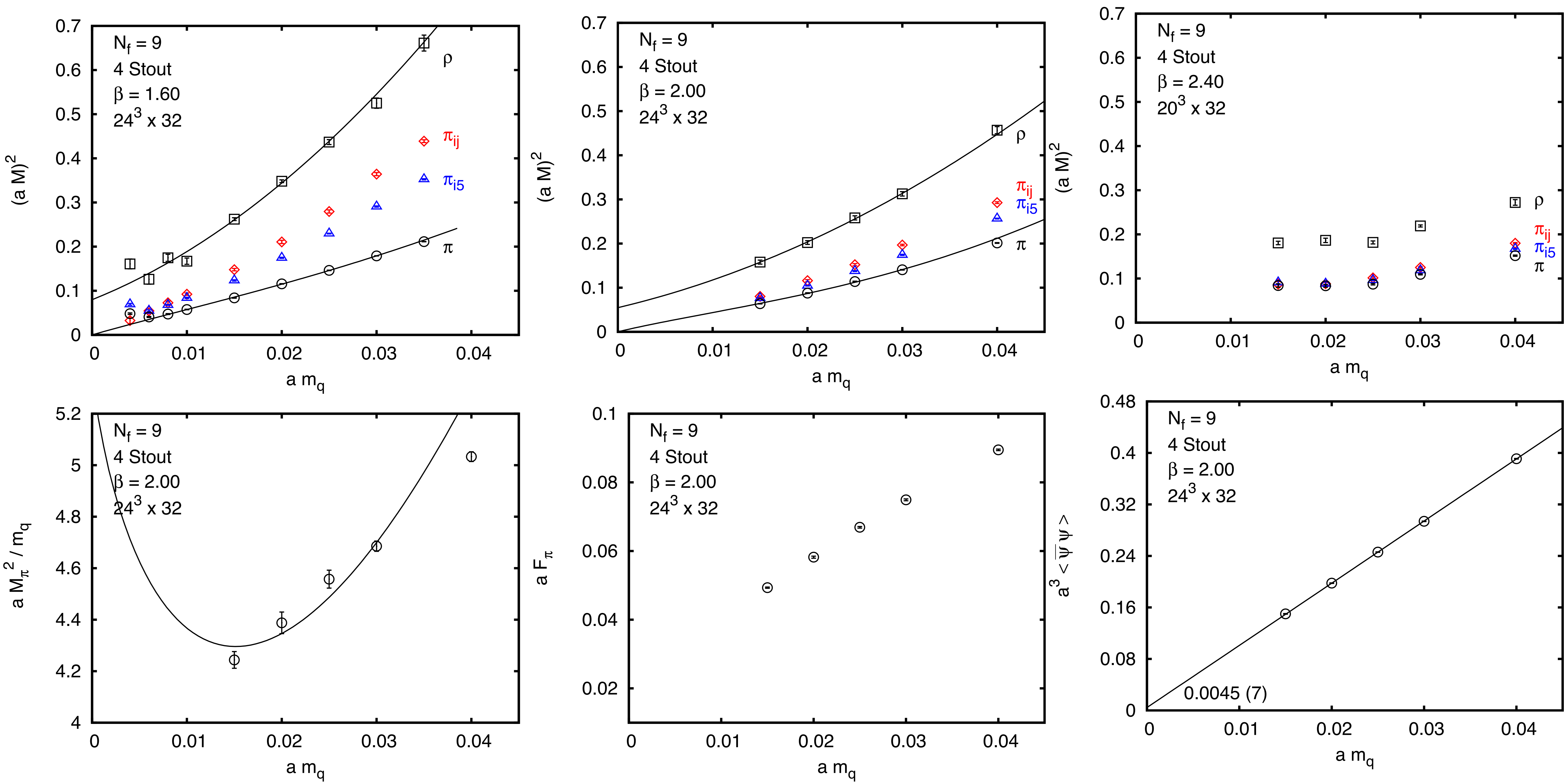
Nf=8 NLO chiral analysis in p-regime:



$$\frac{(m^{1-\mathrm{loop}}_{\pi_5^+})^2}{2m} = \mu\Bigg\{1+\frac{1}{16\pi^2 f^2}\Bigg(\frac{2}{N_F}\Big[\ell(m^2_{\eta'_V})-\ell(m^2_{\pi_V})\Big] + \frac{2}{N_F}\Big[\ell(m^2_{\eta'_A})-\ell(m^2_{\pi_A})\Big]+\frac{1}{2N_F}\ell(m^2_{\pi_I})\Bigg) + \frac{16\mu}{f^2}(2L_8-L_5)(2m)+\frac{32\mu}{f^2}(2L_6-L_4)(4N_F m)+a^2C\Bigg\}$$

Nf=8 staggered NLO

Nf=9 NLO chiral analysis in p-regime:

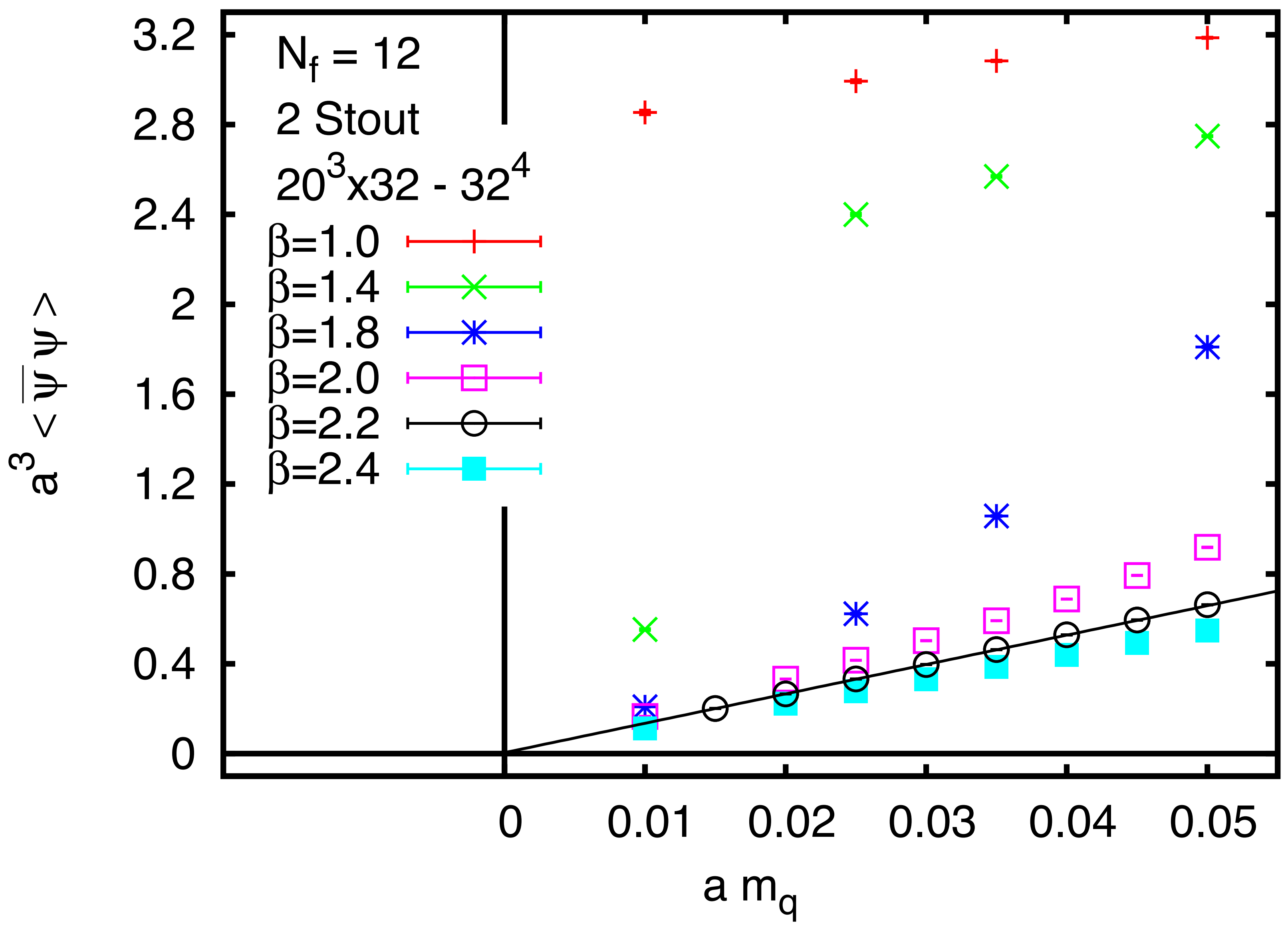


Testing rooting (nothing unusual happens)

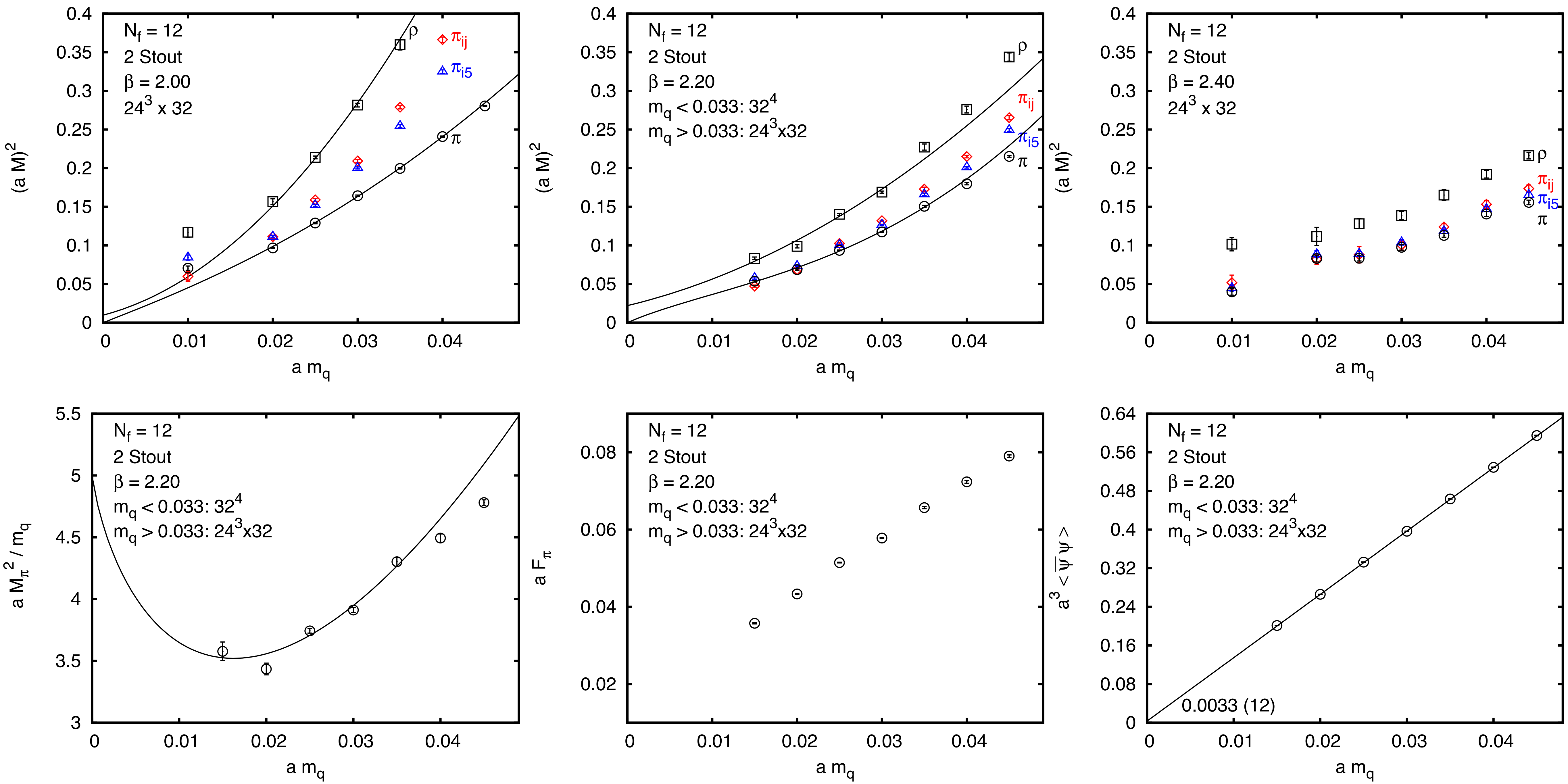
(useful for rooted sextet code, complete and running with Nf=2)

Provides additional independent info on chiral condensate trend

Nf=12 runs are far away from crossover region



Nf=12 NLO chiral analysis in p-regime:



Similar pattern to Nf=8 case!

All features exhibit chiral symmetry breaking

more work is needed

Some features of Nf=4,8,9,12 runs:

Nearly degenerate Goldstone spectra

stout action performs very well

Chiral condensate measured in F unit is enhanced as Nf increases

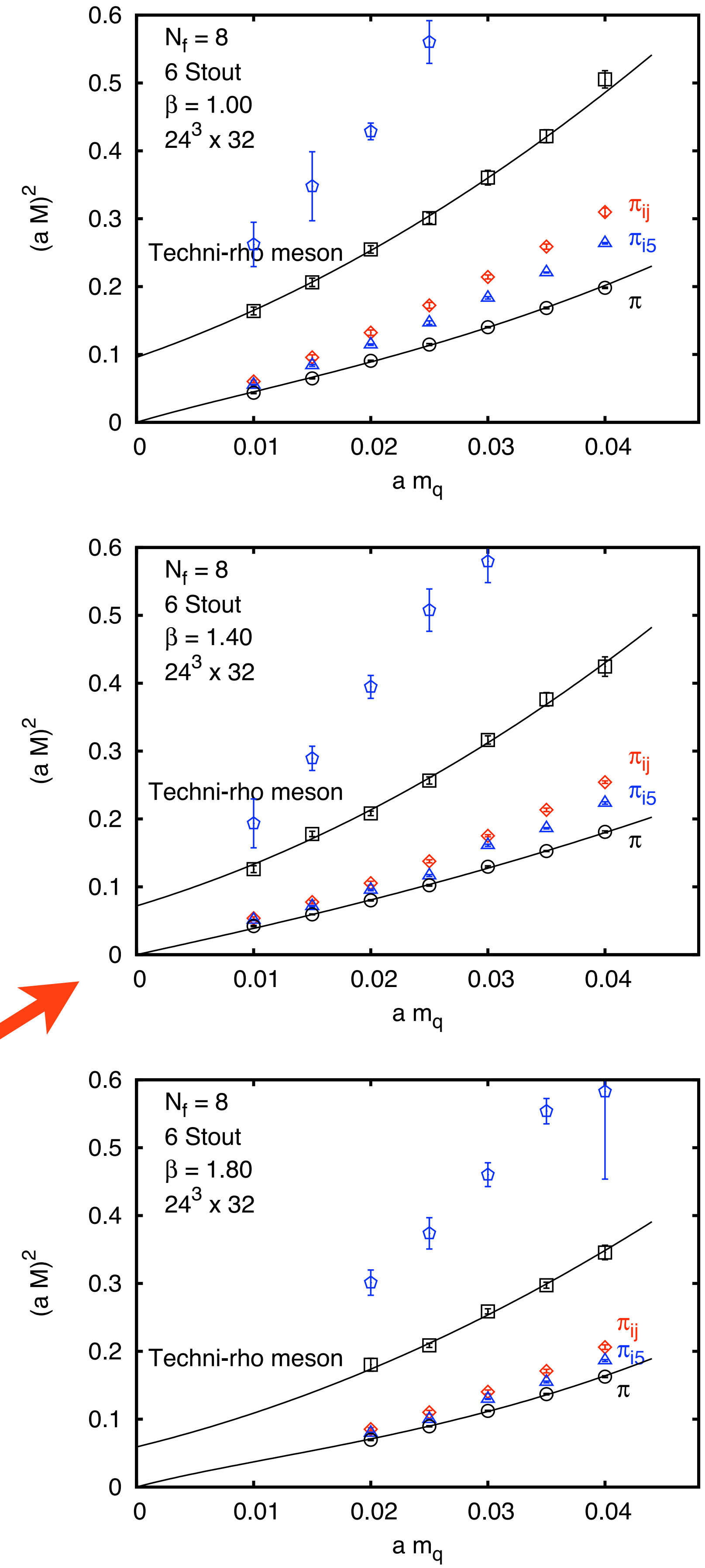
$$\sim \frac{<\bar{\psi}\psi>^{1/3}}{F}$$

renormalization scale is not set

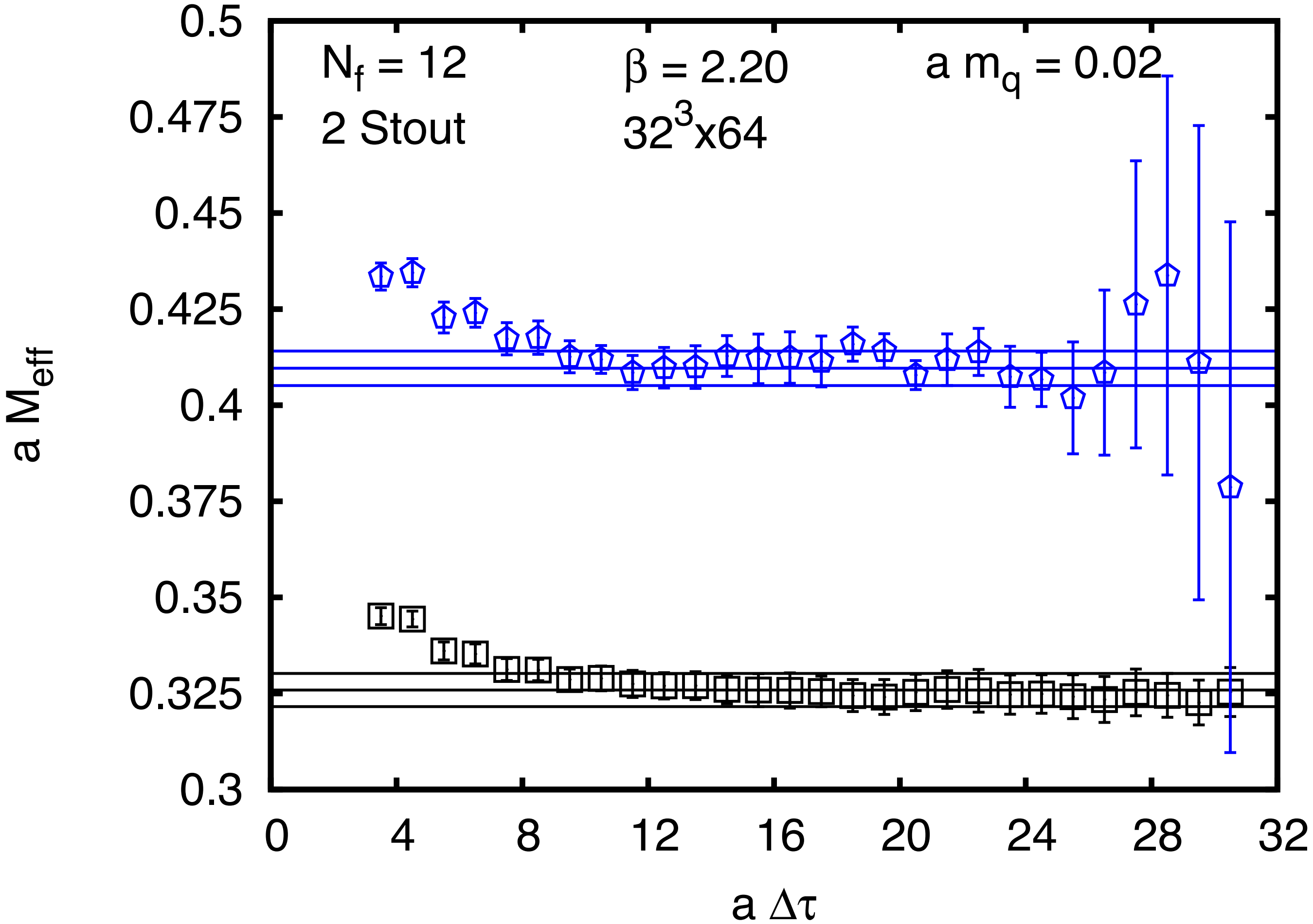
Nf=4	B/F = 53(6)
Nf=8	B/F = 157(17)
Nf=9	B/F = 125(19)
Nf=12	B/F = 209(64)

large errors, preliminary, limited to Ls=32!

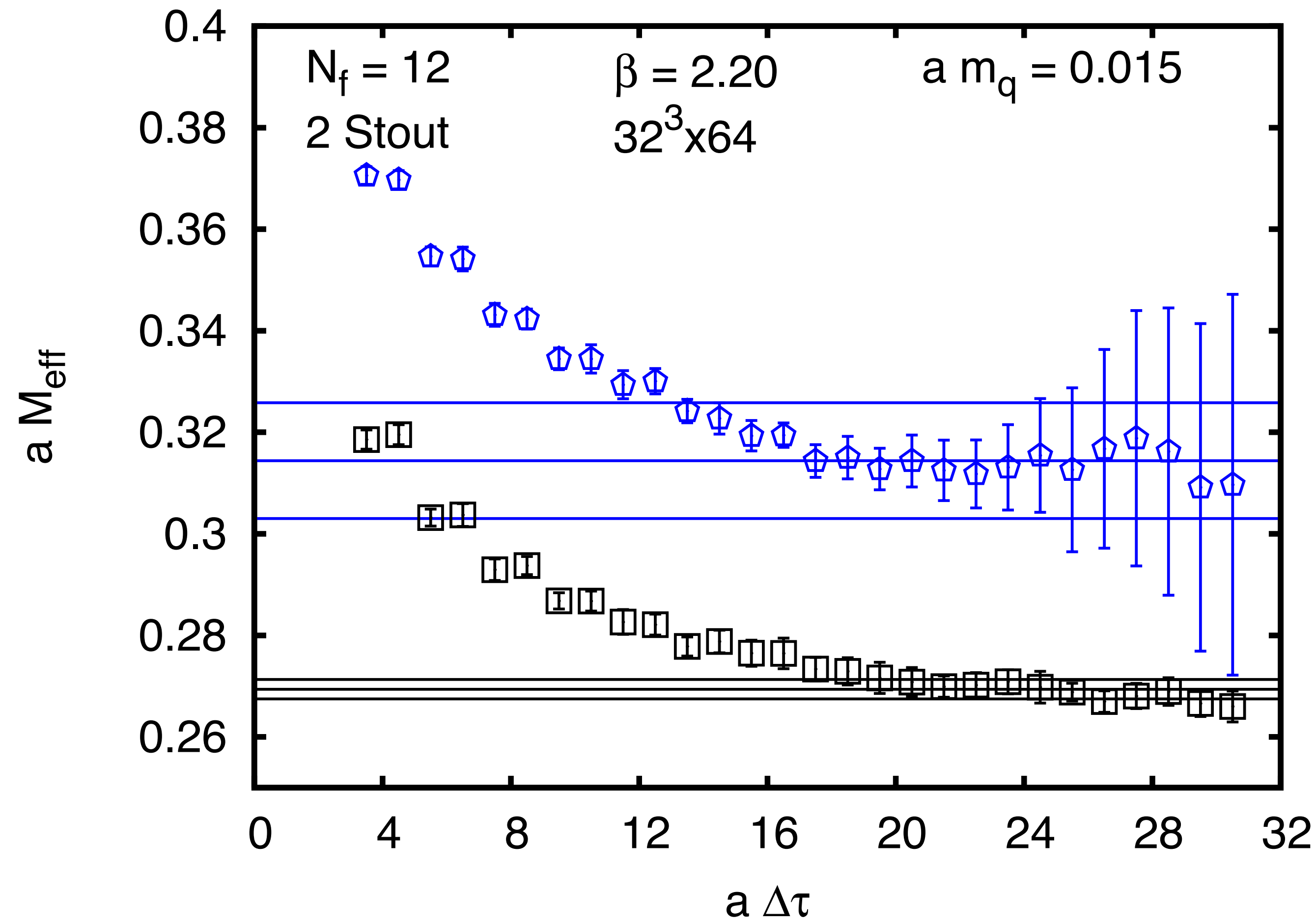
rho - A1 splitting



Better separtaion of rho and Goldstones at Nf=12 would require bigger runs at smaller fermion masses

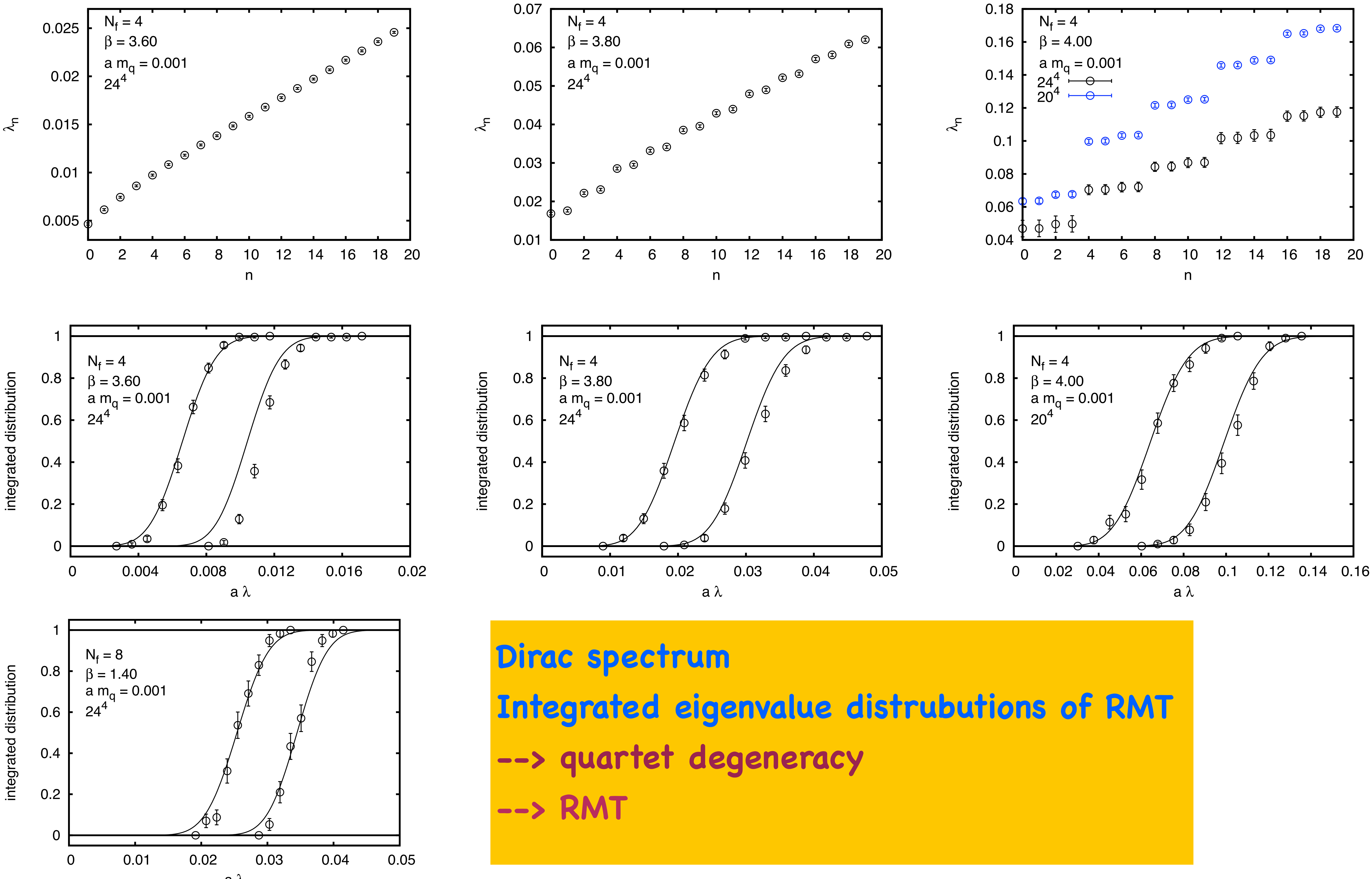


Nf=12 mq=0.02 rho-A1 splitting
pulled out from single correlator
with two parity partners



Nf=12 mq=0.015 rho-A1 splitting
pulled out from single correlator
with two parity partners

Random Matrix Theory tests in epsilon regime:



Dirac spectrum

Integrated eigenvalue distrubutions of RMT

--> quartet degeneracy

--> RMT

First conclusions on our Nf sequence:

Nf=4,8,9,12 all appear to be in chirally broken phase according to several tests:

1. chiral Goldstone dynamics
2. nonvanishing condensate in chiral limit
3. rho-A1 parity doublet splitting close to chiral limit
4. epsilon regime and RMT
5. string tension and running coupling from potential/force ?

Important warning is appropriate related to the size of F*L

Preliminary indications on our Nf=2 sextet model:

It appears to be in chirally broken phase according to several tests like the ones discussed earlier:

1. chiral Goldstone dynamics
2. nonvanishing condensate in chiral limit
3. rho-A1 parity doublet splitting close to chiral limit ?
4. epsilon regime and RMT ?
5. string tension and running coupling from potential/force ?

More favorable to reach large enough F*L values

When is F*L large enough?

This can be quantified

(epsilon, delta and p regimes are all connected)

$$E_l = \frac{1}{2\theta} l(l+2) \text{ with } l = 0,1,2,... \quad \text{rotator spectrum for SU(2)}$$

$$\text{with } \theta = F^2 L_s^3 (1 + \frac{C(N_f = 2)}{F^2 L_s^2} + O(1/F^4 L_s^4)) \quad \text{(P. Hasenfratz and F. Niedermayer)}$$

there is overall factor $\frac{N_f^2 - 1}{N_f}$ for arbitrary N_f

$C(N_f = 2) = 0.45$ expected to grow with N_f

At $FL_s = 0.8$ the correction is 70% and grows with N_f

When expansion collapses in $\delta -$ regime, the p-regime analysis needs more scrutiny

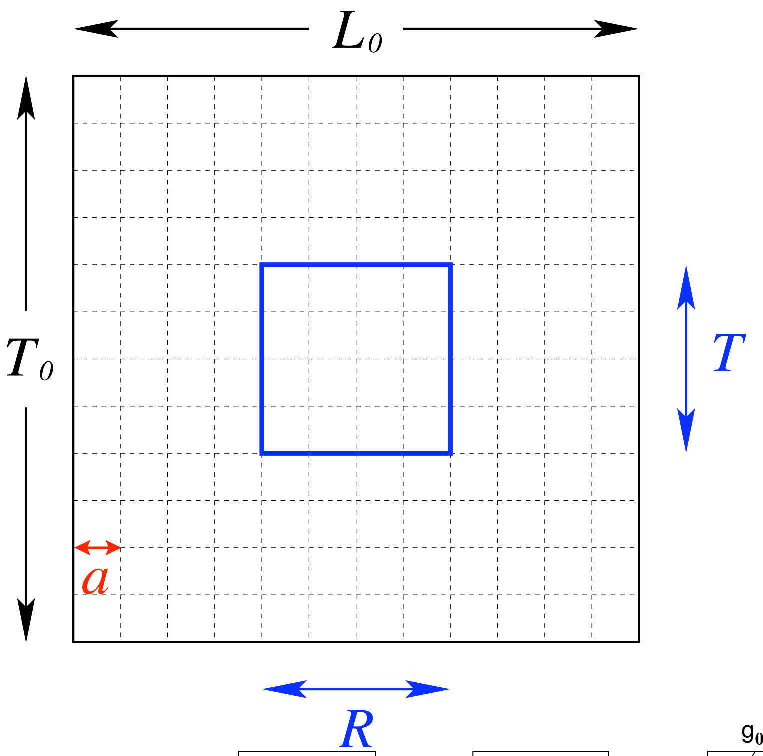
Cross checks from several running coupling schemes is important

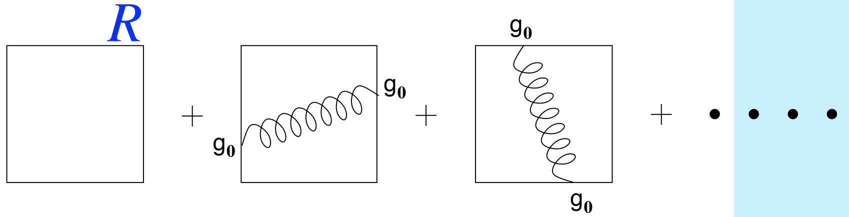
Running gauge coupling from RG on large Wilson loops

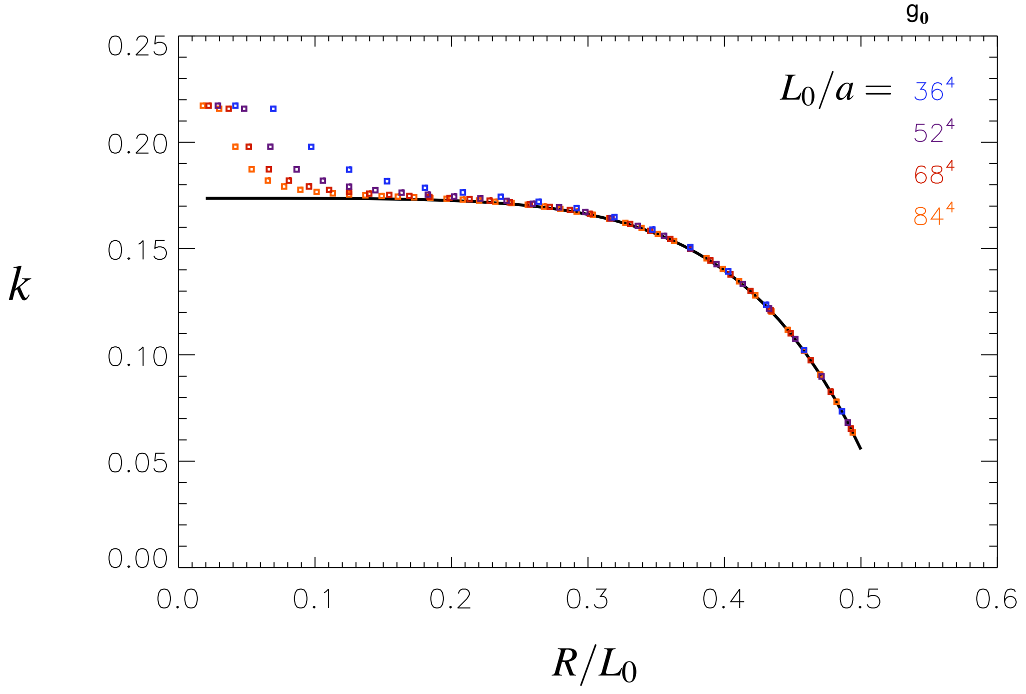
two groups: our group and Bilgici et al. (generalization from earlier work)

problem with Bilgici et al: implementation was not independent of Schrodinger functional method (corrected now)

Important that our implementation is



$W(R,T;L_0,T_0;a;g_0) =$ 



define renormalized coupling from second derivative of Wilson loops running with L if R/L is kept fixed:

$$k \cdot g_R^2\left(L_0, \frac{R}{L_0}\right) = -R^2 \frac{\partial^2}{\partial R \partial T} \ln\langle W(R,T;L_0,T_0)\rangle\Big|_{T=R}$$

k is geometric factor (cutoff dependent on lattice) defined from tree level relation with the bare coupling g_0

$$-R^2 \frac{\partial^2}{\partial R \partial T} \ln\langle W(R,T;L_0)\rangle^{\text{tree}}\Big|_{T=R} = k g_0^2$$

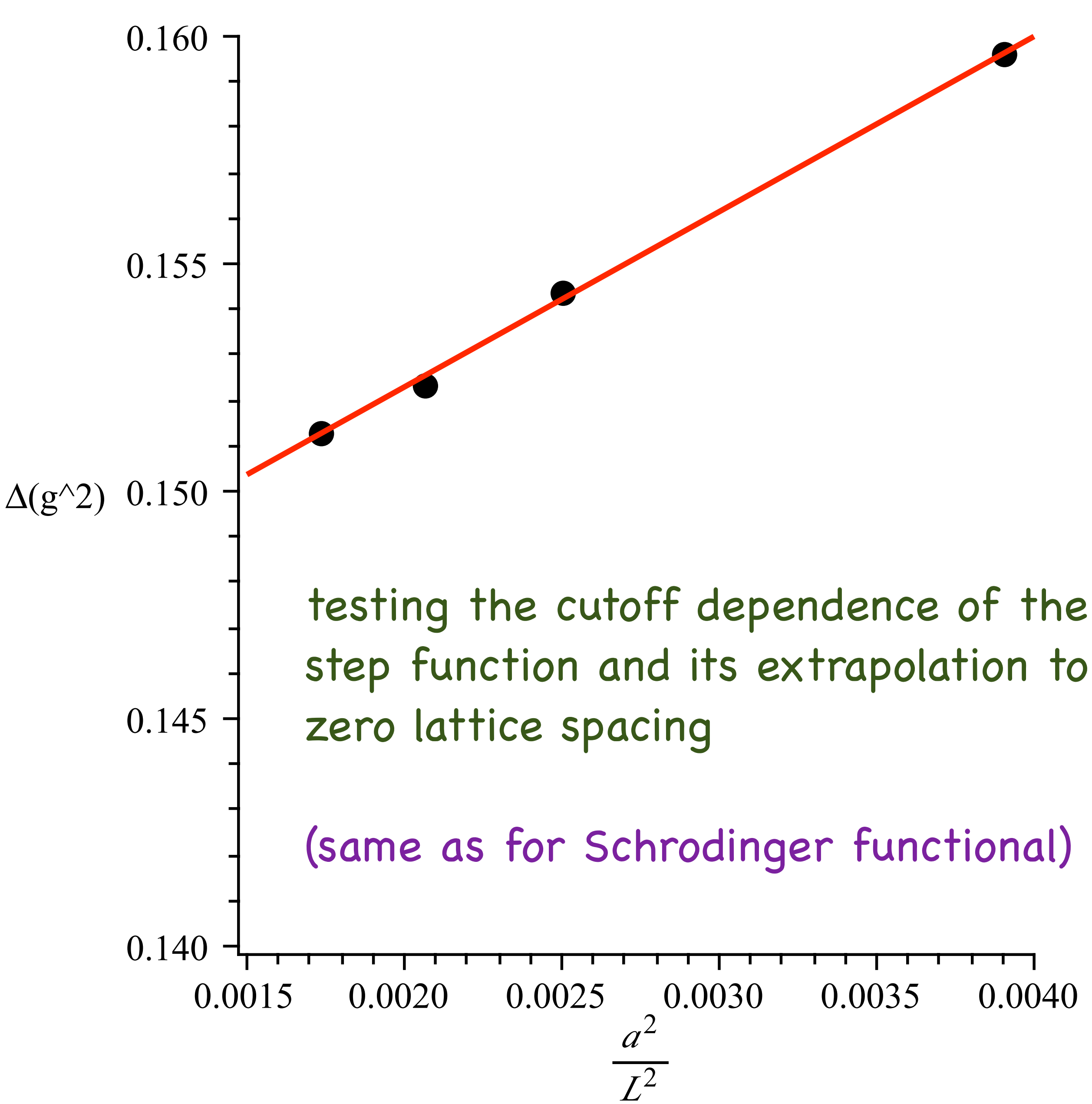
Lattice implementation requires the study of the step function together with its cutoff dependence

Useful alternative to Schrodinger functional?

Wilson loops could be replaced by Polyakov loop correlators

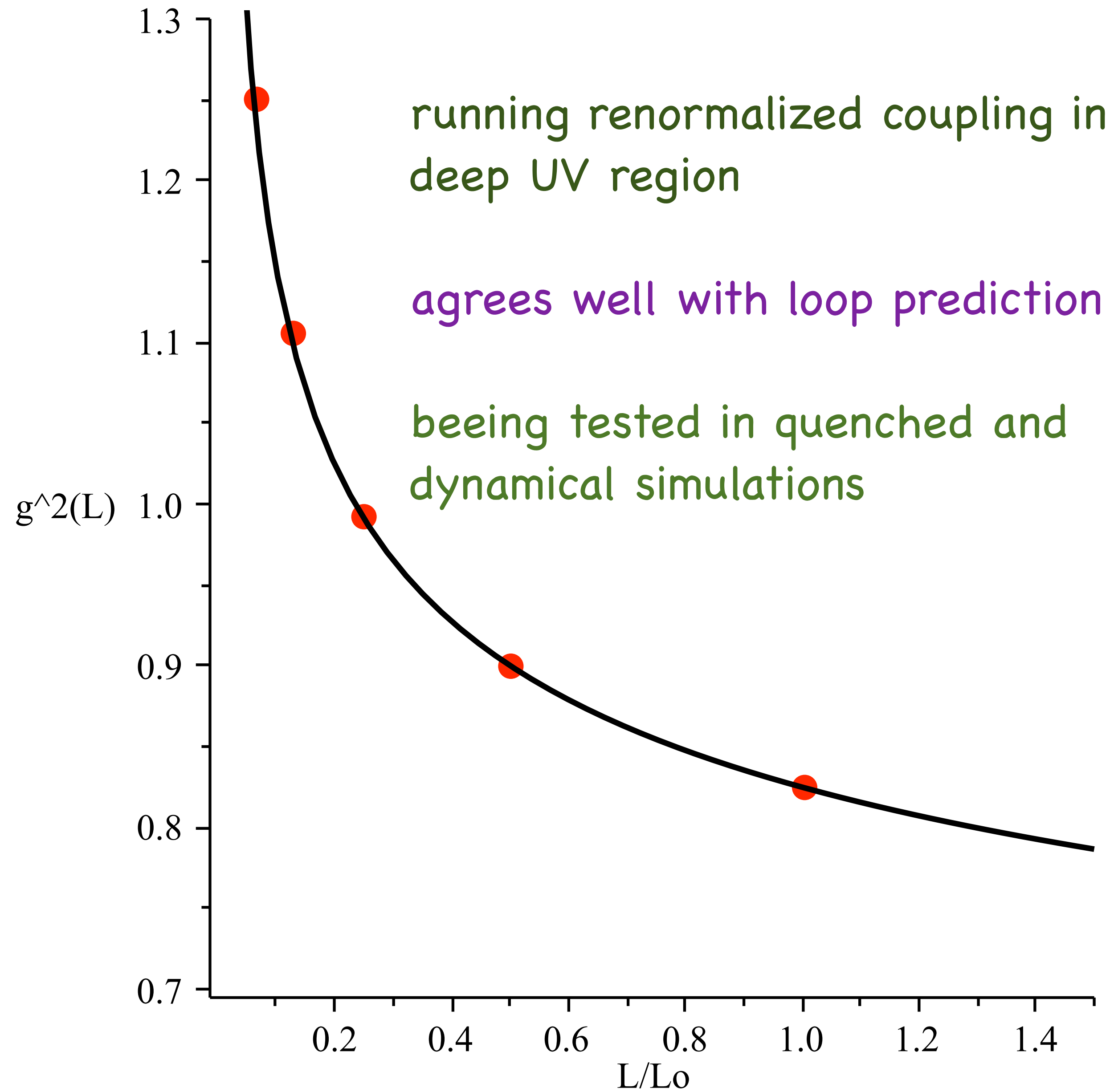
We first tested the method at weak coupling for large Wilson loops. Rather than simulating Wilson loops with Monte Carlo, we calculated (simulated) them analytically using the boosted coupling procedure of Lepage and McKenzie which reproduces even large Wilson loops accurately

The finite volume dependence was obtained from Urs Heller's code who calculated the Wilson loops in bare perturbation theory
(thanks to Urs Heller and Paul Mackenzie for the help they provided)



testing the cutoff dependence of the step function and its extrapolation to zero lattice spacing

(same as for Schrodinger functional)



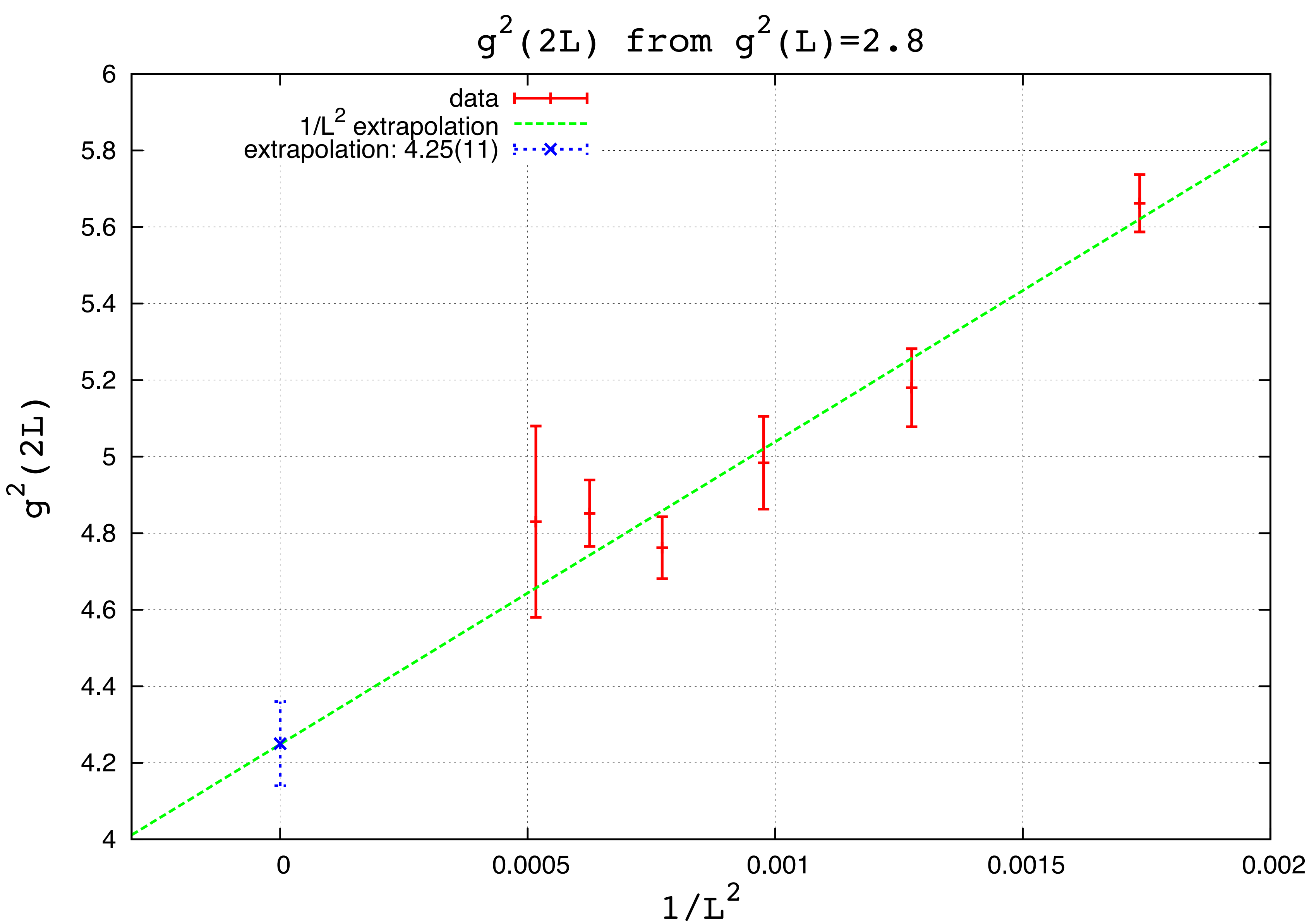
running renormalized coupling in the deep UV region

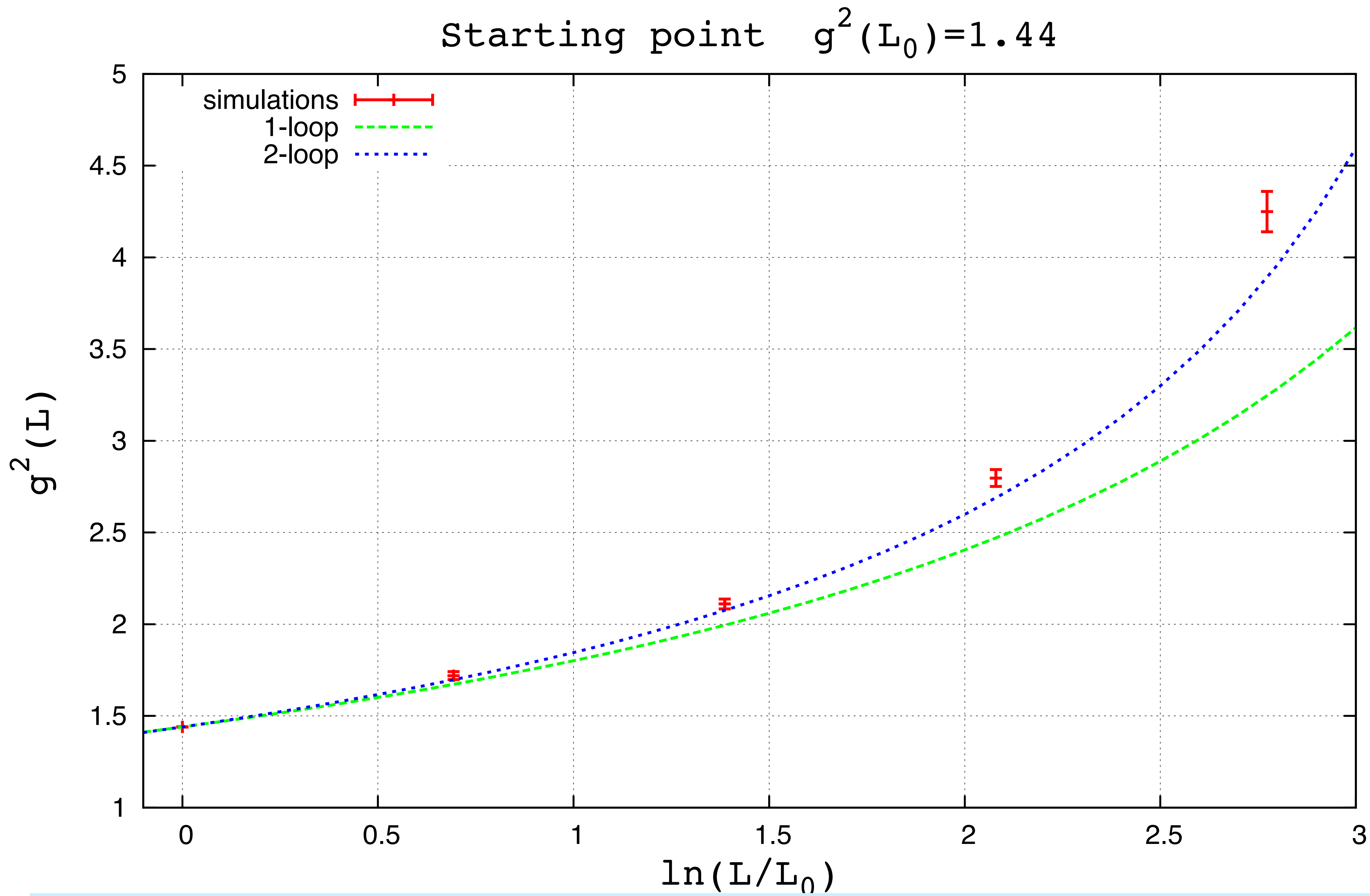
agrees well with loop prediction

beeing tested in quenched and dynamical simulations

Onto Monte Carlo now -->

Quenched SU(3) simulation extrapolating the step function renormalized coupling to zero cutoff at fixed finite physical box size:





The running coupling of our Wilson-Creutz scheme

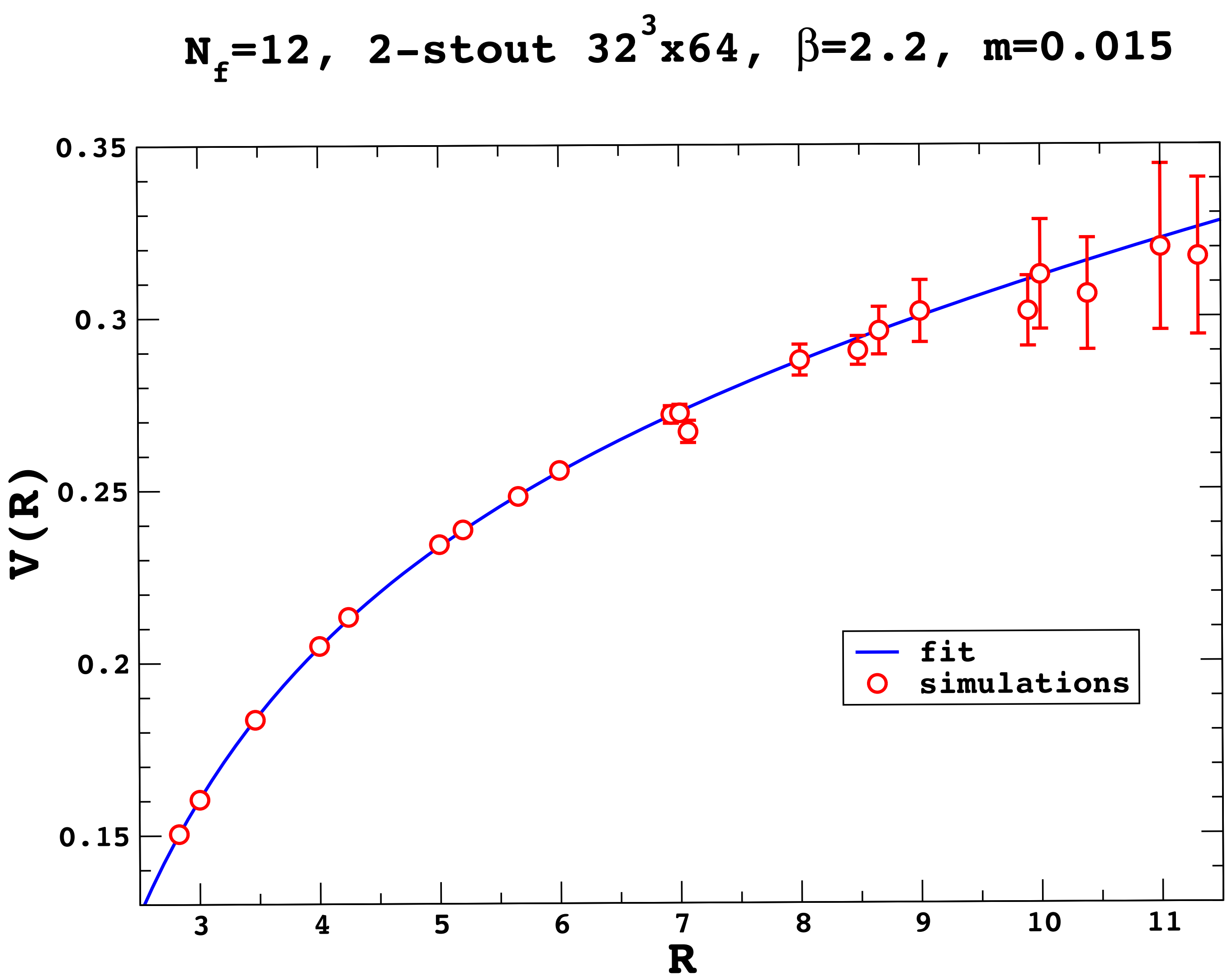
In quenched SU(3) simulation renormalized coupling is running with physical box size L without lattice cutoff effects

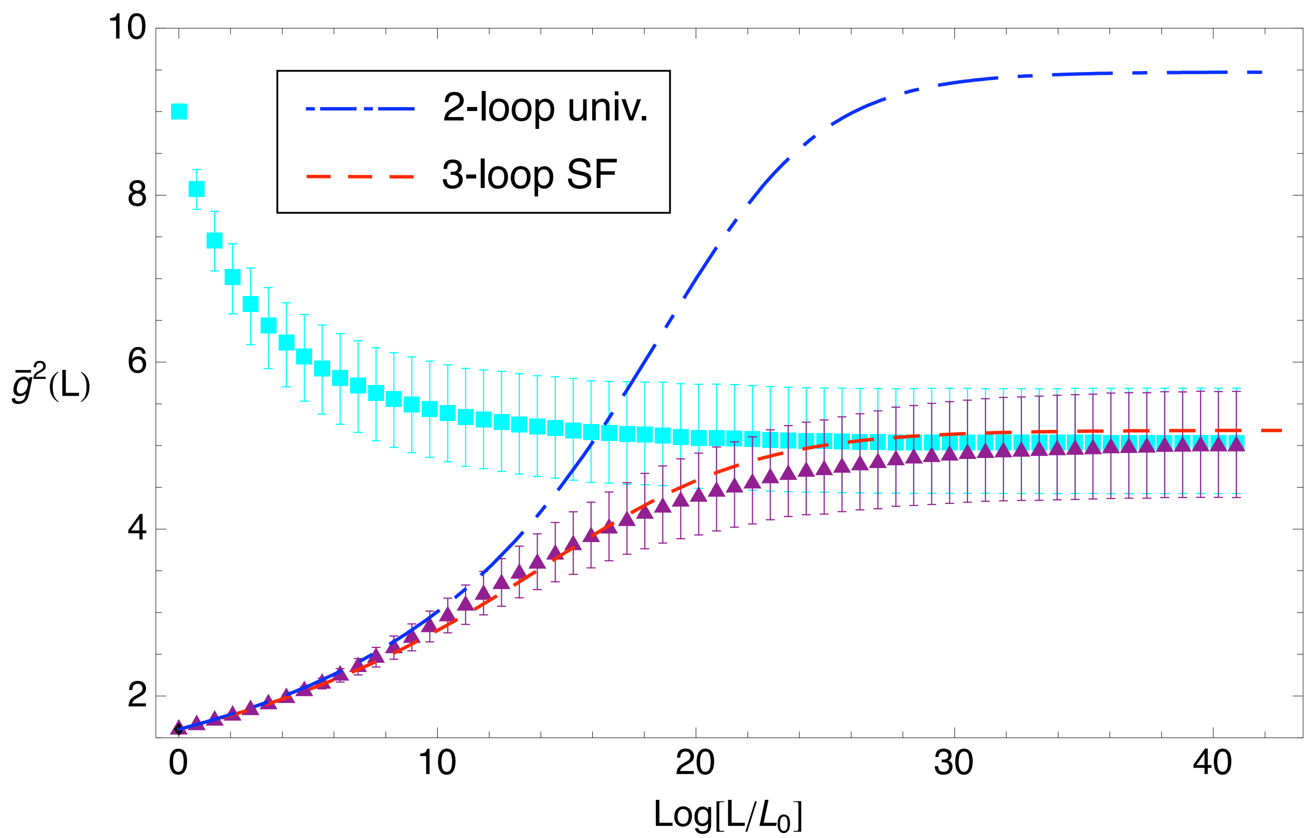
Onto dynamical fermions: fairly strong cutoff effects but no sign of Nf=12 conformal fixed point

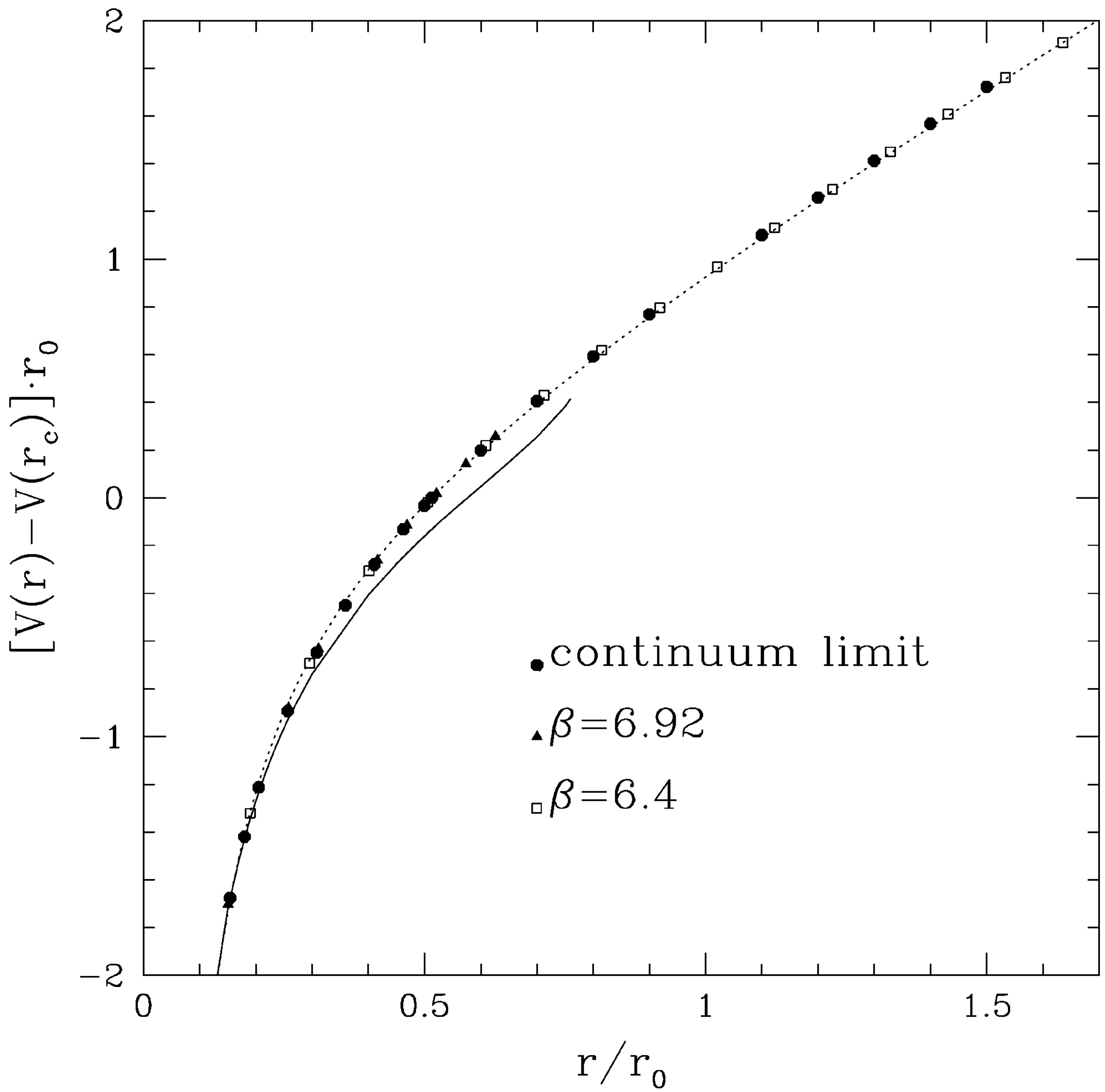
Our new calculation:

use V(r) potential and F(r) force to get traditional running g(r) in several schemes just like in QCD

Looks very promising







Quenched test works Necco-Sommer

$$F(r) = \frac{C_F \bar{g}^2_{qq}(r)}{4\pi r^2}, \qquad C_F = \frac{4}{3},$$

$$-r\frac{d}{dr}\bar{g}_{qq} = \beta(\bar{g}_{qq}) = -\sum_{\nu=0}^{2} b_\nu \bar{g}_{qq}^{2\nu+3}, \qquad b_0 = \frac{11}{16\pi^2}, \qquad b_1 = \frac{102}{(16\pi^2)^2},$$

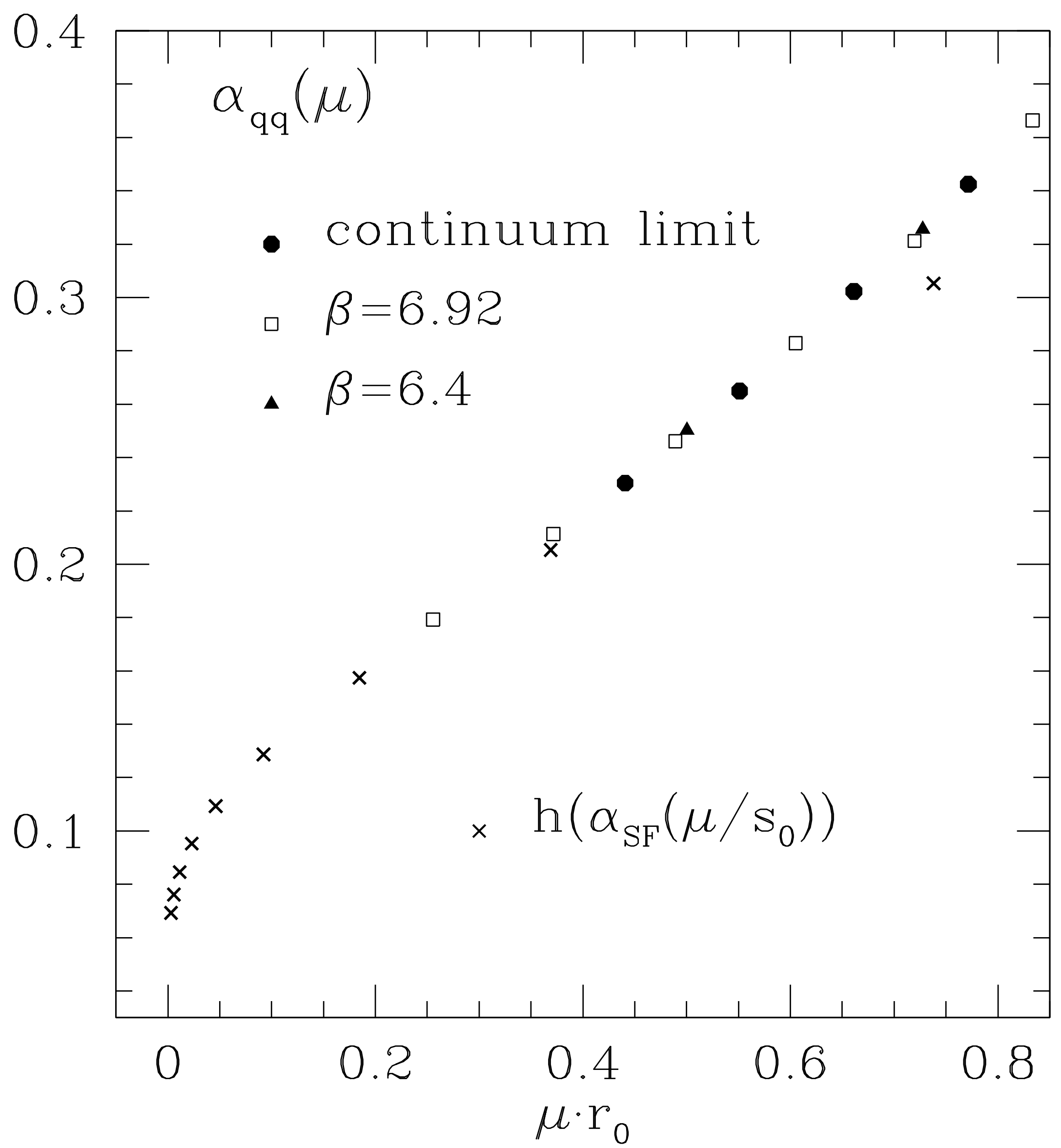
$$b_2 = \frac{1}{(4\pi)^6}\left(-3470 + 2519\frac{\pi^2}{3} - 99\frac{\pi^4}{4} + 726\zeta(3)\right).$$

$$F(r) = \sigma + \frac{\pi}{12r^2}, \qquad \sigma r_0^2 = 1.65 - \frac{\pi}{12}$$

Infinite volume, continuum extrapolated

limited r/ro range between 0.15 and 0.3

we try to run with the volume!



Running coupling from force and SF running nicely match for Nf=0 and Nf=2

we have difficulties to match the running of the two different couplings at Nf=12 in the relevant coupling range

Inside the conformal window Nf=16 case study

Nf=16 is most accessible to analysis

What is the finite volume spectrum?

How does the running coupling $g^2(L)$ evolve with L?

From 2-loop beta function $g^{*2} \approx 0.5$

$$g^2(L) \to g^{*2}, \text{ as } L \to \infty$$

Nontrivial small volume dynamics in QCD turns into large volume dynamics around weak coupling fixed point of conformal window

At small $g^2(L)$ the zero momentum components of the gauge field dominate the dynamics: Born-Oppenheimer approximation

Originally it was applied to pure-gauge system Luscher, van Baal

SU(3) pure-gauge model: 27 inequivalent vacua

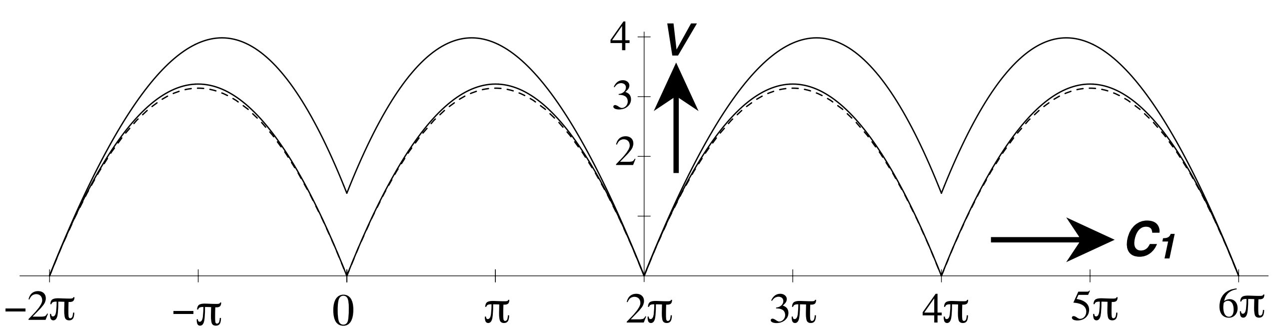
Low excitations of Hamiltonian (Transfer Matrix) scale with $\sim g^{2/3}(L)/L$
will evolve into glueball states for large L

Three scales of dynamics   on smallest scale WF is localized on one vacuum
tunneling accross vacua on second scale
over the barrier: confinement scale (third)

$A_i(\mathbf{x}) = T^a C_i^a / L$ <-- zero momentum mode of gauge field

For $SU(3)$, $T_1 = \lambda_3/2$ and $T_2 = \lambda_8/2$

$$V_{\rm eff}^{\mathbf{k}}(\mathbf{C}^b) = \sum_{i>j} V(\mathbf{C}^b[\mu_b^{(i)} - \mu_b^{(j)}]) - N_f \sum_i V(\mathbf{C}^b \mu_b^{(i)} + \pi\mathbf{k}) \qquad \mu^{(1)} = (1, 1, -2)/\sqrt{12} \text{ and } \mu^{(2)} = \tfrac{1}{2}(1, -1, 0)$$



Effective potential shows the effects of massless fermions

Fermions develop a gap in the spectrum ~ 1/L   k=(0,0,0) periodic
k=(1,1,1) antiperiodic

van Baal

Quantum vacuum is at minimum of Veff(C) when massless fermions are turned on

early work by van Baal, Kripfganz and Michael

Fermions develop a gap $\sim \pi/L$ in the spectrum

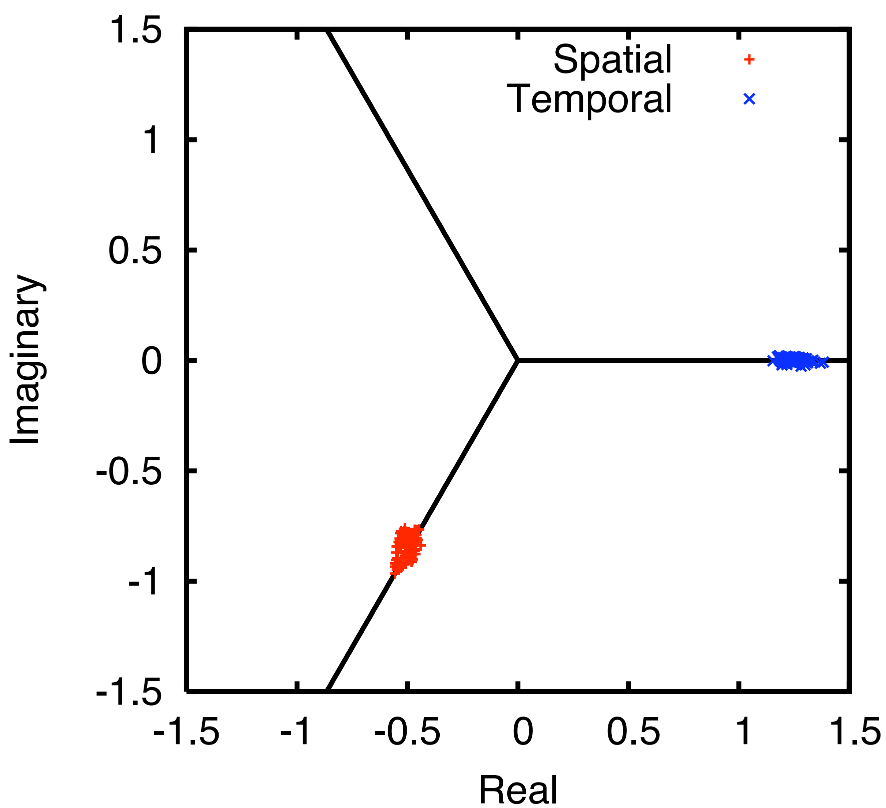
$$\mu_b^{(n)}\mathbf{C}^b = 2\pi\mathbf{l}/N \ (\mathrm{mod}\ 2\pi)$$

$$V_{\mathrm{eff}}^{\mathbf{k}} = -N_f N \breve{V}(2\pi\mathbf{l}/N + \pi\mathbf{k})$$

k=(1,1,1) antiperiodic   minimal when l=0 (mod 2π) A=0

k=(0,0,0) periodic   minimal when $\vec{l} \neq 0$ nontrivial vacua

Polyakov loop distributions probe the vacua



$$P_j = \frac{1}{N}\,\mathrm{tr}\left(\exp(iC_j^b T_b)\right) = \frac{1}{N}\sum_n \exp(i\mu_b^{(n)} C_j^b) = \exp(2\pi i l_j/N)$$

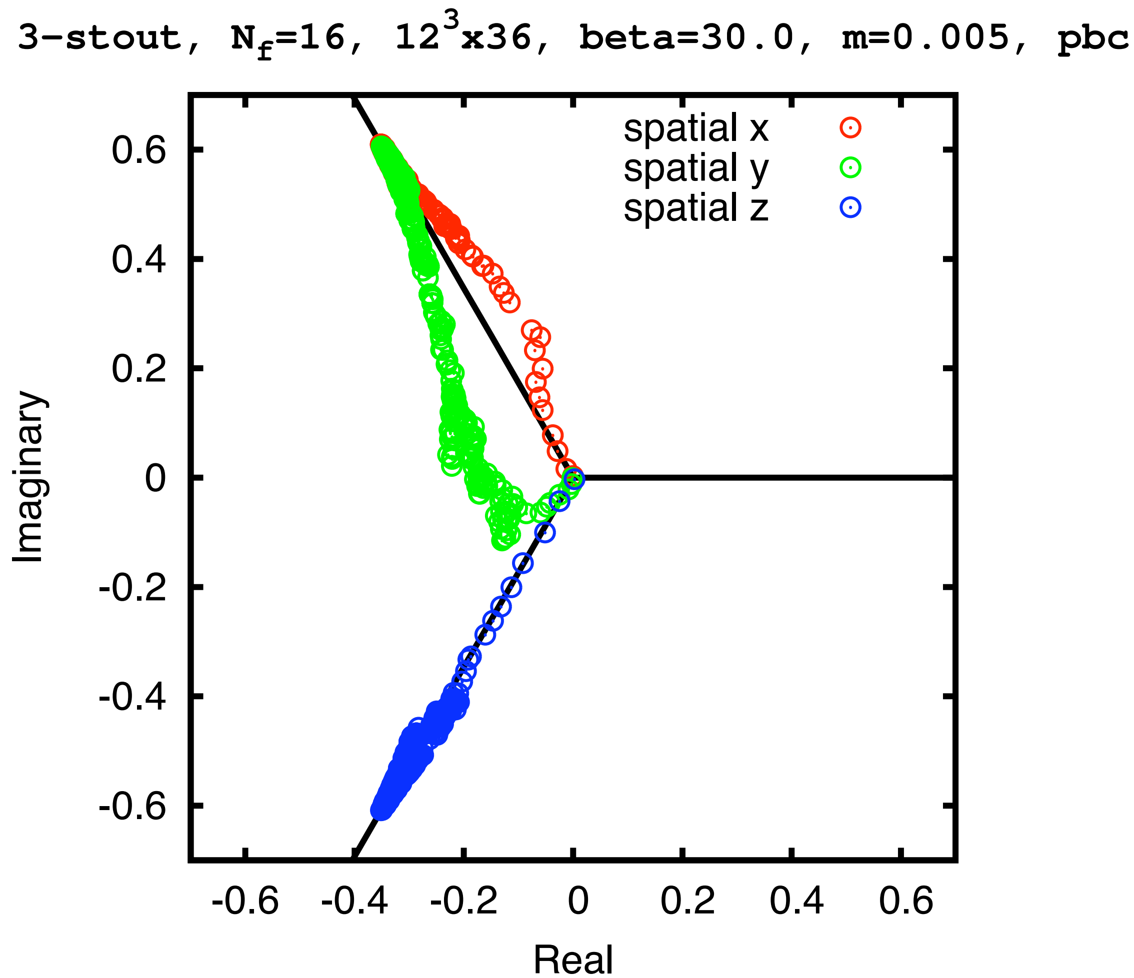
k=(0,0,0) antiperiodic $A = 0 \ (P_j = 1)$

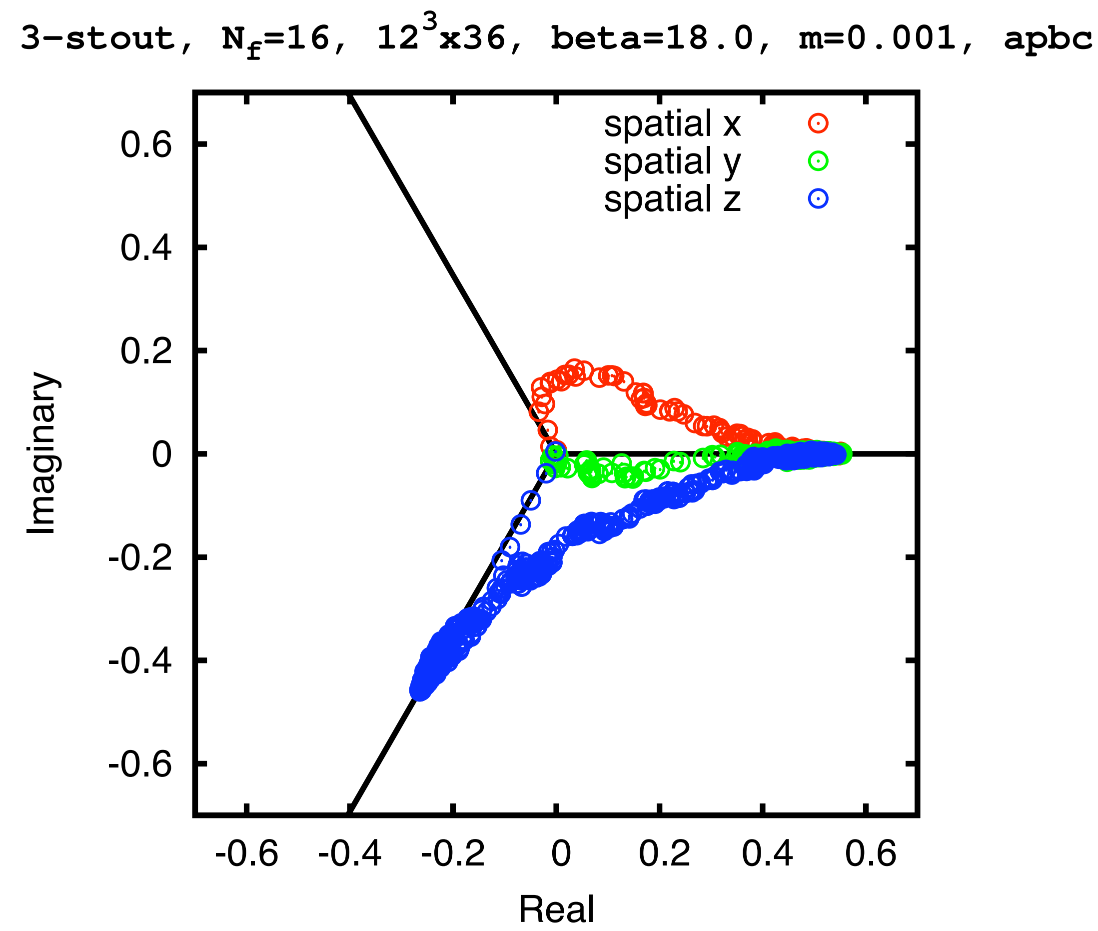
k=(0,0,0) periodic $P_j = \exp(\pm 2\pi i/3)$

16^4 lattice simulation at $\beta = 18$

If there is strong coupling inside the conformal window, transition over the barrier into third regime (confinement in QCD) where this picture qualitatively changes

Nf=16 inside conformal window
femto volume and tunneling volume





Conclusions and Outlook

- Our focus shifted to Nf=10-16 range (and beyond?)

  Nf=12 chiral symmetry breaking and running coupling
  from V(r) and F(r) and Wilson loops

- Nf=12 might be close enough to realize walking technicolor but otherwise (like the S-parameter) it is not unlikely to fail

- What is the fate of the Nf=2 sextet model? Next controversy is brewing? Walking Nf=2 sextet would be a favorite candidate for composite Higgs (mass generation?)

- Zero mode dynamics important at weak coupling inside conformal window

- Reliable EW precision quantities (S/T/U) will be important to get accurately once we settle on the candidate model(s)