Non-Static Extra Dimensions

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- Extra dimensions:
  (1) Large (ADD)
  (2) Warped (RS)
  (3) Infinite: (DGP) or (RS II)
  (4) Fractional (Chris Hill)

- What if the compactified extra dimension is non-static?

What does it mean by "Non-Static"?

- Consider the metric:

$$d\tau^2 = \eta_{ij} dx^i dx^j + dt^2 + 2\, g(u)\, dt\, du - h(u)\, du^2$$

The extra dimension is compactified on S^1 with $L = 2\,\pi\, R$
$\Rightarrow\ u \sim u + L$

- The metric needs to reflect the periodicity:

$$\begin{aligned} g(u+L) &= g(u) \\ h(u+L) &= h(u) \end{aligned}$$

Otherwise two identified points will have different metrics
Simplest case: both g and h are constants

Why Consider Non-Static Extra Dimensions?

- At the beginning, just for curiosity
- But we found unusual (crazy or interesting?) consequences
- This is an exploration of what would happen if there is a non-static extra dimension
- Don't expect me to solve many theoretical issues

Number and Size of the Extra Dimensions?

- Cullen and Perelstein (PRL83,268,1999):
  Most energy from core-collapsed supernova is carried away by neutrinos. Emission of KK gravitons may lead to excessive cooling of the hot core.
- Agreement with neutrino observations implies the bounds:

$$\begin{array}{lll} n=2 & R \lesssim 10^{-7}\, m & M \gtrsim 50\, TeV \\ n=3 & R \lesssim 10^{-10}\, m & M \gtrsim 4\, TeV \\ n=4 & R \lesssim 10^{-11}\, m & M \gtrsim 1\, TeV \end{array}$$

- Add TWO more flat and static extra dimensions:

$$\begin{aligned} d\tau^2 &= \eta_{ij} dx^i dx^j + dt^2 + 2\, g(u)\, dt\, du - h(u)\, du^2 \\ &\quad -(dx^6)^2 - (dx^7)^2 \end{aligned}$$

Without affecting the discussions in the rest of this talk

What are $g(u)$ and $h(u)$?

- Any function with period L can be expressed in Fourier modes:

$$g(u) = g_0 + A - \sum_{n=1}^{\infty} \left\{ a_n \cos\left(\frac{2\pi\, n\, u}{L}\right) + b_n \sin\left(\frac{2\pi\, n\, u}{L}\right) \right\}$$

$A = \sum_{n=1}^{\infty} a_n$ and $g(u=0) = g_0$

- Determinant of metric tensor for one or three extra dimensions:

$$\mathrm{Det}[G_{\mu\nu}] = g^2 + h$$

For two or four extra dimensions, $\mathrm{Det}[G_{\mu\nu}] = -(g^2 + h)$

- Take $g^2 + h = 1$ for simplicity

Why don't we Diagonalize the Metric?

- Yes, we could:

$$
\begin{aligned}
d\tau^2 &= \eta_{ij}\, dx^i dx^j + d\bar{t}^2 - d\bar{u}^2 \\
d\bar{t} &\equiv dt + g(u)\, du \\
d\bar{u} &\equiv \sqrt{g^2(u) + h(u)}\, du \qquad (\text{recall } g^2 + h = 1)
\end{aligned}
$$

- The new coordinates:

$$
\begin{aligned}
\bar{u} &= u \\
\bar{t} &= t + (g_0 + A)\, u - \left(\frac{L}{2\pi}\right) \sum_{n=1}^{\infty} \left(\frac{1}{n}\right) \left\{ a_n \sin\left(\frac{2\pi\, n\, u}{L}\right) \right. \\
&\qquad \left. + b_n \left[1 - \cos\left(\frac{2\pi\, n\, u}{L}\right)\right] \right\}
\end{aligned}
$$

But $\bar{t}$ is Pathological!

- Recall:

$$\bar{t} = t + (g_0 + A)\, u - \left(\frac{L}{2\pi}\right) \sum_{n=1}^{\infty} \left(\frac{1}{n}\right) \left\{ a_n \sin\left(\frac{2\pi\, n\, u}{L}\right) + b_n \left[1 - \cos\left(\frac{2\pi\, n\, u}{L}\right)\right]\right\}$$

  A linear combination of a non-compact variable t and a compact variable u

- For fixed u, $\bar{t}$ is smooth and continuous, with domain $\bar{t} \in [\,-\infty,\, +\infty\,]$
- For fixed t, $u \sim u + L$ implies $\bar{t} \sim \bar{t} + (g_0 + A)\, L$, with domain $\bar{t} \in [\,0,\, (g_0 + A)\, L\,]$

Spinning Cosmic Strings

- The metric for a spinning cosmic string:

$$d\tau^2 = (dt + 4\,G\,J\,d\theta)^2 - dr^2 - (1 - 4\,G\,M)^2\,r^2\,d\theta^2 - dz^2$$

G = Newton's constant J = angular momentum per unit length M = mass per unit length

- Define $\tilde{t} = t + 4\,G\,J\,\theta$ and $\varphi = (1 - 4\,G\,M)\,\theta$:

$$d\tau^2 = d\tilde{t}^2 - dr^2 - r^2\,d\varphi^2 - dz^2$$

- Deser, Jackiw and 't Hooft (Ann.Phys.152,220,1984): $\tilde{t} = t + 4\,G\,J\,\theta$ is pathological because it is BOTH continuous and compactified

Classical Equations of Motion

- Recall:

$$d\tau^2 = \eta_{ij} dx^i dx^j + dt^2 + 2\, g(u)\, dt\, du - h(u)\, du^2$$

- Geodesic equations for t and u:

$$\begin{aligned} \ddot{t}(\tau) &= \frac{1}{2} \frac{-2g'h + gh'}{g^2 + h} \dot{u}^2 \\ \ddot{u}(\tau) &= -\frac{1}{2} \frac{(g^2 + h)'}{g^2 + h} \dot{u}^2 = 0 \end{aligned}$$

"dot" = $\frac{d}{d\tau}$ and "prime" = $\frac{d}{du}$

Recall $g^2 + h = 1$

- Initial conditions:
  $\dot{u}(\tau = 0) = \dot{u}_0$
  $\dot{t}(\tau = 0) = \gamma_0$
  $u(\tau = 0) = t(\tau = 0) = 0$
- Initial velocity of a particle in u-direction:

$$\beta_0 \equiv \frac{\dot{u}_0}{\gamma_0} = \left.\frac{du}{dt}\right|_{t=0}$$

- We obtain:

$$\begin{aligned} u(\tau) &= \dot{u}_0\,\tau \\ t(u) &= \left(\frac{1}{\beta_0} - A\right) u + \left(\frac{L}{2\pi}\right) \sum_{n=1}^{\infty} \left(\frac{1}{n}\right) \left\{ a_n \sin\left(\frac{2\pi\, n\, u}{L}\right) \right. \\ &\qquad \left. + b_n \left[1 - \cos\left(\frac{2\pi\, n\, u}{L}\right)\right] \right\} \end{aligned}$$

So What?

- Suppose that a particle leaves the brane at $u = 0$, traverses N times over the extra dimension and comes back to the brane
  $\Rightarrow \; u = N\,L$
- The time as measured by a stationary observer on the brane:

$$t(u = N\,L) = \left(\frac{1}{\beta_0} - A\right) N\,L \quad < 0 \quad \text{if} \;\; \beta_0 > \frac{1}{A}$$

$\Rightarrow$ Closed-Timelike Curve (CTC) !!!

$\Rightarrow$ Causality Violation:
(1) Locally flat spacetime
(2) Subluminal speed

This CTC is Unique because ...

- Matter content of infinite extend (unphysical)? NO!
- Violation of energy conditions (instability)? NO!

Null Energy Condition:	$T_{AB}\, l^A\, l^B \geq 0$
Weak Energy Condition:	$T_{AB}\, t^A\, t^B \geq 0$
Strong Energy Condition:	$T_{AB}\, t^A\, t^B \geq \frac{1}{2}\, T^A_A\, t^B\, t_B$
Dominant Energy Condition:	$T_{AB}\, t^A\, t^B \geq 0$
	and $T_{AB}\, T^B_C\, t^A\, t^C \leq 0$

Null vector: $G_{AB}\, l^A\, l^B = 0$ Timelike vector: $G_{AB}\, t^A\, t^B > 0$
$T^{AB} =$ energy-momentum tensor $= 0$

- Infinite blue-shift (instability)? NO!

$\Rightarrow$ Non-static extra dimension can lead to a physical and stable CTC!

Could there be any Signature at the LHC?

- At least a qualitative one
- Yes, but we need a model to produce the causality-violating particles ....

- Add the following 5D renormalizable Lagrangian density to standard model:

$$\mathcal{L}^{(5D)} = \frac{G^{AB}}{2}\,\partial_A\,\phi\,\partial_B\,\phi - \frac{m^2}{2}\,\phi^2 \\ -\sqrt{L}\,\alpha\,\phi\,H^\dagger\,H\,\delta(u) - L\,\lambda\,\phi^2\,H^\dagger\,H\,\delta(u)$$

G^{AB} = inverse metric　　A, B = 0,1,2,3,5
ϕ = scalar singlet　　H = SM Higgs doublet
SM particles are confined to the brane $\Rightarrow \delta(u)$

- Can be added:
  (1) ϕ is a tadpole term
  (2) ϕ^3 leads to unbounded Hamiltonian from below
  (3) ϕ^4 is non-renormalizable in 5D
  $\Rightarrow$ neglected

- First proposed by Zee:
  Impose $\phi \to -\phi$ symmetry and so ϕ can be a dark matter candidate
- Asano and Kitano (PRD81,0545026,2010):
  For 114 GeV $< M_h <$ 200 GeV, CDMSII excluded $m <$ 50 GeV

KK Decomposition

- Boundary condition $\phi(u+L)=\phi(u)$ allows the Fourier expansion:

$$\phi(x^\mu,u)=\sum_{n=-\infty}^{\infty}\phi_n(x^\mu)\,\frac{e^{i\,n\,u/R}}{\sqrt{L}}$$

$\phi_{-n}(x^\mu)=\phi_n^*(x^\mu)$ to ensure that $\phi(x^\mu,u)$ is a real field

- Use the Fourier representation of the delta function:

$$\delta(u)=\frac{1}{L}\sum_{q=-\infty}^{\infty}e^{i\,q\,u/R}$$

- Integrate $\mathcal{L}^{(5D)}$ over u to get $\mathcal{L}^{(4D)}$:

$$
\begin{aligned}
\mathcal{L}^{(4D)} \quad = \quad & \frac{1}{2} \sum_n \left\{ \partial_0 \, \phi_n \, \partial_0 \, \phi_n^* - \nabla \, \phi_n \cdot \nabla \, \phi_n^* \right. \\
& \left. - \left(m^2 + \frac{n^2}{R^2} \right) \phi_n \, \phi_n^* \right\} + \text{ugly terms} \\
& -\alpha \, H^\dagger \, H \sum_n \phi_n \\
& -\lambda \, H^\dagger \, H \left(\sum_n \phi_n \, \phi_n^* + \sum_{n_1, n_2 \neq n_1} \phi_{n_1} \, \phi_{n_2}^* \right)
\end{aligned}
$$

- EWSB occurs when Higgs gets a vev:

$$H^\dagger \, H \rightarrow \left(\frac{v + h}{\sqrt{2}} \right)^2$$

- We obtain:

$$\mathcal{L}^{(4D)} = \frac{1}{2}\sum_n \Big\{ \partial_0\,\phi_n\,\partial_0\,\phi_n^* - \nabla\,\phi_n\cdot\nabla\,\phi_n^* - \Big(m^2 + \frac{n^2}{R^2} + \lambda\,v^2\Big)\,\phi_n\,\phi_n^* \Big\} + \text{ugly terms}$$
$$-\frac{1}{2}\lambda\,v^2\left(\sum_{n_1,n_2\neq n_1}\phi_{n_1}\phi_{n_2}^*\right) - \alpha\,v\,h\sum_n\phi_n$$
$$-\lambda\,v\,h\left(\sum_n\phi_n\,\phi_n^* + \sum_{n_1,n_2\neq n_1}\phi_{n_1}\phi_{n_2}^*\right) + \ldots\ldots$$

Possible Physical Processes

- Mixing between h and ϕ:

$$-\alpha\, v\, h \sum_n \phi_n$$

- Higgs decaying into a pair of $\phi_n \phi_{-n}$ or $\phi_n \phi_n$:

$$-\lambda\, v\, h \left(\sum_n \phi_n \phi_n^* + \sum_{n_1, n_2 \neq n_1} \phi_{n_1} \phi_{n_2}^* \right)$$

Recall $\phi_n^* = \phi_{-n}$

$\delta(u)$ breaks translational invariance in the fifth dimension and so KK numbers need not be conserved
$\Rightarrow \; h\, \phi_{n_1}\, \phi_{n_2}^*$

- SM Higgs decays into a pair of $\phi_n \phi_{-n}$ or $\phi_n \phi_n$
  $\Rightarrow$ This is the primary vertex
- One of the ϕ_n's may traverse the compactified non-static extra dimension which forms the CTC to violate causality
  $\Rightarrow$ Missing energy at the LHC
- It will return to the brane at an earlier time, and may mix with the SM Higgs and produce a SM Higgs
  $\Rightarrow$ This is the secondary vertex

The Unique Signatures

- The secondary vertex can pre-appear before the primary vertex
- Energy is covariantly conserved (not shown):
  The two vertices are correlated by energy conservation
- The amount of missing energy detected at t will be observed at $t - \Delta t$

Too Crazy?

- I am sorry ....
- Let's do something usual!
- No more CTC!

- Consider:

$$d\tau^2 = \eta_{ij} dx^i dx^j + dt^2 + 2\, g_0\, dt\, du - h_0\, du^2$$

Assume that g_0 and h_0 are constants $\Rightarrow$ No more CTC

- Take $g_0^2 + h_0 = 1$ again
- Take $h_0 > 0$ to ensure that u is spacelike
  $\Rightarrow 0 < h_0 < 1$ and $|g_0| < 1$
- Without loss of generality, assume $0 < g_0 < 1$
- What would particle physics look like on this spacetime?

- Imposing the symmetry $\phi \leftrightarrow -\phi$ removes all terms odd in ϕ from $\mathcal{L}^{(5D)}$:

$$\mathcal{L}^{(5D)} = \frac{G^{AB}}{2} \partial_A \phi \, \partial_B \phi - \frac{m^2}{2} \phi^2 - L \lambda \phi^2 H^\dagger H \delta(u)$$

- What would be the dark matter candidate in $\mathcal{L}^{(5D)}$? zeroth KK mode?

4D Effective Lagrangian Density after EWSB

- Similar to previous consideration, we obtain:

$$
\begin{aligned}
\mathcal{L}^{(4D)} \;=\;& \frac{1}{2} \sum_n \Bigg\{ h_0\, \partial_0\, \phi_n\, \partial_0\, \phi_n^* - \nabla\, \phi_n \cdot \nabla\, \phi_n^* \\
& - \; (\, m^2 + \frac{n^2}{R^2} + \lambda\, v^2\,)\, \phi_n\, \phi_n^* - 2\, i\, g_0 \left(\frac{n}{R}\right) (\, \partial_0\, \phi_n\,)\, \phi_n^* \Bigg\} \\
& - \frac{1}{2}\, \lambda\, v^2 \left(\sum_{n_1, n_2 \neq n_1} \phi_{n_1} \phi_{n_2}^* \right) \\
& - \lambda\, v\, h \left(\sum_n \phi_n\, \phi_n^* + \sum_{n_1, n_2 \neq n_1} \phi_{n_1} \phi_{n_2}^* \right) + \ldots\ldots
\end{aligned}
$$

Dispersion Relation?

- Recall:

$$\mathcal{L}^{(4D)}_{\text{free}} = \frac{1}{2} \sum_n \Big\{ h_0\, \partial_0\, \phi_n\, \partial_0\, \phi_n^* - \nabla\, \phi_n \cdot \nabla\, \phi_n^* - (m^2 + \frac{n^2}{R^2})\, \phi_n\, \phi_n^* - 2\, i\, g_0 \left(\frac{n}{R}\right) (\partial_0\, \phi_n)\, \phi_n^* \Big\}$$

- Calculate Euler-Lagrange equation for ϕ_n^* and apply the Fourier transform $\phi_n(x^\mu) = \int d^4p\, \Phi_n(p^\mu)\, e^{-i\,E\,t}\, e^{i\,\vec{p}\cdot\vec{r}}$, and solve for E:

$$E = \frac{-g_0\,\frac{n}{R} + \sqrt{g_0^2 + h_0}\,\sqrt{h_0\,(p^2 + m^2) + \frac{n^2}{R^2}}}{h_0}$$

Negative root deleted, and recall $g_0^2 + h_0 = 1$

What's so Special about E?

- Recall that $0 < g_0, h_0 < 1$:

$$E = \frac{-g_0 \frac{n}{R} + \sqrt{h_0 \left(p^2 + m^2\right) + \frac{n^2}{R^2}}}{h_0}$$

The metric breaks reflection symmetry in u
$\Rightarrow$ E not symmetric in n

- Zeroth mode energy:

$$E_{n=0} = \frac{\sqrt{p^2 + m^2}}{\sqrt{h_0}}$$

- The term $-g_0 \frac{n}{R}$ gives a negative contribution for $n > 0$, so some $n > 0$ modes may have energy:

$$E_{n>0} < E_{n=0} \quad !!!$$

What is the Minimum of E?

- Recall that $0 < g_0, h_0 < 1$:

$$E = \frac{-g_0 \frac{n}{R} + \sqrt{h_0 \left(p^2 + m^2\right) + \frac{n^2}{R^2}}}{h_0}$$

- For a fixed momentum p, minimum E occurs at:

$$n_\star = g_0 \sqrt{p^2 + m^2}\, R > 0$$

For $\sqrt{p^2 + m^2} \sim 100$ GeV and $R \sim 10^{-10}$m, $n_\star \gg 1$

- Minimum E:

$$E_{n_\star} = \sqrt{p^2 + m^2}$$

What is $n_\star$ in our Model?

- Consider $h \to \phi_{n_\star}\, \phi_{n_\star}$. Energy conservation in the center-of-mass frame requires:

$$M_h = 2E_{n_\star} = 2\sqrt{p^2 + m^2}$$

- Recall: $n_\star = g_0 \sqrt{p^2 + m^2}\, R$
$\Rightarrow\ n_\star = \frac{1}{2}\, g_0\, M_h\, R,$ but this is generally not true!

- The $n_\star$ mode may not be kinematically allowed

- If $n_\star = \frac{1}{2}\, g_0\, M_h\, R$ happens to hold, it will be kinematically allowed
- OR maybe the $n_\star$ mode is kinematically allowed by some other models which produce it (not by SM Higgs decay)
- $n_\star$ mode as an unconventional DM candidate:
  $n_\star \gg 1$ in contrast to the $k = 1$ mode DM candidate in UED theories with KK parity $(-1)^k$

Decay Rates for Non-Static Extra Dimensions

- KK number conserving process:

$$\Gamma_{h\to\phi_n\,\phi_{-n}} = \frac{\lambda\,v^2}{16\,\pi\,M_h}\,\frac{1}{\sqrt{h_0}}\,\sqrt{1-\frac{4\,\bar{M}_n^2}{h_0^2\,M_h^2}}$$

$M_h =$ Higgs mass $\qquad \bar{M}_n^2 = h_0\,m^2 + \frac{n^2}{R^2}$

- KK number violating process:

$$\Gamma_{h\to\phi_n\,\phi_n} = \frac{\lambda^2\,v^2}{8\,\pi\,M_h}\,\frac{1}{\sqrt{h_0}}\,\sqrt{1-\frac{4\,\bar{M}_n^2}{\left(h_0\,M_h+\frac{2\,g_0\,n}{R}\right)^2}}$$

In the limit $g_0 \to 0$ and $h_0 \to 1$, one gets back the usual results for flat and static extra dimensions

- For flat and static spacetime, the calculation for Γ involves the Lorentz invariant integral:

$$\int d^4p\,\delta(\,G^{\mu\nu}\,p_\mu\,p_\nu - m^2\,)\;\theta(p_0) = \int \frac{d^3\vec{p}}{2\,E}$$

- For non-static spacetime, this must be promoted to a generally covariant integral:

$$\int \sqrt{|\mathrm{Det}(\mathrm{G_{AB}})|}\;d^4p\,\delta(\,G^{AB}\,p_A\,p_B - m^2\,)\;\theta(p_0)$$
$$= \int \frac{d^3\vec{p}}{2\,(\,h_0\,E_n + \frac{g_0\,n}{R}\,)}$$

Consider $h \to \phi_0 \, \phi_0$

- Higgs decaying into a pair of ϕ_0:

$$\Gamma_{h \to \phi_0 \, \phi_0} = \frac{\lambda^2 \, v^2}{8 \, \pi \, M_h} \, \frac{1}{\sqrt{h_0}} \, \sqrt{1 - \frac{4 \, m^2}{h_0 \, M_h^2}}$$

- This is possible only if:

$$M_h > \frac{2 \, m}{\sqrt{h_0}}$$

$\Rightarrow$ If $h_0 \ll 1$, M_h needs to much heavier than m to produce a pair of ϕ_0

- If Higgs is light, say $M_h \sim 120$ GeV, then even if $m \sim 50$ GeV and $\sqrt{h_0} \sim 0.5$ would forbid this process

Consider $h \longrightarrow \phi_{n_\star}\, \phi_{n_\star}$

- Higgs decaying into a pair of $\phi_{n_\star}$:

$$\Gamma_{h \to \phi_{n_\star}\, \phi_{n_\star}} = \frac{\lambda^2\, v^2}{8\,\pi\, M_h} \sqrt{1 - \frac{4\, m^2}{M_h^2}}$$

Are $h \to \phi_0\, \phi_0$ and $h \to \phi_{n_\star}\, \phi_{n_\star}$ subdominant?

- Comparison to $\Gamma_{h\to\tau^+\tau^-}$:

$$\frac{\Gamma_{h\to\phi_0\,\phi_0}}{\Gamma_{h\to\tau^+\tau^-}} \sim \frac{\lambda^2\, v^4}{M_h^2\, m_\tau^2}\, \frac{1}{\sqrt{h_0}}\, \sqrt{1 - \frac{4\, m^2}{h_0\, M_h^2}}$$

$$\frac{\Gamma_{h\to\phi_{n_\star}\,\phi_{n_\star}}}{\Gamma_{h\to\tau^+\tau^-}} \sim \frac{\lambda^2\, v^4}{M_h^2\, m_\tau^2}\, \sqrt{1 - \frac{4\, m^2}{M_h^2}}$$

Neglected terms of order $(\frac{m_\tau}{M_h})^2$

- For $M_h > \frac{2\, m}{\sqrt{h_0}}$, both ratios could be large even for perturbatively small λ

The Question

- How to tell if the extra dimension is non-static or not?

A Quick Reminder for Static Extra Dimensions

- For a flat and static extra dimension:

$$E^{\mathrm{static}} = \sqrt{p^2 + m^2 + \frac{n^2}{R^2}} \quad \Rightarrow \quad E^{\mathrm{static}}_{n=0} = \sqrt{p^2 + m^2}$$

- The decay rate:

$$\Gamma^{\mathrm{static}}_{h \to \phi_0 \, \phi_0} = \frac{\lambda^2 \, v^2}{8 \, \pi \, M_h} \sqrt{1 - \frac{4 \, m^2}{M_h^2}}$$

- The ratio:

$$\frac{\Gamma^{\text{non-static}}_{h\to\phi_0\,\phi_0}}{\Gamma^{\text{static}}_{h\to\phi_0\,\phi_0}} = \frac{\frac{1}{\sqrt{h_0}}\sqrt{1-\frac{4\,m^2}{h_0\,M_h^2}}}{\sqrt{1-\frac{4\,m^2}{M_h^2}}}$$

- Two Cases:

$$\frac{\Gamma^{\text{non-static}}_{h\to\phi_0\,\phi_0}}{\Gamma^{\text{static}}_{h\to\phi_0\,\phi_0}} < 1 \quad \text{if} \quad \frac{2\,m}{\sqrt{h_0}} < \sqrt{1+h_0}\,\frac{2\,m}{\sqrt{h_0}} < M_h$$

$$\frac{\Gamma^{\text{non-static}}_{h\to\phi_0\,\phi_0}}{\Gamma^{\text{static}}_{h\to\phi_0\,\phi_0}} > 1 \quad \text{if} \quad \frac{2\,m}{\sqrt{h_0}} < M_h < \sqrt{1+h_0}\,\frac{2\,m}{\sqrt{h_0}}$$

$\Gamma^{\text{non-static}}_{h\to\phi_{n_\star}\,\phi_{n_\star}}$ versus $\Gamma^{\text{static}}_{h\to\phi_0\,\phi_0}$

- Equivalence:

$$E^{\text{non-static}}_{n_\star} = E^{\text{static}}_{n=0} = \sqrt{p^2 + m^2}$$

$$\Gamma^{\text{non-static}}_{h\to\phi_{n_\star}\,\phi_{n_\star}} = \Gamma^{\text{static}}_{h\to\phi_0\,\phi_0}$$

- Seems to be no way to distinguish in this case?
- Perhaps the first step:
  How to put a bound on g_0 and h_0?

Some Other (Maybe) Interesting Questions

- KK gravitons in non-static extra dimensions?
- Implications of the $n_\star$ mode dark matter candidate?
- LHC signatures?
- Other interesting phenomenology of non-static extra dimensions?

Conclusions

- Compactified non-static extra dimensions can lead to causality violations
- Secondary vertex can pre-appear before primary vertex, and they are correlated by energy conservation
- Particles carry an unconventional energy dispersion
- Positive KK modes can have lower energy than the zeroth mode
- $n_\star \gg 1$ mode can be the dark matter candidate