Giant $K$ factors

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Work performed with Mathieu Rubin and Sebastian Sapeta, arXiv:1006.2144

Fermilab Theory Seminar
19 May 2011
What kinds of searches at colliders?

New resonance (e.g. $Z'$) where you see all decay products and reconstruct an invariant mass

QCD may:
- swamp signal
- smear signal

leptonic case easy; hadronic case harder
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What kinds of searches at colliders?

New resonance (e.g. R-parity conserving SUSY), where undetected new stable particle escapes detection.

Reconstruct only part of an invariant mass \( \rightarrow \) kinematic edge.

QCD may:

- swamp signal
- smear signal
What kinds of searches at colliders?

Unreconstructed SUSY cascade. Study *effective* mass (sum of all transverse momenta).

Broad excess at high mass scales.

Knowledge of backgrounds is crucial is declaring discovery.

QCD is *one way* of getting handle on background.
Signal ?

$\frac{d\sigma}{dm}$ [log scale]

mass
This talk
SUSY example: gluino pair production

Signal

$\tilde{g} \rightarrow q \chi^0$

$\chi^0 \rightarrow q q$

$g \rightarrow \tilde{g} \tilde{q}$
SUSY example: gluino pair production

Signal

\[ \begin{align*}
g & \rightarrow \tilde{g} \rightarrow q \chi^0 \\
\tilde{g} & \rightarrow \tilde{q} \rightarrow jet \\
\chi^0 & \rightarrow E_T \\
E_T & \rightarrow jet
\end{align*} \]
SUSY example: gluino pair production

**Signal**

- $\tilde{g} \rightarrow \tilde{q} \chi^0$
- $\tilde{q} \rightarrow j e t$
- $\chi^0 \rightarrow E_T$

**Background**

- $g \rightarrow q \bar{q}$
- $Z \rightarrow \nu \bar{\nu}$
- $E_T$

*Giant $K$-factors (@ FNAL)*

2011-05-19
SUSY example: gluino pair production

Signal

\[ \tilde{g} \rightarrow q \chi^0 \]

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\[ \tilde{g} \rightarrow q \nu \]

\[ \tilde{g} \rightarrow q \nu \]

\[ \tilde{g} \rightarrow q Z \]

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Background

\[ E_T \]

\[ E_T \]

\[ E_T \]

\[ E_T \]
SUSY searches: what excesses?

Atlas selection [all hadronic]
- no lepton
- MET > 100 GeV
- $1^{st}$, $2^{nd}$ jet > 100 GeV
- $3^{rd}$, $4^{th}$ jet > 50 GeV
- MET / $m_{\text{eff}}$ > 20%

CMS selection [leptonic incl.]
(optimized for 10fb$^{-1}$, using genetic algorithm)
- 1 muon $p_T$ > 30 GeV
- MET > 130 GeV
- $1^{st}$, $2^{nd}$ jet > 440 GeV
- $3^{rd}$ jet > 50 GeV
- $-0.95 < \cos(\text{MET},1^{st}\text{jet}) < 0.3$
- $\cos(\text{MET},2^{nd}\text{jet}) < 0.85$

Christian Autermann
SUSY08 16.-21.6.08
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SUSY searches: what excesses?

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- \( \cos(\text{MET}, 2\text{nd jet}) < 0.85 \)

**SUSY ≈ factor 5–10 excess**
How accurate is perturbative QCD?

\[ \sigma = c_0 + c_1 \alpha_s + c_2 \alpha_s^2 + \ldots \]

\[ \alpha_s \approx 0.1 \]

That implies LO QCD (just \( c_0 \)) should be accurate to within 10%

It isn’t

Rules of thumb:
LO good to within factor of 2
NLO good to within scale uncertainty

This talk is about an example where these rules fail spectacularly, the lessons we learn, and the solutions we can apply.
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Control samples

We don’t always have NLO for the background (e.g. $Z+4$ jets, a $2 \rightarrow 5$ process).

Though amazing recent progress
$2 \rightarrow 4$: Blackhat, Rocket, Helac-NLO, BDDP
$2 \rightarrow 5$ ($W+4j$): Blackhat

Must then rely on LO (matched with parton showers). How does one verify it?

Common “data-driven” procedure:
[roughly]

▷ Get control sample at low $p_t$
▷ SUSY should be small(er) contamination there
▷ Once validated, trust LO prediction at high-$p_t$
Is this safe?

A conservative QCD theory point of view:

It’s hard to be sure: since we can’t (yet) calculate $Z+4$ jets beyond LO.

But we would tend to think it is safe, as long as control data are within usual factor of two of LO prediction.

Illustrate issues with toy example: $Z$+jet production

- It’s known to NLO and a candidate for “first” $2 \rightarrow 2$ NNLO
  \[ e^+e^- \rightarrow \gamma^*/Z \rightarrow 3 \text{ jets}, \text{NNLO: Gehrman et al '08, Weinzierl '08} \]

- But let’s pretend we only know it to LO, and look at the $p_t$ distribution of the hardest jet (no other cuts — keep it simple)

![Diagram of the process $g + q$ producing a $Z$ boson and a jet](Diagram)
Toy data, control sample

stage 1: get control sample

Check LO v. data at low $p_t$

- normalisation off by factor 1.5 (consistent with expectations)

So renormalise LO by K-fact

- shape OKish

Don’t be too fussy: SUSY could bias higher $p_t$
Toy data, control sample

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Toy data, high $p_t$  

stage 2: look at high $p_t$  
- good agreement at low $p_t$, by construction  
- excess of factor $\sim 10$ at high $p_t$  
- check scale dependence of LO  

[Note: not always done except e.g. Alwall et al. 0706.2569]  
still big excess
Toy data, high $p_t$

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still big excess

$Z + \text{jet}$ cross section (LHC)

$\mu^2 = m_Z^2 + p_{t,Z}^2$

$k_t$ alg., $R=0.7$

MCFM 5.2

CTEQ6M
Toy data, high $p_t$

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What’s in the toy data?

Is it:

- QCD + extra signal?
- just QCD? But then where does a $K$-factor of 10 come from?

Here it’s just a toy illustration. Later this year it may be for real:

- Do Nature / Science / PRL accept the paper?
  
  **Discovery of New Physics at the TeV scale**
  
  We report a 5.7$\sigma$ excess in MET + jets production that is consistent with a signal of new physics . . .

- Do we proceed immediately with a linear collider?
  
  It’ll take 10–15 years to build; the sooner we start the better

- At what energy? It would be a shame to be locked in to the wrong energy . . .
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It’ll take 10–15 years to build; the sooner we start the better

▷ At what energy? It would be a shame to be locked in to the wrong energy...
Unlike for SUSY multi-jet searches, in the $Z+$jet case we do have NLO.

Once NLO is included the excess disappears.

The “toy data” were just the upper edge of the NLO band.

Example based on background work for Butterworth, Davison, Rubin & GPS ’08.

Related observations also by Bauer & Lange ’09; Denner, Dittmaier, Kasprzik & Muck ’09.

Hold on a second: how does QCD give a $K$-factor $O(5 – 10)$?

NB: DYRAD, MCFM consistent.
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Hold on a second: how does QCD give a $K$-factor $\mathcal{O}(5−10)$?

NB: DYRAD, MCFM consistent
What about other observables?

\[ p_t \text{ of } Z\text{-boson} \]

\[ p_t \text{ of jet 1} \]

\[ H_{T,jets} = \sum_{jets} p_{t,j} \]

\[ K \approx 1.5 \]

\[ K \approx 5 \]

\[ K \approx 50 \]

"Giant K-factors"
Why the large $K$-factor?

Leading Order

\[ \alpha_s \alpha_{EW} \]

Next-to-Leading Order

\[ \alpha_s^2 \alpha_{EW} \]

LHC probes scales $\gg$ EW scale, $\sqrt{s} \gg M_Z$. EW bosons are light. New logarithmically enhanced topologies appear.

Giant $K$-factors (© FNAL) 2011-05-19 14 / 31
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**LHC probes scales** $\gg$ **EW scale**, $\sqrt{s} \gg M_Z$. EW bosons are **light**.

**New logarithmically enhanced topologies appear.**

NLO driven in part by $qq$ parton luminosity: large at $pp$ colliders.
Is this example not a little contrived? After all, experiments would surely notice unexpected event topology such as that here.

We actually first saw the problem in a more complex process: $Wb\bar{b}$ as a background to boosted Higgs searches (with “wrong” cuts). The more complicated the process, the trickier the diagnosis of the problem.

It’s enough to get this wrong once, leading to “unwarranted” press-releases and major subsequent embarassment.
In day-to-day work the experiments don’t just use LO.
Instead, they

- Take $Z + \text{jet}$, $Z + 2 \text{jet}$ samples, etc.,
- Attach a parton shower to each of them
- and combine with procedures such as MLM matching or CKKW matching.
Testing Alpgen + Herwig + MLM Matching

$\rho_t$ of Z-boson

$\frac{d\sigma}{dp_t,Z}$ [fb / 100 GeV]

$pp, 14$ TeV

LO

NLO

Giant $K$-factors (@ FNAL)
Testing Alpgen + Herwig + MLM Matching

$p_t$ of Z-boson

$\frac{d\sigma}{dp_t,Z}$ [fb / 100 GeV]

pp, 14 TeV

LO
NLO
Alpgen+HW6 Z+jet
Testing Alpgen + Herwig + MLM Matching

$\rho_t$ of Z-boson

$\frac{d\sigma}{dp_t,Z} [\text{fb} / 100 \text{ GeV}]

pp, 14 TeV

All predictions similar and stable

LO

NLO

$\text{Alpgen+HW6 Z+jet}$

$\text{Alpgen+HW6 Z+jet/Z+2jets}$

2011-05-19
Testing Alpgen + Herwig + MLM Matching

\( p_t \) of jet 1

\[ \frac{d\sigma}{dp_{t,j1}} \text{ [fb / 100 GeV]} \]

pp, 14 TeV

\( p_{t,j1} \) [GeV]

200 300 400 500 600 700 800 900 1000

\[ \times 10^{-2} \times 10^{-1} 1 10 10^2 10^3 10^4 \]

LO

NLO
Testing Alpgen + Herwig + MLM Matching

$p_t$ of jet 1

$pp, 14$ TeV

Showered $Z+j \simeq LO$

$\frac{d\sigma}{dp_{t,j1}}$ [fb / 100 GeV]

$200$  $300$  $400$  $500$  $600$  $700$  $800$  $900$  $1000$

$p_{t,j1}$ [GeV]

LO

NLO

Alpgen+HW6 $Z$+jet
Testing Alpgen + Herwig + MLM Matching

$p_t$ of jet 1

Showered $Z+j \simeq$ LO
Showered $Z+j/Z+2j \simeq$ NLO
Testing Alpgen + Herwig + MLM Matching

\[ H_{T,jets} = \sum_{jets} p_{t,j} \]

![Graph showing the distribution of \( d\sigma/dH_{T,jets} \) for pp collisions at 14 TeV, comparing LO and NLO predictions.](graph.png)
Testing Alpgen + Herwig + MLM Matching

\[ H_{T,jets} = \sum_{jets} p_{t,j} \]

\[ \frac{d\sigma}{dH_{T,jets}} [\text{fb} / 100 \text{ GeV}] \]

pp, 14 TeV

Showered Z+j ≠ LO

cf. also de Aquino, Hagiwara, Li & Maltoni '11

Giant \( K \)-factors (@ FNAL)
Testing Alpgen + Herwig + MLM Matching

\[ H_{T,jets} = \sum_{jets} p_{t,j} \]

Showered $Z+j \neq$ LO
Showered $Z+j/Z+2j \sim$ NLO

cf. also de Aquino, Hagiwara, Li & Maltoni '11
It’s great that (the widely-used) MLM/CKKW matching procedure correctly approximates the NLO giant $K$-factor.

But what happens at NNLO?

a natural question when LO $\rightarrow$ NLO convergence is poor

Despite 10 years’ calculation, the answer is not yet known

Our strategy: get an approximation to NNLO

specifically, we will approximate a subset of the loop contributions, with a method dubbed “LoopSim”
Ingredients of (N)NLO

LO

Giant $K$-factors (© FNAL)
Ingredients of (N)NLO

NLO

- Giant $K$-factors (@ FNAL)
Ingredients of (N)NLO

NNLO

[Diagrams of NNLO processes]
Approximations to (N)NLO

Our naming scheme:
For each loop that we approximate, replace $N \rightarrow \bar{n}$

- $\bar{n}$LO: approx 1-loop diagrams

Approximate

Exact

Approximate
Approximations to \((N)NLO\)

\[\bar{n}\bar{n}LO\]

\[\begin{aligned}
\text{\begin{tikzpicture}
\draw (0,0) -- (1,1);
\draw (0,0) -- (-1,1);
\draw (0,0) -- (0,-1);
\end{tikzpicture}}\quad & \quad \begin{tikzpicture}
\draw (0,0) -- (1,1);
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\end{aligned}\]

Our naming scheme:
For each loop that we approximate, replace \(N \rightarrow \bar{n}\)

- \(\bar{n}LO\): approx 1-loop diagrams
- \(\bar{n}\bar{n}LO\): approx 1- and 2-loops
Approximations to (N)NLO

Our naming scheme:
For each loop that we approximate, replace $N \rightarrow \bar{n}$

- $\bar{n}$LO: approx 1-loop diagrams
- $\bar{n}\bar{n}$LO: approx 1- and 2-loops
- $\bar{n}$NLO: approx 2-loop only
First try $Z + \text{jet} @ \bar{n}\text{LO}$:

Take the “leading” process

$[Z + \text{jet} @ \text{LO}]$

and add in process with one extra jet.

[i.e. include $Z + 2 \text{jets} @ \text{LO}$]

**approximate** the 1-loop $Z + \text{jet}$ term, by requiring cancellation of all divergences

[those from singly unresolved limit of $Z + 2 \text{jets}$]
The LoopSim idea: \( \bar{n}\text{LO} \)

- Identify softest or most collinear parton [with help of a jet algorithm]
- “Loop” it \( \equiv \) remove it from event, reshuffle other momenta; weight of looped event is \((-1)\times\) weight of tree-level event
The LoopSim idea: $\bar{n}$LO

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$Z + 2$ partons

\[ |M^2(p_1, p_2, p_3)| \]
The LoopSim idea: $\bar{n}$LO

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\[ Z + 2 \text{ partons} \]
\[ |M^2(p_1, p_2, p_3)| \]

\[ Z + 1 \text{ parton} + 1 \text{ sim. loop} \]
\[ -|M^2(p_1, p_2, p_3)| \]
The LoopSim idea: \( \bar{n}\text{LO} \)

- Identify softest or most collinear parton [with help of a jet algorithm]
- "Loop" it \( \equiv \) remove it from event, reshuffle other momenta; weight of looped event is \( (-1) \times \) weight of tree-level event

This cancels all the "single-unresolved" divergences in the \( Z+2 \) events
When the $K$-factors are large, $\tilde{n}$LO agrees well with NLO

Works with similar “quality” to MLM matching
MLM/CKKW matching also effectively provide $\bar{n}$LO type accuracy.

How does LoopSim compare to MLM/CKKW?

1. Does not rely on shower (✔: simplicity; ✗: not easily integrated with shower MCs)

2. Does not need arbitrary separation of $Z+1/Z+2$ etc. samples with (hard-to-choose) momentum cutoff

3. Can easily be extended beyond LO matching
add tree-level $Z+3$, 
cancel divergences in single + doubly unresolved limits: $\tilde{n}\tilde{n}$LO

\[ |M^2(p_1, p_2, p_3, p_4)| \]
add tree-level $Z+3$, cancel divergences in single + doubly unresolved limits: $\bar{n}\bar{n}$LO

\[ |M^2(p_1, p_2, p_3, p_4)| - |M^2(p_1, p_2, p_3, p_4)| - |M^2(p_1, p_2, p_3, p_4)| \]

Separately loop either of the 2 softest emissions: provides approx of 1-loop
add tree-level Z+3,
cancel divergences in single + doubly unresolved limits: $\bar{n}\bar{n}$LO

\[
|M^2(p_1, p_2, p_3, p_4)| - |M^2(\tilde{p}_1, p_2, p_3, p_4)| - |M^2(p_1, \tilde{p}_2, p_3, p_4)| + |M^2(\tilde{p}_1, \tilde{p}_2, p_3, p_4)|
\]

Simultaneously loop each of the 2 softest emissions: provides approx of 2-loop
Total of tree plus approx 1- and 2-loop pieces gives zero
add tree-level Z+3,
cancel divergences in single + doubly unresolved limits: \( \bar{n}\bar{n}\)LO

\[
|M^2(p_1, p_2, p_3, p_4)| - |M^2(p_1, p_2, p_3, p_4)| - |M^2(p_1, p_2, p_3, p_4)| + |M^2(p_1, p_2, p_3, p_4)|
\]

Simultaneously loop each of the 2 softest emissions: provides approx of 2-loop
Total of tree plus approx 1- and 2-loop pieces gives zero

add in (exact Z+2 @ 1-loop) − (approximate Z+2 @ 1-loop)
+ extra simulated 2-loop piece to cancel new Z+2@1-loop divergences

This is \( \bar{n}NLO \)
The 2-loop piece has the topology of the LO diagram.

The “mistake” we make in approximating it should therefore be a “pure” $\mathcal{O}(\alpha_s^2)$ correction, without any large enhancements from new NLO type topologies.

$$\sigma_{\bar{n}\text{NLO}} = \sigma_{\text{NNLO}} + \mathcal{O}(\alpha_s^2 \sigma_{\text{LO}})$$

$$= \sigma_{\text{NNLO}} \left( 1 + \mathcal{O} \left( \frac{\alpha_s^2}{K_{\text{NNLO}}} \right) \right)$$

$$K_{\text{NNLO}} = \frac{\sigma_{\text{NNLO}}}{\sigma_{\text{LO}}} \sim K_{\text{NLO}} \gg 1$$

The relative contribution of the neglected piece is suppressed by the large $K$-factor.

$\bar{n}\text{NLO}$ should be a good approximation to NNLO when the $K$-factor is large and due to new higher-order topologies.
Testing $\bar{n}\text{NLO}$, in 3 processes

[making use of existing NLO calculations]

1. $Z@\text{NLO}$ and $Z+j@\text{NLO} \rightarrow Z@\bar{n}\text{NLO}$
   with MCFM; compare to exact NNLO from DYNNLO

2. $Z+j@\text{NLO}$ and $Z+2j@\text{NLO} \rightarrow Z+j@\bar{n}\text{NLO}$
   with MCFM

3. $2j@\text{NLO}$ and $3j@\text{NLO} \rightarrow 2j@\bar{n}\text{NLO}$
   with NLOjet++
Validation: Drell-Yan lepton $p_t, \tilde{n}\text{NLO} \text{ v. } \text{NNLO}$

For $p_{t,\ell} \gtrsim \frac{1}{2} M_Z + \Gamma_Z$ (giant $K$-factor!) it had to work
For $p_{t,\ell} \lesssim \frac{1}{2} M_Z + \Gamma_Z$ it's remarkable that it still works
Validation: Drell-Yan lepton $p_t$, $\bar{n}$NLO v. NNLO

**$\bar{n}$LO v. NLO**

- LO
- NLO
- $\bar{n}$LO

**$\bar{n}$NLO v. NNLO**

- NLO
- $\bar{n}$NLO ($\mu$ dep)
- $\bar{n}$NLO ($R_{LS}$ dep)
- NNLO

**K factor wrt NLO**

**For $p_{t,\ell} \gtrsim \frac{1}{2}M_Z + \Gamma_Z$ (giant $K$-factor!) it had to work**

**For $p_{t,\ell} \lesssim \frac{1}{2}M_Z + \Gamma_Z$ it’s remarkable that it still works**

**NNLO from DYNNLO, Z (i.e. DY) with Z+j from MCFM & LoopSim**

- $pp, 14\text{ TeV}$
- $66 < m_{e^+e^-} < 116\text{ GeV}$
Validation: Drell-Yan lepton $p_t$, $\bar{n}$NLO v. NNLO

**$\bar{n}$LO v. NLO**

- LO
- NLO
- $\bar{n}$LO

**$\bar{n}$NLO v. NNLO**

- NLO
- $\bar{n}$NLO ($\mu$ dep)
- $\bar{n}$NLO ($R_{LS}$ dep)
- NNLO

For $p_{t,\ell} \gtrsim \frac{1}{2} M_Z + \Gamma_Z$ (giant K-factor!) it had to work
For $p_{t,\ell} \lesssim \frac{1}{2} M_Z + \Gamma_Z$ it’s remarkable that it still works

**NNLO from DYNNLO, Z (i.e. DY) with Z+j from MCFM & LoopSim**
\[ \tilde{n}\text{NLO for } Z+\text{j observables} \]

\textbf{\( p_t \) of \( Z \)-boson}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{ptZ_distribution}
\caption{\( p_t \) distribution of \( Z \)-boson at different orders of \( \tilde{n}\text{NLO} \).}
\end{figure}

- \( p_{tZ} \) distribution didn’t have giant \( K \)-factors.
- \( \tilde{n}\text{NLO} \) brings no benefit
  
  To get improvement you would need exact 2-loop terms.
\[ \bar{n}\text{NLO for } Z+j \text{ observables} \]

- \( p_t \) of jet 1

\[
\begin{array}{c}
\text{LO} \\
\text{NLO} \\
\bar{n}\text{NLO (} \mu \text{ dep)} \\
\bar{n}\text{NLO (} R_{LS} \text{ dep)}
\end{array}
\]

- \( p_{t,j1} \) distribution seems to converge at \( \bar{n}\text{NLO} \)
- scale uncertainties reduced by \( \sim \) factor 2
\[ H_{T,jets} = \sum_{jets} p_{t,j} \]

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Significant further enhancement for \( H_{T,jets} \)

\[ H_{T,jets} \] is not a good observable!
What’s the problem with $H_T$?

$H_T$ (effective mass) type observables are widely used in searches

- $H_T$ has a steeply falling distribution (like $p_{tj}$, $p_{tZ}$)
- At each order (NLO, NNLO), an extra (soft) jet contributes to the $H_T$ sum
- That shifts $H_T$ up, which translates to a substantial increase in the cross section

We can test this hypothesis for plain jet events, using a truncated sum,

$$H_{T,n} = \sum_{i=1}^{n} p_{t,\text{jet } i}$$
A clear message:

for a process with \( n \) objects at lowest order, use \( H_{T,n} \)

Do you know what gets used in your experiment’s searches?
Some take-home messages

Be aware that giant $K$-factors exist
Always look one order beyond the leading order, for example with
MLM/CKKW matching

New tool to get good predictions in such cases: **LoopSim**
Basically an “operator” to generate approximations to unknown loops
Combine $Z+j@NLO$, $Z+2j@NLO$ to get “$\bar{n}NLO$” $Z+$jet
It sometimes works even beyond “giant-$K$-factor” regions

Watch out for $H_T$
Even for simple processes, it converges very poorly
unless you define it carefully (limit number of objects in sum)
EXTRAS
Add $Z+1\text{jet}$, $Z+2\text{jet} + \text{shower}$
Add Z+1jet, Z+2jet + shower

shower Z+parton
Add $Z+1$jet, $Z+2$jet + shower

$\text{shower} \ Z+\text{parton}$  \hspace{1cm}  $Z+2\text{partons}$
Add $Z+1\text{jet}$, $Z+2\text{jet} + \text{shower}$

**Diagram:**

- **shower $Z+\text{parton}$**
- **shower $Z+2\text{partons}$**
Add $Z+1jet$, $Z+2jet + shower$

\[\text{shower } Z+\text{parton} \quad + \quad \text{shower } Z+2\text{partons} \quad \text{v.} \quad \text{shower of } Z+\text{parton generates hard gluon}\]
Add $Z+1$jet, $Z+2$jet + shower

- shower $Z+$parton
- shower $Z+2$partons
- shower of $Z+$parton generates hard gluon
Add $Z+1$jet, $Z+2$jet + shower

**DOUBLE COUNTING**

$Z$ + parton implicitly includes part of $Z + 2$ partons

It’s just that the 2nd parton isn’t always explicitly “visible”
MLM merging relies on parton shower to help figure out what fraction of $Z + \text{parton}$ is really $Z + 2 \text{partons}$.

In a few slides, we will try to do that without the parton shower.
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In a few slides, we will try to do that without the parton shower.
For every $Z + 2$ parton ($2 \rightarrow 3$) event, figure out what event it would really have come from. "Loop" the softest parton. [Don't actually explicitly calculate any loop diagrams: simulate the loops]

- Subtract that $2 \rightarrow 2$ event

Unlike MLM, no cutoffs on $2 \rightarrow 3$ events. If done properly, divergences will cancel.
Use jet algorithm to assign a branching structure to event \( \text{à la CKKW} \)

The particles that are softest are the ones that will be “looped”
Define operators:

\[ U_{\ell}(\text{event } E) \equiv \text{all simulated } \ell\text{-loop events from } E \]

\[ U_{\forall}(\text{event}) \equiv \sum_{\ell=0} U_{\ell}(\text{event}) \]

“U” stands for unitarisation (cancellation of all divergences)
sum of all diagrams (essentially) adds up to zero

To combine \( Z+j \) with \( Z+2j \) take

\[ Z+j@\tilde{nLO} \equiv Z+j@LO + U_{\forall}(Z+2j@LO) \]

we use “\( \tilde{nLO} \)” to emphasize that this is a crude approximation

to an actual NLO calculation — the exact loops are missing

NB: \( U_{\forall} \) here includes \( \ell = 0, 1 \)
Just replace simulated loops with exact loops
Apply LoopSim to exact 1-loop to get (e.g.) simulated 2-loop terms

\[ E_{n,\ell} \equiv \text{event with } n \text{ partons and } \ell \text{ exact loops} \]

\[ U_{\forall,\ell} \equiv \text{operator to apply when } \ell \text{ exact loops known} \]

\[ U_{\forall,1}(E_{n,0}) = U_{\forall}(E_{n,0}) - U_{\forall}(U_1(E_{n,0})) \]

\[ U_{\forall,1}(E_{n,1}) = U_{\forall}(E_{n,1}) \]

\[ Z+j@\tilde{n}\text{NLO} = Z+j@NLO + U_{\forall,1}(Z+2j@NLO_{\text{only}}) \]

Extension to NLO, NNLO, multi-leg, etc. is almost trivial in LoopSim

Not the case in methods that merge with parton showers too