Standard-Model Prediction of ε_K at NNLO

Fermilab Theory Seminar, November 3, 2011

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In collaboration with Martin Gorbahn

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arXiv:1108.2036

Tension in the Data?

Experiment:

ε_K is measured precisely:

$|\varepsilon_K| = 2.228(11)\times 10^{-3}$.

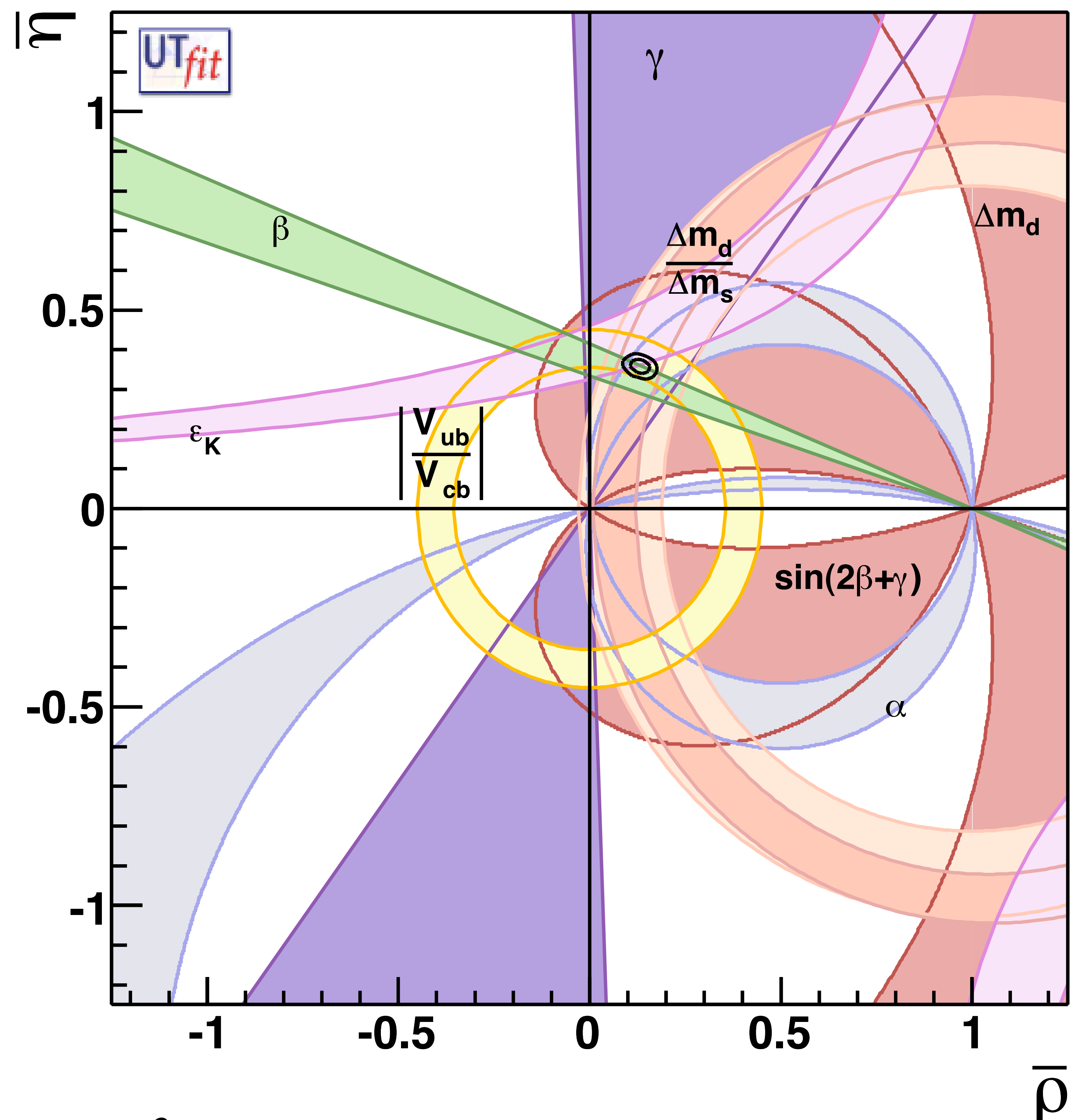
[PDG2010]

Theory:

ε_K can be calculated reliably:

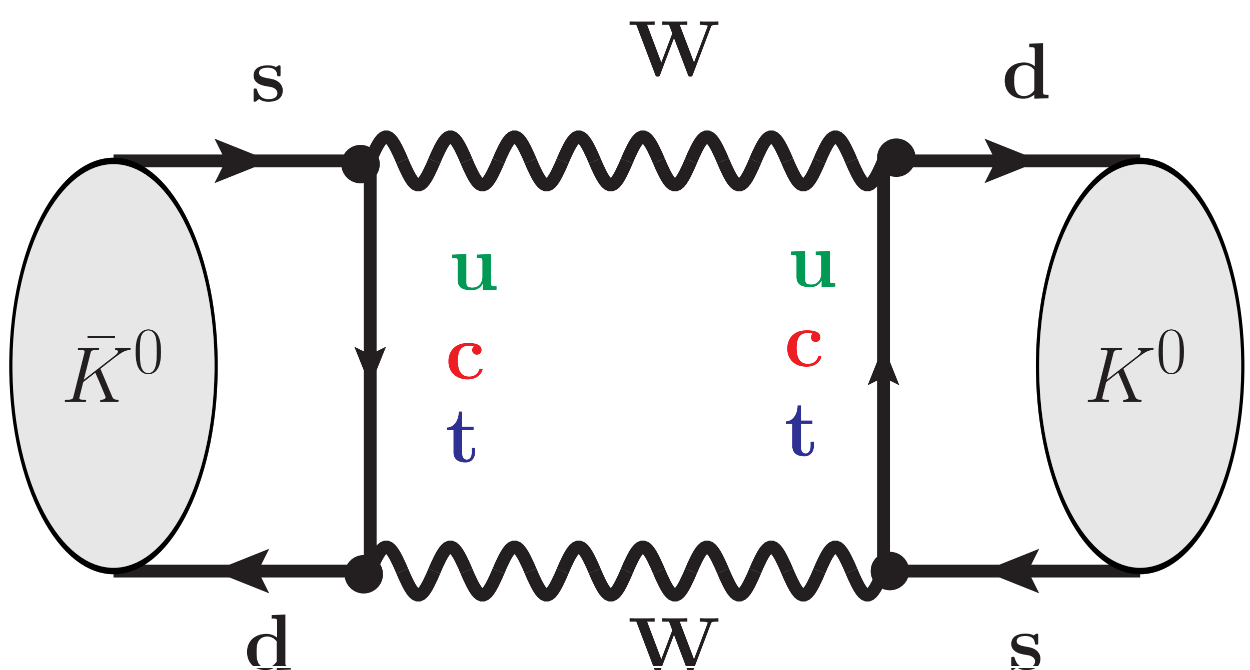
$|\varepsilon_K| = 1.83(27)\times 10^{-3}$.

[NLO SM prediction]



ε_K in the Standard Model

- Neutral Kaon mixing is a FCNC transition
- Loop-induced in the Standard Model (SM)



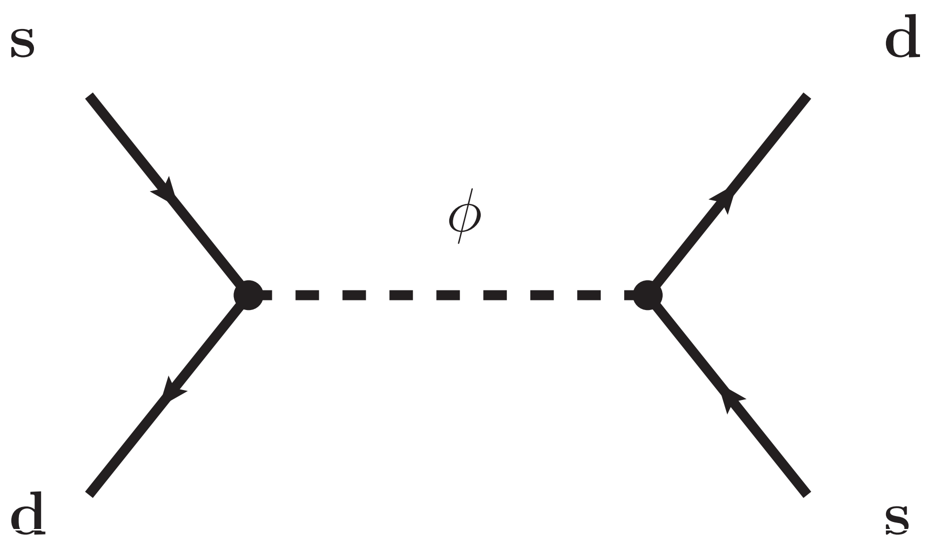
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

SM → CP violation by Complex Phase in the CKM matrix

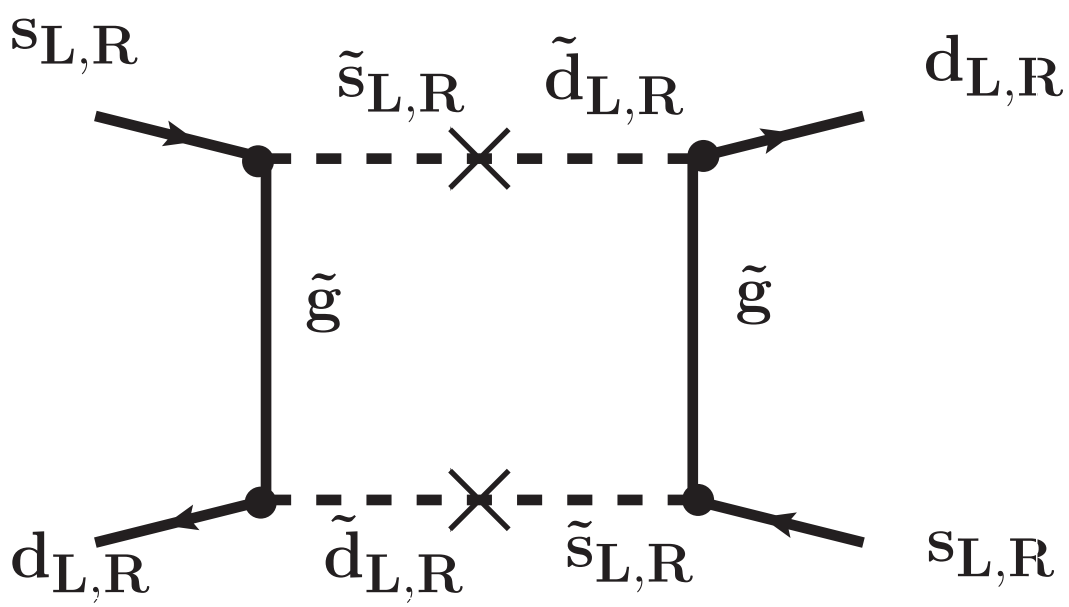
Top-quark contribution CKM suppressed
⇒ sensitivity to deviations from MFV

ε_K beyond the Standard Model

Many more sources of CP violation:



2HDM Type III



SUSY models

+Technicolour, extra dimensions, ...

Chiral Enhancement of non-SM operators

Strong constraints from ε_K!

Outline

- Introduction: Effective Field Theory
- Calculation of ε_K
  - Long-distance contributions
  - Short-distance contributions
- Result and error budget
- Summary and outlook

ε_K - Definition

ε_K describes indirect CP violation in the neutral Kaon system

$$\langle K^0 | H^{|\Delta S|=2} | \bar{K}^0 \rangle$$

$$\frac{\mathrm{Im}\langle (\pi\pi)_{I=0} | K^0 \rangle}{\mathrm{Re}\langle (\pi\pi)_{I=0} | K^0 \rangle}$$

$$\boxed{\epsilon_K = \frac{\langle (\pi\pi)_{I=0} | K_L \rangle}{\langle (\pi\pi)_{I=0} | K_S \rangle} = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im}(M_{12})}{\Delta m_K} + \xi \right)}$$

$$\phi_\epsilon = \arctan \frac{\Delta m_K}{\Delta \Gamma_K / 2}$$

Effective Field Theory

Factorize different energy scales:
electroweak, charm, hadronic

- Wilson coefficients
- Short-distance
- Perturbative

$$\langle H^{|\Delta S|=2} \rangle = C \times \langle \tilde{Q}^{|\Delta S|=2} + Q^{|\Delta S|=1} Q^{|\Delta S|=1} \rangle$$

- Hadronic matrix elements
- Long-distance
- Non-perturbative QCD - lattice, ChPT

Effective Field Theory: Logs

Generic structure of perturbation series:

1

α_s $\alpha_s \log(m_c/M_W)$

α_s^2 $\alpha_s^2 \log(m_c/M_W)$ $\alpha_s^2 \log^2(m_c/M_W)$

...

Effective Field Theory: Logs

Generic structure of perturbation series:

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$\alpha_s \qquad \alpha_s \log(m_c/M_W)$

$\alpha_s^2 \qquad \alpha_s^2 \log(m_c/M_W) \qquad \alpha_s^2 \log^2(m_c/M_W)$

...

$\alpha_s^n \log^n$

$\alpha_s \, \alpha_s^n \log^n$

$\alpha_s^2 \, \alpha_s^n \log^n$

Renormalisation Group sums all terms proportional to

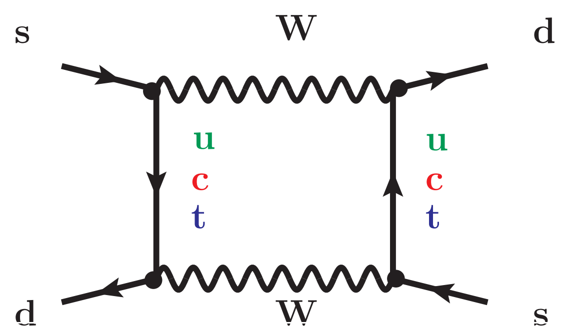
Effective Field Theory: Running

Solution to the Renormalisation Group Equation:
Running Coupling

$$C(\mu) = U(\mu, \mu_0) C(\mu_0)$$

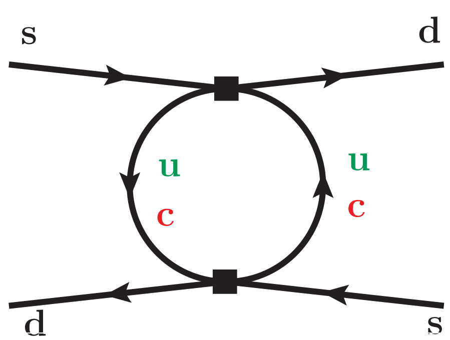
$$U(\mu, \mu_0) = T_g \exp \int_{g(\mu_0)}^{g(\mu)} dg' \, \frac{\gamma^T(g')}{\beta(g')}$$

Effective Field Theory



Standard Model

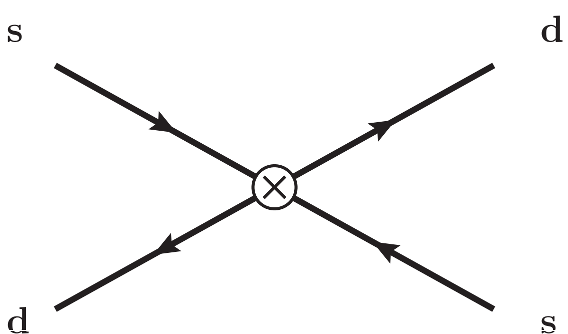
$\mu_{W t}$



$$\mathcal{H}^{\mathrm{eff}}_{f=5} = \frac{4G_F}{\sqrt{2}} \sum_{i=+,-,3}^{6} C_i \left[\sum_{j=+,-} Z_{ij} \sum_{k,l=u,c} V^*_{ks} V_{ld} Q^{kl}_j \right.$$

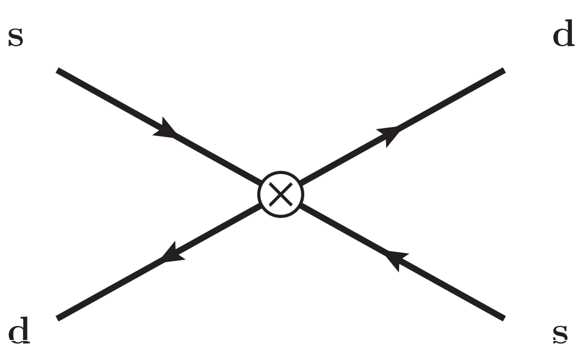
$$\left. -\lambda_t \sum_{j=3}^{6} Z_{ij} Q_j \right] + \frac{G_F^2}{4\pi^2} \lambda_t^2 \tilde{C}^t_{\mathrm{S2}} \tilde{Z}_{\mathrm{S2}} \tilde{Q}_{\mathrm{S2}}$$

μ_b



$$+8G_F^2 \lambda_c \lambda_t \left[\sum_{k=+,-} \sum_{l=+,-,3}^{6} C_k C_l \hat{Z}_{kl,7} + \tilde{C}_7 \tilde{Z}_{77} \right] \tilde{Q}_7 + \mathrm{h.c.}\,.$$

μ_c



$$\mathcal{H}^{|\Delta S|=2}_{f=3} = \frac{G_F^2}{4\pi^2} \left[\lambda_c^2 \tilde{C}^c_{S2}(\mu) + \lambda_t^2 \tilde{C}^t_{S2}(\mu) + \lambda_c \lambda_t \tilde{C}^{ct}_{S2}(\mu) \right] \tilde{Z}_{S2} \tilde{Q}_{S2}$$

Definition of B_K

$$\boxed{B_K = \frac{3}{2}\frac{\langle \bar{K}^0 | Q_{S2} | K^0 \rangle}{f_K^2 M_K^2}}$$

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$$\hat{B}_K = \frac{3}{2} b(\mu_L) \frac{\langle \bar{K}^0 | \tilde{Q}_{S2} | K^0 \rangle}{f_K^2 M_K^2}$$

$$\eta_{ij}\, S(x_i, x_j)$$

Definition of B$_K$

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$$\eta_{ij}\, S(x_i, x_j)$$

Bare lattice → RI-(S)MOM → MS-bar [Aoki et. al '08,'10]

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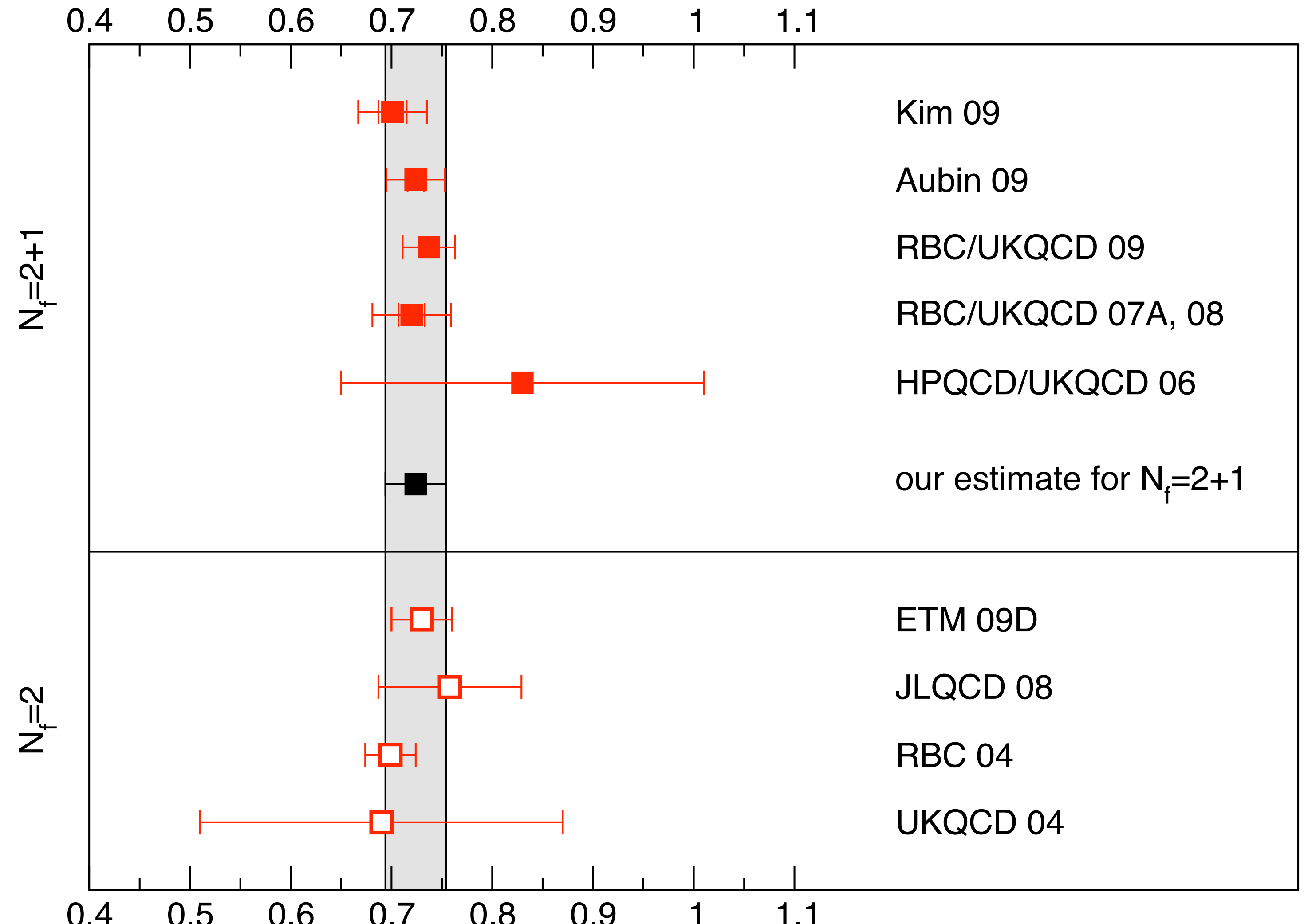
Determination of B_K

$$\langle K^0|\tilde{Q}^{|\Delta S|=2}|\bar{K}^0\rangle \propto \widehat{B}_K \begin{array}{l} = 0.749(27) \text{ [Aoki et al '10]} \\ = 0.725(26) \text{ [Aubin et al '09]} \end{array}$$

Huge progress:

- Unquenched calculations
- Small pion masses
- Different lattice spacings
- Main error: matching to perturbative renormalisation



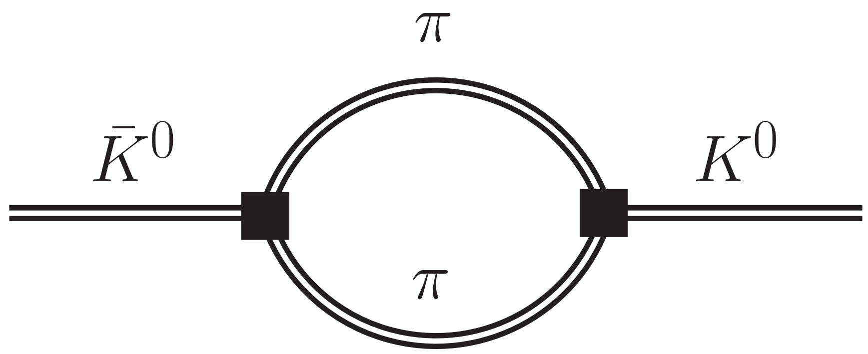
[FLAG review '10]

LD-Contributions from ΔS=1 Hamiltonian

$$\epsilon_K = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im}(M_{12})}{\Delta m_K} + \xi \right)$$

Dispersive (real) and

absorptive (imaginary) part of



$$\int d^4x \langle K^0 | H^{|\Delta S|=1}(x) H^{|\Delta S|=1}(0) | \bar{K}^0 \rangle$$

- Estimate ξ from ε'/ε: -6% [Nierste '02, Buras et al. '09]
- Estimate absorptive part in ChPT: +2.4% [Buras et al. '10]

Combine with prefactor to κ_ε = 0.94(2)

LD-Contributions from ΔS=2 Hamiltonian

$$\epsilon_K = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im}(M_{12})}{\Delta m_K} + \xi \right)$$

$$\langle K^0 | H^{|\Delta S|=2} | \bar{K}^0 \rangle$$

Higher-dimensional operators contribute less than 1%

[Cata et al. '03,'04]

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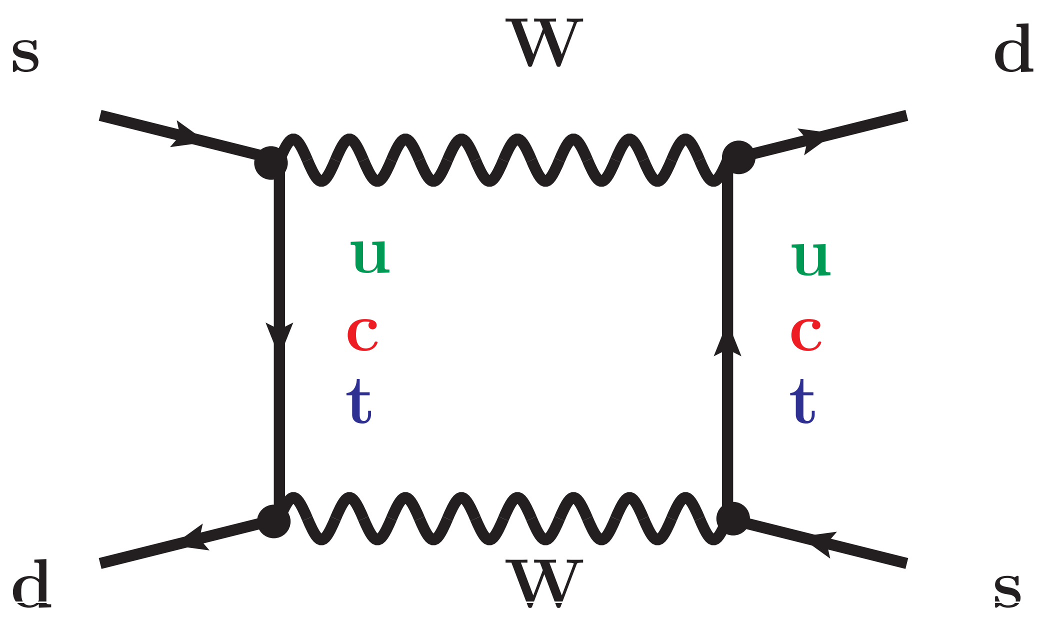
Too deep we delved there, and woke the nameless fear.

[J. R. R. Tolkien, The Lord of the Rings]

$|\Delta S|=2$ Hamiltonian

$$H^{|\Delta S|=2} \propto \left[\lambda_c^2 \eta_{cc} S\left(\frac{m_c^2}{M_W^2}\right) + \lambda_t^2 \eta_{tt} S\left(\frac{m_t^2}{M_W^2}\right) + \lambda_c \lambda_t \eta_{ct} S\left(\frac{m_c^2}{M_W^2}, \frac{m_t^2}{M_W^2}\right)\right] \tilde{Q}^{|\Delta S|=2}$$



CKM parameters:

$$\lambda_i = V_{is}^* V_{id}$$

$$\lambda_u = -\lambda_c - \lambda_t$$

Re $\langle K^0 | H^{|\Delta S|=2} | \bar{K}^0 \rangle \Rightarrow \Delta m_K$ Im $\langle K^0 | H^{|\Delta S|=2} | \bar{K}^0 \rangle \Rightarrow \varepsilon_K$

|ΔS|=2 Hamiltonian

$$H^{|\Delta S|=2} \propto \left[\lambda_c^2 \eta_{cc} S\left(\frac{m_c^2}{M_W^2}\right) + \lambda_t^2 \eta_{tt} S\left(\frac{m_t^2}{M_W^2}\right) + \lambda_c \lambda_t \eta_{ct} S\left(\frac{m_c^2}{M_W^2}, \frac{m_t^2}{M_W^2}\right)\right] \tilde{Q}^{|\Delta S|=2}$$

$$\approx \lambda_{\text{Cabibbo}}^{10} \times \eta_{tt} \times \frac{m_t^2}{M_W^2}$$

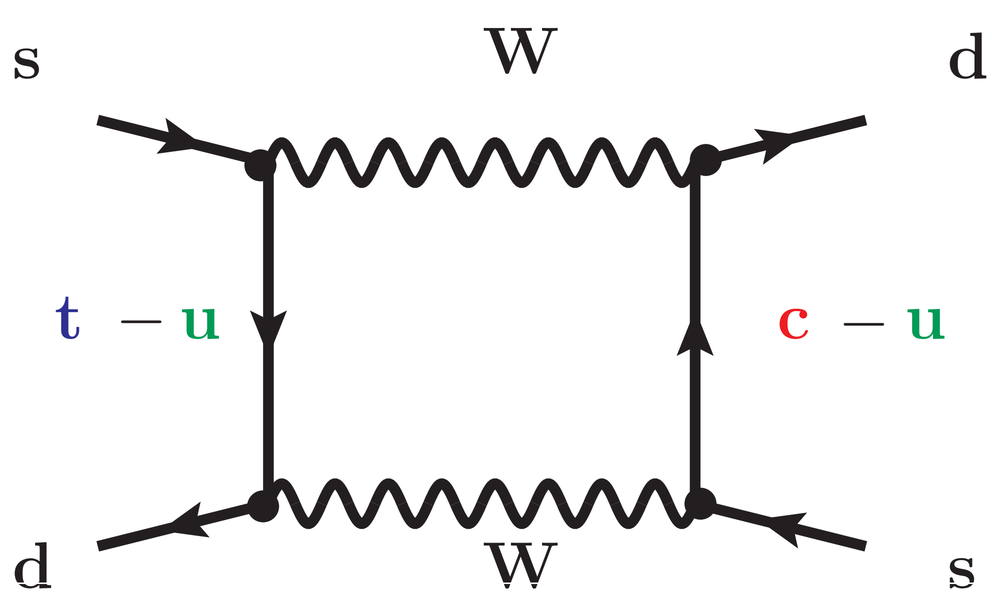
$\lambda_{\text{Cabibbo}} \approx 0.2$

Dominant contribution to ε_K (≈+74%)

η_{tt} = 0.5765(65) at NLO QCD [Buras et al. '90]

|ΔS|=2 Hamiltonian

$$H^{|\Delta S|=2} \propto \left[\lambda_c^2 \eta_{cc} S\left(\frac{m_c^2}{M_W^2}\right) + \lambda_t^2 \eta_{tt} S\left(\frac{m_t^2}{M_W^2}\right) + \lambda_c \lambda_t \eta_{ct} S\left(\frac{m_c^2}{M_W^2}, \frac{m_t^2}{M_W^2}\right)\right] \tilde{Q}^{|\Delta S|=2}$$



$$\approx \lambda_{\text{Cabibbo}}^6 \times \eta_{ct} \times \frac{m_c^2}{M_W^2} \log \frac{m_c^2}{M_W^2}$$

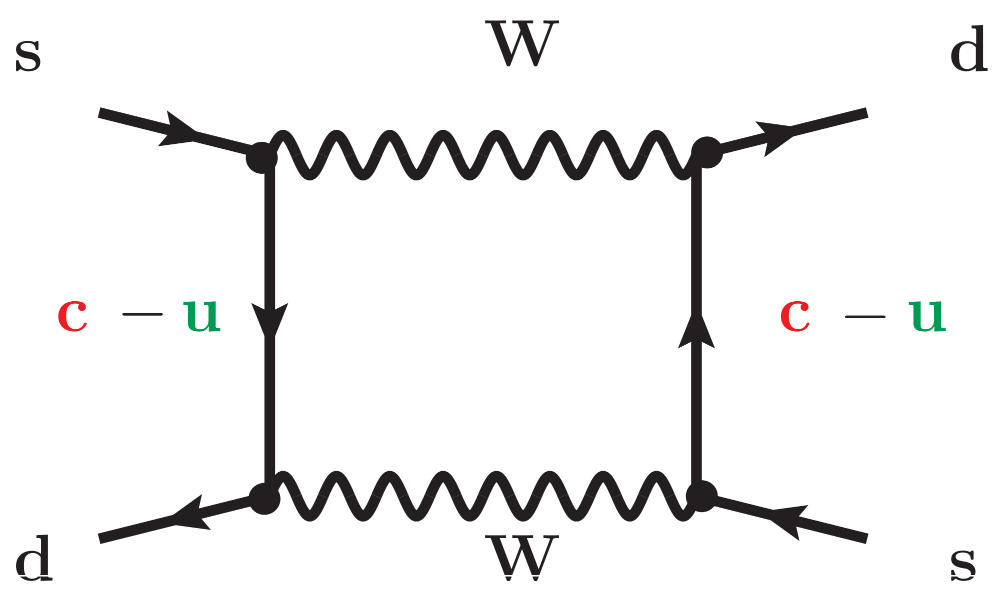
Contributes ≈+40% to ε_K (NLO)

$\eta_{ct} = 0.47(4)$ at NLO QCD [Herrlich, Nierste '96]

|ΔS|=2 Hamiltonian

$$H^{|\Delta S|=2} \propto \left[\lambda_c^2 \eta_{cc} \boxed{S\left(\frac{m_c^2}{M_W^2}\right)} + \lambda_t^2 \eta_{tt} S\left(\frac{m_t^2}{M_W^2}\right) + \lambda_c \lambda_t \eta_{ct} S\left(\frac{m_c^2}{M_W^2}, \frac{m_t^2}{M_W^2}\right)\right] \tilde{Q}^{|\Delta S|=2}$$

$$\approx \lambda_{\mathrm{Cabibbo}}^6 \times \eta_{cc} \times \frac{m_c^2}{M_W^2}$$

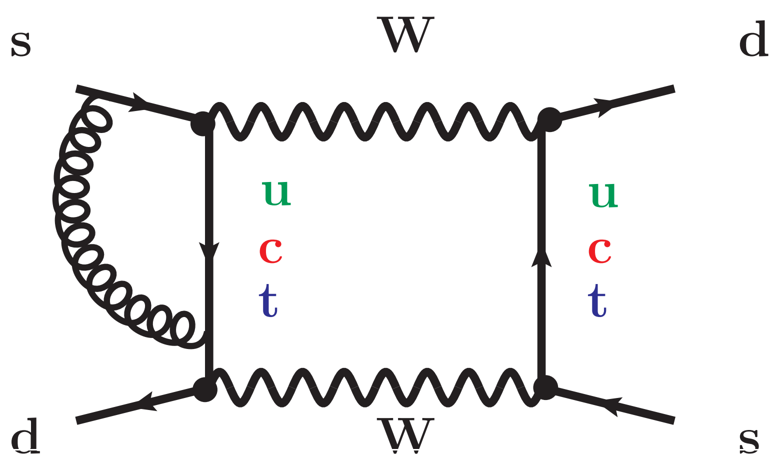


Smallest contribution to ε_K (≈-14% at NLO)

η_{cc} = 1.40(35) at NLO QCD [Herrlich, Nierste '94, '03]

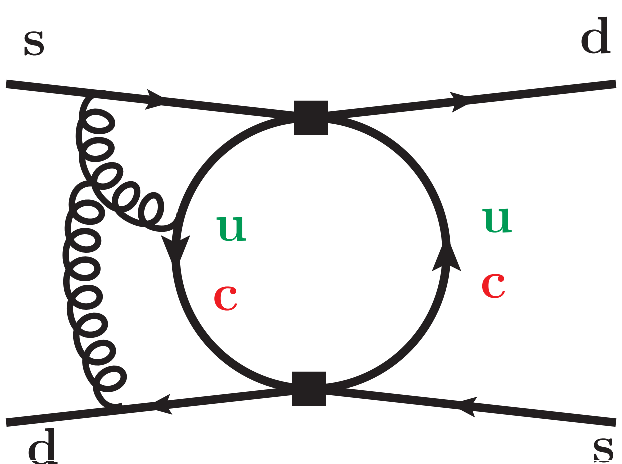
η_{ct} at NNLO: Calculation

μ_{NP}



Matching at M_W: initial condition

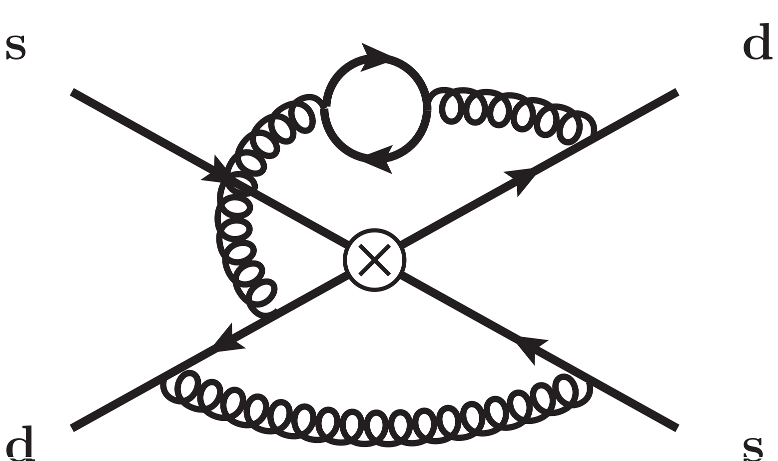
μ_{Wt}



Running to m_c

- O(100 000) diagrams
- RGE for double insertions
- Include threshold corrections at m_b

μ_b



Matching at m_c
to effective 3-flavour theory

μ_c

Λ_{QCD}

Computation of the diagrams

We had to compute O(100 000) one-, two-, and three-loop vacuum Feynman diagrams. We use

- qgraf[Nogueira '93], MATAD[Steinhauser '01]
- Our own FORM[Vermaseren '00] routines
- FeynArts [Hahn '01], FIRE [Smirnov '08], Mathematica

Two fully independent setups, renormalisation, numerics

η_{ct} at NNLO: Result...

$$\Rightarrow \eta_{ct} = 0.496$$

Corresponds to +7% shift!

η_{ct} at NNLO: Result...

$$\Rightarrow \eta_{ct} = 0.496 \pm ???$$

Corresponds to +7% shift!

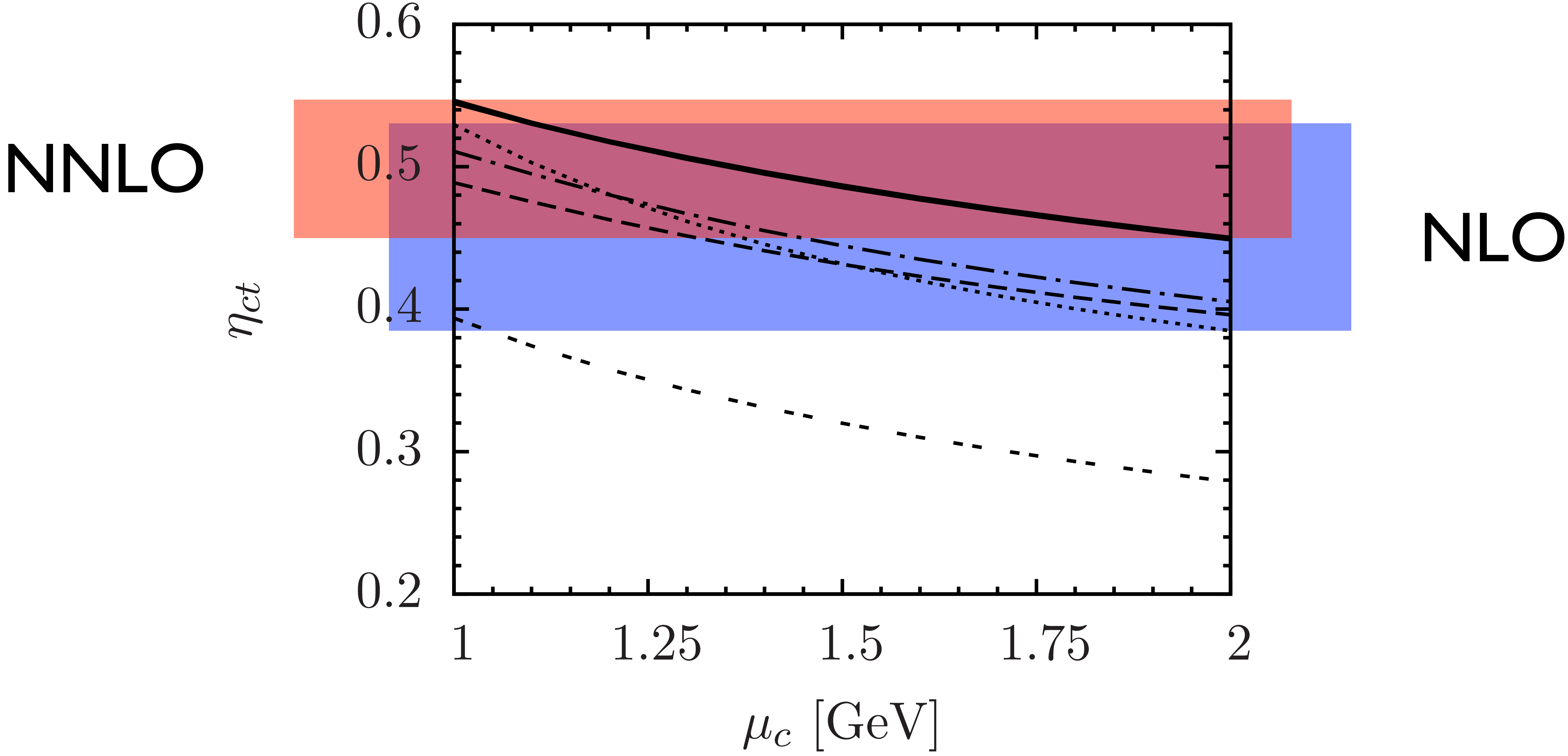
Theory uncertainty

Consequence of the truncation
of the perturbative expansion

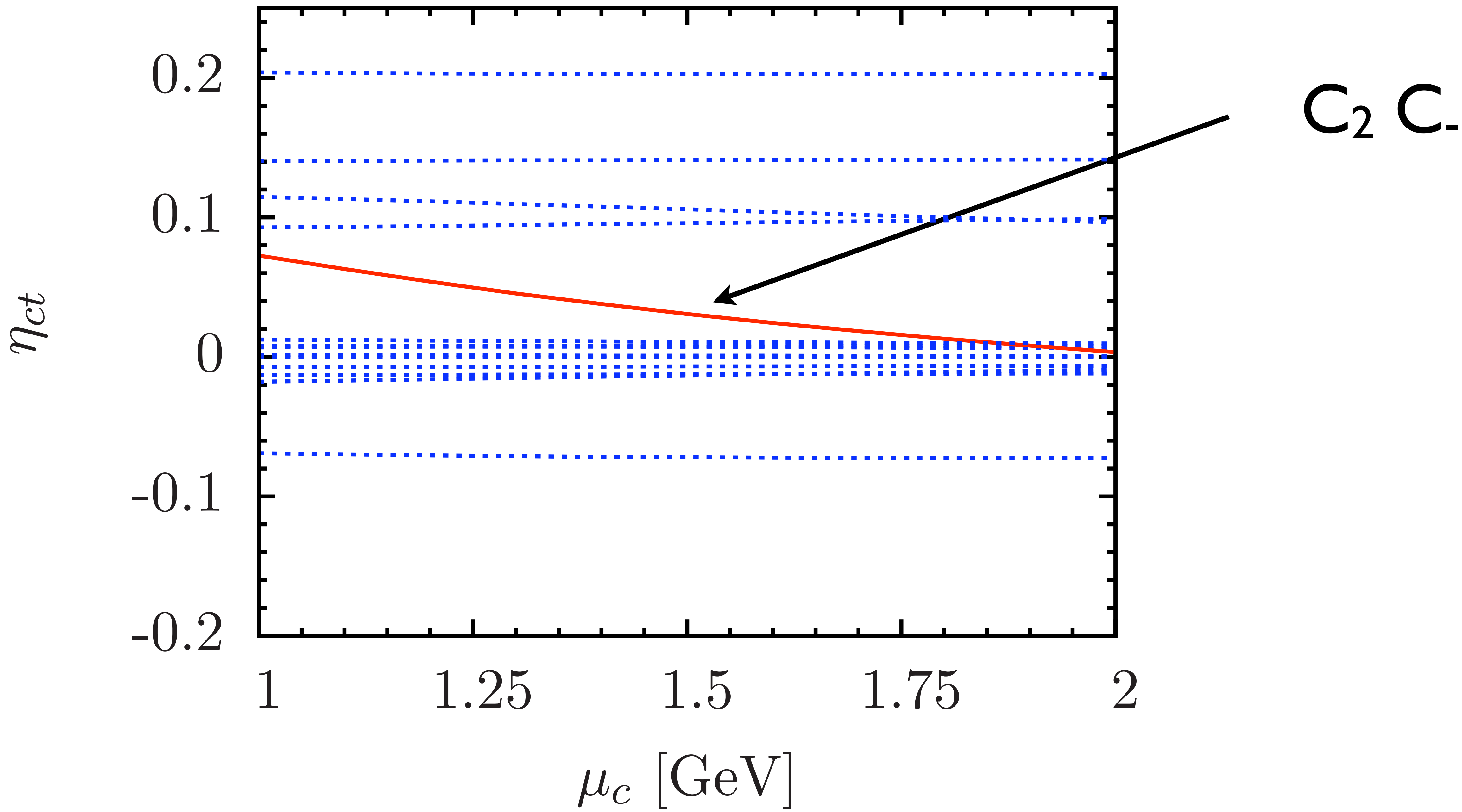
η_{ct} (and η_{cc}) is scale- and scheme independent!

- Study residual scale dependence
- Size of NNLO corrections

η_{ct} at NNLO: Scale Dependence



η_{ct} at NNLO: Scale Dependence

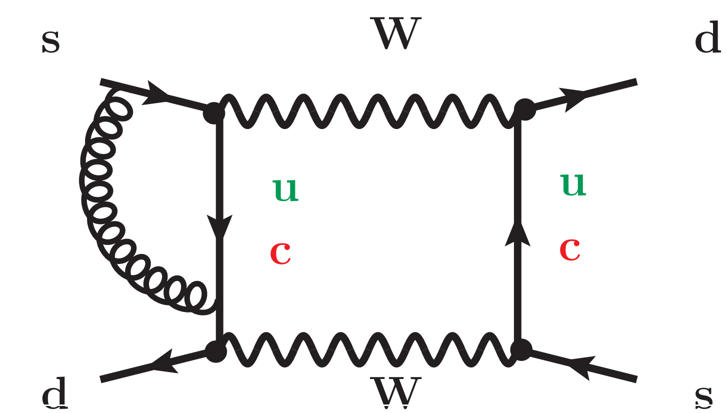


η_{ct} at NNLO: Result

$$\Rightarrow \eta_{ct} = 0.496 \pm 0.047$$

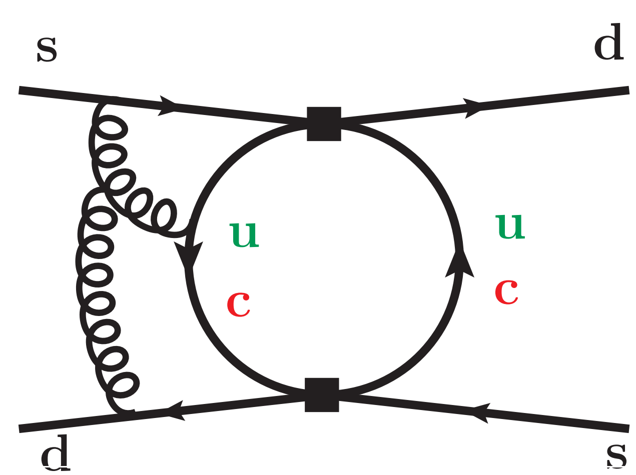
η_{cc} at NNLO: Calculation

μ_{NP}



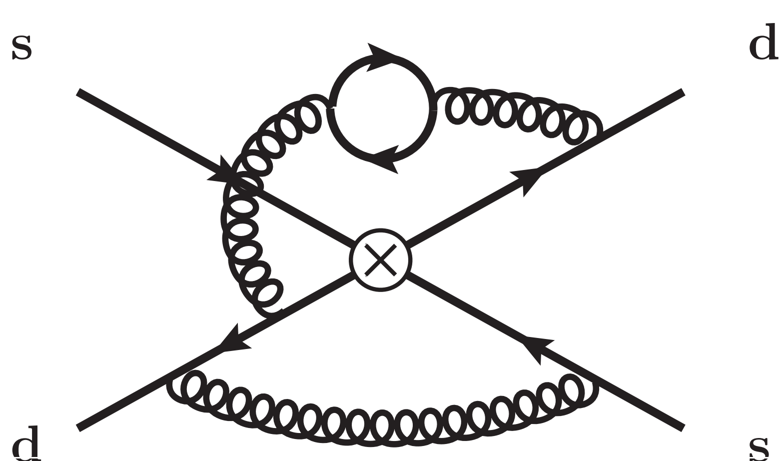
Matching at M_W: vanishes by GIM [Witten '76]

μ_{Wt}



Running to m_c:
only $\Delta S=1$ operators contribute

μ_b



Matching at m_c:

μ_c

- Match to effective 3-flavour theory
- O(100 000) 3-loop Feynman diagrams

Λ_{QCD}

η_{cc} at NNLO: Result...

$$\langle Q_2 Q_2 \rangle = \frac{69738523}{113400} + \frac{47407}{8505}\pi^2 - \frac{1733}{810}\pi^4 + \frac{1872}{35}\sqrt{3}\,\mathrm{ImLi}_2((-1)^{1/3})$$
$$+24(\mathrm{ImLi}_2((-1)^{1/3}))^2 - \frac{32}{27}\pi^2\log(2)^2 + \frac{32}{27}\log(2)^4 + \frac{563}{18}\log\frac{\mu_c^2}{m_c^2}$$
$$+\frac{32}{3}\pi^2\log\frac{\mu_c^2}{m_c^2} + \frac{193}{3}\log^2\frac{\mu_c^2}{m_c^2} + \frac{256}{9}\mathrm{Li}_4(1/2) - \frac{15145}{54}\zeta_3$$

η_{cc} at NNLO: Result...

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$$\Rightarrow \eta_{cc} = 1.87$$

Corresponds to +36% shift!

η_{cc} at NNLO: Result...

$$
\begin{aligned}
\langle Q_2 Q_2 \rangle = \frac{69738523}{113400} + \frac{47407}{8505}\pi^2 - \frac{1733}{810}\pi^4 + \frac{1872}{35}\sqrt{3}\,\mathrm{ImLi}_2((-1)^{1/3}) \\
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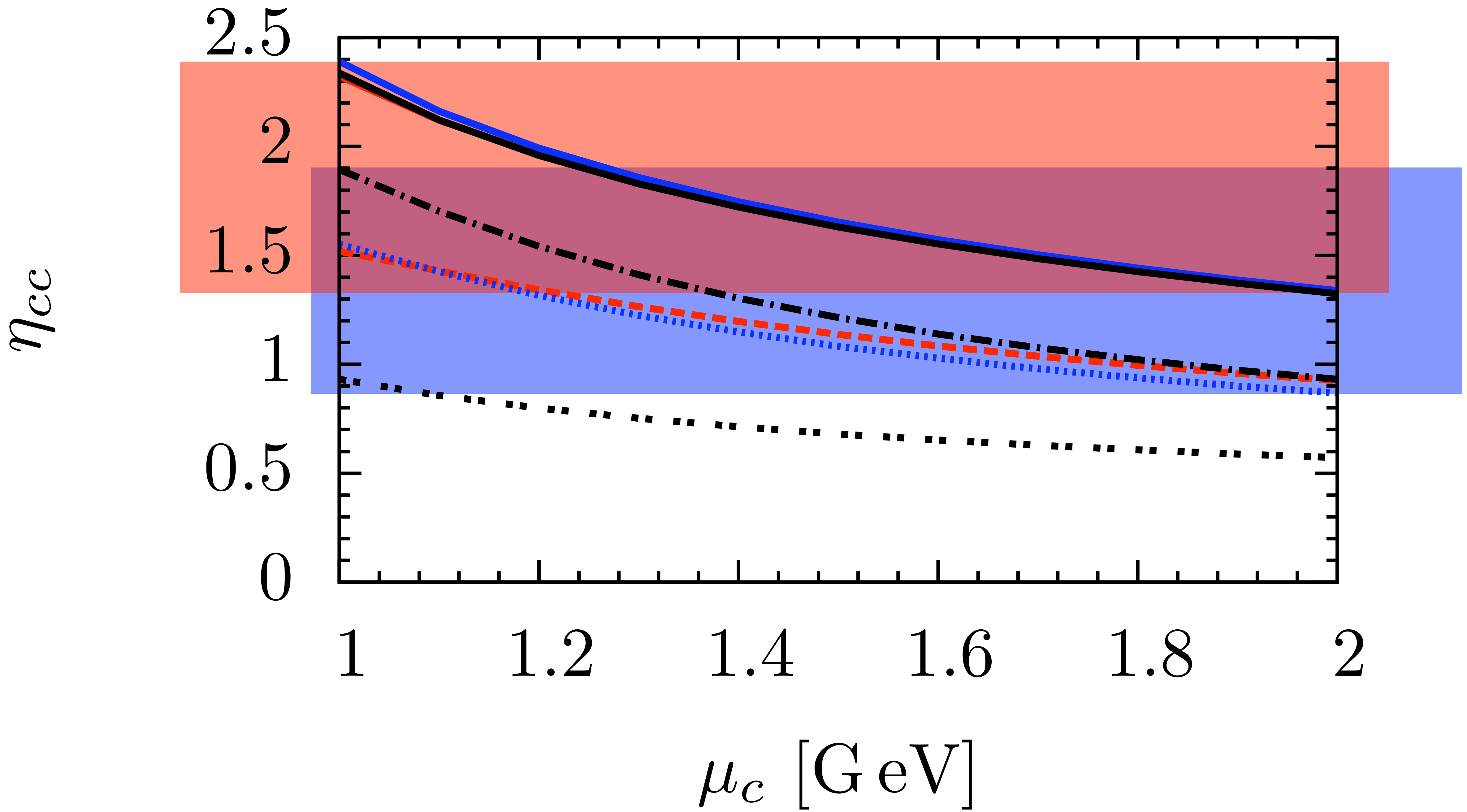
\end{aligned}
$$

$\Rightarrow \eta_{cc} = 1.87 \pm ???$

Corresponds to +36% shift!

η_{cc} at NNLO: Scale Dependence

Matching @ mc(mc)



η_{cc} at NNLO: Convergence

$$\eta_{cc}/\alpha_s\text{^}(2/9) = 1 + \alpha_s\ (0.25 + 0.64\ L_c) + \alpha_s\text{^}2\ (1.20 + 0.44\ L_c + 1.08\ L_c\text{^}2)$$

$L_c = \log(m_c/M_W) = -\ 4.14$ $\alpha_s = \alpha_s(m_c) = 0.35$

$$\eta_{cc}/\alpha_s\text{^}(2/9) = 1 + (0.09 + 0.9) + (0.15 - 0.2 + 2.3)$$

η_{cc} at NNLO: Uncertainty

- Divergence of perturbation series?
- Make use of RI-SMOM schemes, at NNLO?
- Compute charm on the lattice???

Preliminary prescription:
Add NNLO shift and scale uncertainty in quadrature

$$\boxed{\eta_{cc} = 1.87 \pm 0.76}$$

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„Data! Data! Data!“ he cried impatiently.

[Sherlock Holmes, The adventure of the copper beaches]

$|\varepsilon_K|$ - Numerics

using only experimental and lattice input:

$$|\varepsilon_K| \propto \kappa_\varepsilon B_K |V_{cb}|^2 \xi_s \sin\beta$$

$$\times(|V_{cb}|^2 \xi_s \cos\beta \; \eta_{tt} S(x_t) + \eta_{ct} S(x_c,x_t) - \eta_{cc} S(x_c))$$

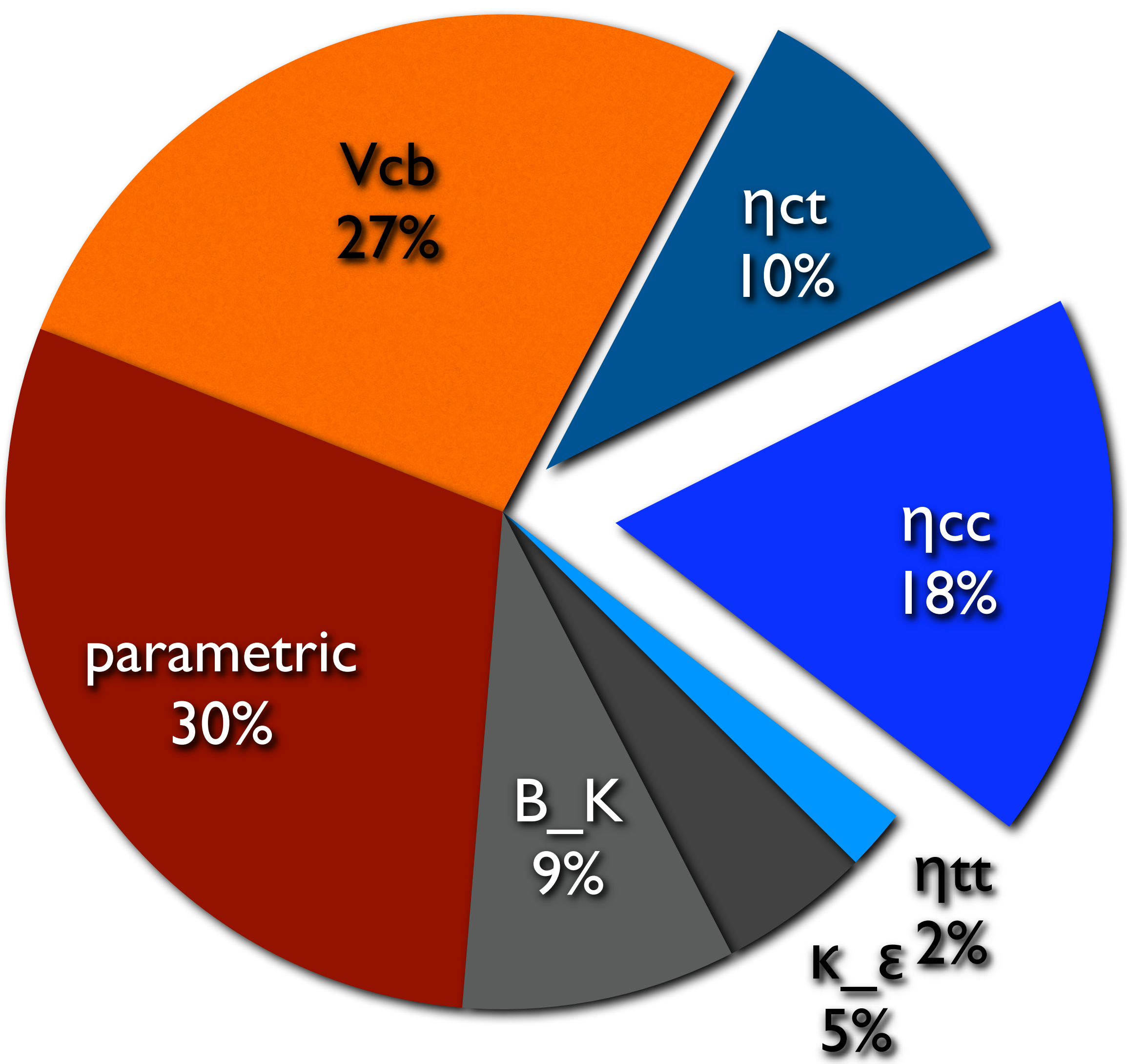
$\lvert V_{cb}\rvert$	0.0406(13)
$\sin(2\beta)$	0.671(23)
ξ_s	1.243(28)
B_K	0.725(26)
κ_ε	0.94(2)

η_{ct}	0.496(47)
η_{cc}	1.87(76)
η_{tt}	0.5765(65)

$$\xi_s = \frac{F_{B_s}\sqrt{\hat{B}_s}}{F_{B_d}\sqrt{\hat{B}_d}} \qquad x_q = \frac{m_q^2}{M_W^2}$$

$|\varepsilon_K|$ - Result & Error Budget

$|\varepsilon_K| = 1.81(28) \times 10^{-3}$ using $\eta_{cc} = 1.87(76)$ and $\eta_{ct} = 0.496(47)$



cf. $|\varepsilon_K| = 1.83(27) \times 10^{-3}$ (NLO)

cf. $|\varepsilon_K| = 1.53(23) \times 10^{-3}$
using $|V_{cb}| = 0.0387(11)$ (exclusive)

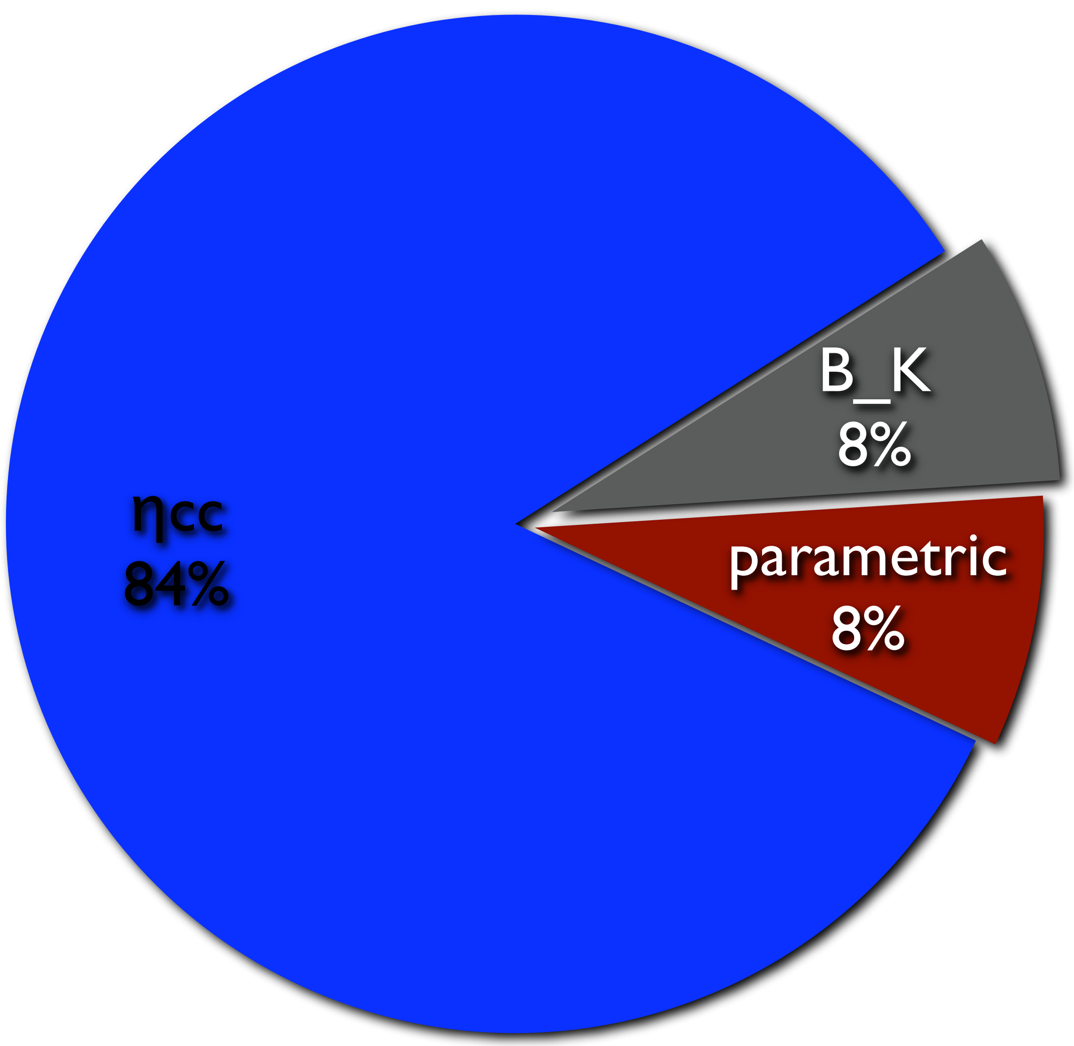
Experiment [PDG '10]:

$$|\epsilon_K| \stackrel{\text{exp.}}{=} 2.228(11) \times 10^{-3}$$

$\Delta M_K(SD)$ -- Numerics

$\Delta M_K(SD) = (3.0 \pm 1.2) \times 10^{-15}$ GeV using $\eta_{cc} = 1.87(76)$

cf. $\Delta M_K(SD) = (2.2 \pm 0.9) \times 10^{-15}$ GeV using $\eta_{cc} = 1.38(53)$ (NLO!)



$\Delta M_K(SD)$
$= 3.483(6) \times 10^{-15}$ GeV
[PDG '10]

$(86 \pm 34)\%$ short distance

Outline

- Introduction: Effective Field Theory
- Calculation of ε_K
  - Long-distance contributions
  - Short-distance contributions
- Result and error budget
- Summary and outlook

In five minutes you will say that it is all so absurdly simple.

[Sherlock Holmes, The adventure of the dancing men]

- ε_K is a powerful constraint on models of New Physics.
- Progress in non-perturbative calculations (in particular B_K)
- NNLO calculation yields +7% shift of charm-top-quark contribution to ε_K, leading to $\eta_{ct} = 0.496(47)$
- NNLO calculation yields +36% shift of charm-quark contribution to ε_K, leading to $\eta_{cc} = 1.87(76)$
- SM prediction for ε_K is shifted by ≈-1% to $\varepsilon_K = 1.81(28)\times10^{-3}$.

We might already see new physics --

but first we need to understand the Standard Model!