In Memoriam Robert Brout...

Why Neutrinos are different …

• Very low mass
• Large leptonic mixing
• Leptonic number conserved or not ?
  ….With link to matter-antimatter asymmetry

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Based on work with Maxim Libanov, Serguey Troitsky, Emin Nugaev, Ling Fu-Sin
What we now from oscillations

Our Generic prediction: large mixings, inverted hierarchy, suppressed neutrinoless double beta decay
Generic prediction: large mixings, inverted hierarchy, suppressed neutrinoless double beta decay.

Neutrino-less double beta decay controlled by weighted sum of masses (with phases/signs of mixings entering)

Inverted Hierarchy

Mass scale

Automatically get
Why Neutrinos are Different

How LHC could confirm (but not exclude) the model

How LHC can compete with fixed target Lepton Flavour Violation expts

From now on …

An « ordinary » Z’ (with same couplings as Z)

e+ e- pairs

for later …

« exotic » Z’

\[ \mu^+ e^- \text{ pairs} \quad (\gg \mu^- e^+) \]

Fig. 1. Number of events as a function of invariant mass \( M \), with \( \kappa = M/(100\text{TeV}) \).
In a nutshell:

Previous work:
- One family in 6D and proper boundary conditions → 3 families in 6D
- At lowest order in Cabibbo mixing Charged fermion masses are
  - Diagonal
  - Strongly hierarchical

NOW
- At same order, we get 4D Majorana neutrinos with Antidiagonal mass matrix
- This yields, in a generic way:
  - Large mixings in the neutrino sector
  - Inverted Hierarchy
  - Pseudo-Dirac structure (further suppression of neutrinoless double beta decay)
- Not as automatic, but typical: measurable $\Theta_{13}$

At LHC, this can result in std model like $Z'$ and later more exotic signals ($Z'^{\pm} \rightarrow \mu^+\mu^- \rightarrow \mu^-\mu^+$)
A very few words about extra dimensions … start with ONE extra spatial dim.

**What are Zero Modes?**

Dirac equation in N+1 dimensions,
For a fermion interacting with a field $\Phi$:

For ONE compact extra dim

$$0 \leq y \leq 2\pi R$$

$$\Psi(x^\mu, y) = \sum \Psi_n(x^\mu)e^{in\frac{y}{2\pi R}}$$

$$i\partial_\nu \gamma^\nu \Psi_n(x^\mu)e^{in\frac{y}{2\pi R}} = \left(\frac{n}{2\pi R}i\gamma_5 + \Phi\right)\Psi_n(x^\mu)e^{in\frac{y}{2\pi R}}$$

$\rightarrow$ **Kaluza 2a Klein tower**

$\Downarrow$ effective mass

$$\approx \frac{1}{1 \text{ TeV}}$$

$= 0 \Rightarrow$ zero modes.
For 2 compact extra dim

\[
\Psi(x^\mu, x^4, x^5) = \sum_{n,l} \psi_{n,l}(x^\mu) f_{n,l}(x^4, x^5)
\]

\[
i\partial_\nu \gamma^\nu \psi_{n,l}(x^\mu) f_{n,l}(x^4, x^5) = \psi_{n,l}(x^\mu)(\Phi - i\partial_4 \gamma^4 - i\partial_5 \gamma^5) f_{n,l}(x^4, x^5)
\]

Large effective mass \[\gg 1 \text{ TeV} \]

\[f = 0 \rightarrow \text{zero mode}\]

Look for zero modes …
Use of dimensional reduction obtain 3+1-dim chiral spinors (L) : use of topological singularities in the extra dimensions to get zero modes, break LR symmetry.

Solitonic background: index theorem localizes one chiral Fermion ; Alternatively, orbifold

Vortex with winding number n localizes n chiral massless fermion modes in 3+1
3 families from one in 5+1 dim

\[ \Phi = e^{i n \phi} \]

we assume a background scalar field \( \Phi \) providing a vortex in the 2 extra dimensions; It vanishes at the origin – where we live!

The 3 fermion modes have different shapes, and different winding properties in the extra dimension variable \( \phi \)

For some reason, \( n=3 \) !!!
The transition from 6-D to the zero-modes of 4D dramatically affects

Each « zero » (massless) mode only has 2-spinor degrees of freedom:

For instance,

\[ L \sim \sum_n \begin{pmatrix} \psi_{+R} \\ \psi_{+L} \\ \psi_{-L} \\ \psi_{-R} \end{pmatrix} \]

\[ \begin{pmatrix} f_{3-n}(r) e^{i(3-n)\phi} \psi_{L_n}(x^\mu) \\ f_{n-1}(r) e^{i(1-n)\phi} \psi_{L_n}(x^\mu) \end{pmatrix} \]
# Field Content

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<tr>
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<td>$\Phi$ $F(r)e^{i\theta}$</td>
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<td>$F(0) = 0, F(\infty) = v_{\Phi}$</td>
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<td>axial $+3/2$</td>
<td>$+1/6$</td>
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<td>$U$ 3 R zero modes</td>
<td>axial $-3/2$</td>
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<td>fermion</td>
<td>$E$ 3 R zero modes</td>
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<td>$-1$</td>
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Table 1: Scalars and fermions with their gauge quantum numbers. For convenience, we describe here also the profiles of the classical scalar fields and fermionic wave functions in extra dimensions.
The 3 fermion modes have different shapes, and different winding properties in the extra dimension variable $\phi$.

The 4D mass matrices are obtained by integrating $r$ and $\phi$, and are the convolution of these curves.
For Quarks and Dirac fermions, we get a mass matrix like:

\[
\begin{pmatrix}
\text{small} & \varepsilon \\
\text{medium} & \varepsilon \\
\text{large} & \varepsilon
\end{pmatrix}
\]

Additional couplings involving the vortex field, with winding $e^{i\phi}$ can give the small Cabibbo mixings $\varepsilon$.

The scheme is very constrained, as the profiles are dictated by the equations, instead of being imposed by hand, like in multilocalisation.
Neutrinos ARE different

In the same context (0th order in Cabibbo mixing), we will get indeed (see later):

\[ M_\nu \sim \begin{pmatrix} m & \cdot & \cdot \\ \cdot & \mu & \cdot \\ m & \cdot & \cdot \end{pmatrix} \]

Where \( m >> \mu \)

After 45° 1-3 rotation and 23 permutation, this leads to an inverted hierarchy, (minute solar mass difference found between the heavier neutrinos)

\[ M_\nu \sim \begin{pmatrix} m & \cdot & \cdot \\ \cdot & -m & \cdot \\ \cdot & \cdot & \mu \end{pmatrix} \]

The – sign may be absorbed in the mixing matrix, but contributes destructively to the effective mass for neutrinoless double beta decay (Pseudo-Dirac structure when full Cabibbo-like mixing is introduced)
WHY the difference? --- return in more detail to the 6D spinors,

$$\Psi = \begin{pmatrix} \psi_{+R} \\ \psi_{+L} \\ \psi_{-L} \\ \psi_{-R} \end{pmatrix}$$

Zero modes

$$\frac{i}{4} \phi_1 = \frac{\sqrt{2}}{2} \frac{i}{4} \phi_1$$

$$\Rightarrow \begin{pmatrix} 0 \\ \phi_1 \\ \phi_1 \\ 0 \end{pmatrix}$$

Solutions:

$$\begin{pmatrix} \phi_1 \\ \phi_1 \end{pmatrix}$$

$$\begin{pmatrix} \phi_1 \\ -\phi_1 \end{pmatrix}$$
WHY the difference? --- return in more detail to the 6D spinors,

\[ \Psi = \begin{pmatrix} \psi^+ R \\ \psi^+ L \\ \psi^- L \\ \psi^- R \end{pmatrix} \]

In each case, the massless mode only has 2-spinor degrees of freedom:
For instance,

\[ L \sim \sum_n \begin{pmatrix} 0 \\ f_{3-n}(r) e^{i(3-n)\phi} \psi_{Ln}(x^\mu) \\ f_{n-1}(r) e^{i(1-n)\phi} \psi_{Ln}(x^\mu) \\ 0 \end{pmatrix} \]
WHY the difference?

It comes from rotation invariance in the 2 extra dimensions...
WHY the difference? --- return in more detail to the 6D spinors, For the charged spinors, we have both L and R spinors bound to the vortex.

\[ L \sim \sum_n \begin{pmatrix} 0 \\ f_{3-n}(r) \ e^{i(3-n)\phi} \psi_{Ln}(x^\mu) \\ f_{n-1}(r) \ e^{i(1-n)\phi} \psi_{Ln}(x^\mu) \\ 0 \end{pmatrix}, \quad R \sim \sum_n \begin{pmatrix} f_{n-1}(r) \ e^{i(1-n)\phi} \chi_{Rn}(x^\mu) \\ 0 \\ 0 \\ f_{3-n}(r) \ e^{i(3-n)\phi} \chi_{Rn}(x^\mu) \end{pmatrix} \]

Effective Lagrangian: integrate over \( r \) and \( \phi \).

\[ \bar{R} \ L = \sum_{n,n'} \bar{R}_n \cdot L_{n'} \]

\[ \int_{0}^{2\pi} d\phi \int dr f_{3-n} f_{3-n'} \chi \left( \frac{i(n-n')}{\phi} \right) \]

\( S(n-n') \) diagonal
For neutrinos (using only Majorana-type 4D mass term) we will get (through see-saw discussed later)

\[ L^c L \Rightarrow \sum_n (L^c_n) L_n \]

\[ L \sim \sum_n \begin{pmatrix}
0 \\
\int f_{3-n}(r) e^{i(3-n)\phi} \psi_L(x^\mu) \\
\int f_{n-1}(r) e^{i(1-n)\phi} \psi_L(x^\mu) \\
0
\end{pmatrix} \]

\[ \Rightarrow \int d\phi e^{i(4-n-n')\phi} \]

\[ \Rightarrow \delta(n+n'-4) \]

\[ \Rightarrow \left( \begin{array}{c}
m^1 \\
m^2 \\
m^3
\end{array} \right) \]
The Majorana term can be traced to the « Majorana mass term » in the Lagrangian (not to be confused with a non-existent 6D Majorana spinor) through a see-saw. It leads to a contribution proportional to the effective propagator after summing over a large number of proper modes of the bulk spinor $N$:

$$\gamma_\nu W N \rightarrow M \overline{N} c N$$

Each mode $n$ of the bulk spinor $N$

$$\mathcal{L} = \frac{1}{2} \langle \mathcal{P}^2 - \left(\frac{2m_n}{R}\right)^2 - M^2 \rangle$$

Small $m$ OR $M \ll \frac{1}{R} \text{ (GeV OK)}$
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<tr>
<td>fermion (N)</td>
<td>massive modes (\chi_{\lambda, m}, \xi_{\lambda, m})</td>
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<td>0</td>
</tr>
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Table 1: Field content of the model (scalars and leptons only).
We can then introduce some extra terms, carrying winding, to generate the Cabibbo-like mixings,

For charged leptons

$$\frac{\mathcal{L}_E}{\sqrt{-\det g_{AB}}} = \sum_{S_+} Y_{l_+}^+(S_+^l) S_+^l H \bar{L} \frac{1 + \Gamma_7}{2} E + \sum_{S_-} Y_{l_-}^-(S_-^l) S_-^l H \bar{L} \frac{1 - \Gamma_7}{2} E + \text{h.c.}$$

For neutrinos

$$\frac{\mathcal{L}_D}{\sqrt{-\det g_{AB}}} = \sum_{S_+} Y_{\nu_+}^+(S_+^l) \tilde{H} S_+ \bar{L} \frac{1 + \Gamma_7}{2} N + \sum_{S_-} Y_{\nu_-}^-(S_-^l) \tilde{H} S_- \bar{L} \frac{1 - \Gamma_7}{2} N + \text{h.c.}$$

$$S_+ = \Phi^*, X^*, X^{*2}\Phi, \ldots$$

$$S_- = X^2, X\Phi, \Phi^2, \ldots$$
Example: for charge leptons, on a sphere:

\[
\begin{pmatrix}
\frac{\delta^4 \beta}{3} & \frac{E \delta^3}{3 \beta} & F \delta^2 \\
\frac{E \delta^3}{3} & \frac{\delta^2}{2} & E' \delta^2/2 \\
G \delta^2 & E' \delta^2/2 & 1
\end{pmatrix}
\]

\[3C \left( \begin{array}{c}
\sim 1 \\
\text{vertex size} \rightarrow \sim 1 \\
E, \text{ Yukawa off-diag} \rightarrow 7 \\
E', \text{ Yukawa off-diag} \rightarrow 2 \\
F, G \rightarrow 0
\end{array} \right)
\]

\[C: \text{ main Yukawa } = 1.6\]
Neutrino matrix:

\[
\begin{pmatrix}
0 & 25 & -45.1 \\
25 & -3 & 0 \\
-45.1 & 0 & 0 \\
\end{pmatrix}
\text{meV}
\]

\[\text{Note: } |\mu_{23}/\mu_{13}| \text{ given by dynamics.}\]
Results of this simple approximation:

\[ m_e \sim 6 \text{ MeV} \]
\[ m_\mu \sim 85 \text{ MeV} \]
\[ m_\tau \sim 1.7 \text{ GeV} \]

\[ \{ -51.9358, 51.2292, -2.29345 \} \]

\[ U_{\text{PMNS}} = \begin{pmatrix} 0.78 & 0.62 & 0.1 \\ -0.47 & 0.68 & -0.57 \\ -0.42 & 0.40 & 0.82 \end{pmatrix} \]

\[ m_\nu = \begin{Bmatrix} -51.9 \\ 51.2 \\ -2.3 \end{Bmatrix} \text{ meV} \]

\[ \langle m_{3\mu} \rangle = 19.3 \text{ meV} \]
Some other developments:

- compactification of the 2 extra dim on a sphere instead of a plane (avoid localisation of gauge bosons) – spinors modified, but conclusions kept (already mentioned) with extra scale $1/R$

- phenomenological implications of the excited modes..

- constraint on B-E-H boson (Libanov and Nugaev: LIGHT)
IMPORTANT: « family number » (n) is approximatively conserved! - $e^{in\phi}$ plays somewhat like a U(1) horizontal symmetry

2 extra dim: $\rightarrow$ ll gauge bosons, possess 2 types of Kaluza-Klein excitations in particular, Z and Gluons

- radial $Z'_0$ (approx. flavour conserving)
- angular: $Z'\pm_1$ behaves like $e^{i\phi}$

$Z'\pm_1$ thus carries « family number »

Almost the same couplings as Z

From now on ...

for later ...

Flavour violating
« family number » (n) is approximatively conserved! - somewhat like U(1) horizontal symmetry $e^{i\phi}$

$Z_{1}$ thus carries « family number »

Flavour conserving

Flavour violating,
Typical limit

$$\kappa_L \rightarrow \mu^- e^+ \text{ or } \mu^+ e^- \quad \text{B.R.} < 10^{-12}$$

Expect thus typical mass scale $M_{Z_1}/\kappa > (10^{12})^{1/4} M_Z = \kappa 100 \text{ TeV}$

In fact, the small overlap of wave functions implies

some suppression of the coupling; \hspace{1cm} \kappa \ll 1

\rightarrow \text{ bound becomes } M(Z_1) > \kappa 100 \text{ TeV}

Take $\kappa$ from .01 to 0.5 \rightarrow \text{Plot for } M(Z_1) > 1 \text{ TeV}--
At LHC,

\[ d + \bar{s} \rightarrow \mu^+ e^- \]

\[ pp \rightarrow \bar{\nu} e + s + x \rightarrow \Lambda, \bar{K}, \ldots \]
We saturate the bound on $\kappa$

$$\kappa = 100 \text{ TeV}/M_{Z_1}$$

(100 fb$^{-1}$, 14 TeV)

numbers for $\mu^-\, e^+$ are ONE ORDER below at LHC, due to quark content of protons

Fig. 1. Number of events for $(\mu^+e^-)$ pairs production as a function of the vector bosons mass $M$, with $\kappa = M/(100 \text{ TeV})$. (also s left in underlying event)

LHC thus has the potential (in a specific model, of course) to beat even the very sensitive fixed-target $K \rightarrow \mu e$ limit!

$t + c$ or $\bar{b} + s$ are similarly produced by the gluon excitations,

Expect a few 1000’s events --- but must consider background!
From now on ...

\[ M(Z'_0) = M(Z'\pm) > \kappa \ 100 \ \text{TeV} \]
\[ \kappa \ 0.01\ldots0.3 \]

Find the \(Z'_0, W'_0, \)
...also expect gluon recurrences

An « ordinary » \(Z'\)
(with same couplings as \(Z\))

No \(\kappa\) suppression

for later ...

Find the \(Z'\pm,\)
\(\kappa\) suppressed

Fig. 1. Number of events for \((\mu^+e^-)\) pairs production as a function of the vector bosons mass \(M,\) with \(\kappa = M/(100\text{TeV})\).

(100 fb\(^{-1}\), 14TeV)
In a nutshell:

• One family in 6D and proper boundary conditions $\rightarrow$ 3 families in 6D
• At lowest order in Cabibbo mixing, Charged fermion masses are diagonal strongly hierarchical

At LHC, this can result in exotic signals ($Z' \rightarrow \mu^+e^- \gg \mu^-e^+$)

• At same order, we get 4D Majorana neutrinos with Antidiagonal mass matrix
• This yields, in a generic way:
  Large mixings in the neutrino sector
  Inverted Hierarchy
  Pseudo-Dirac structure (further suppression of neutrinoless double beta decay)
• Not as automatic, but typical: measurable $\Theta_{13}$