Research report from the RBC collaboration

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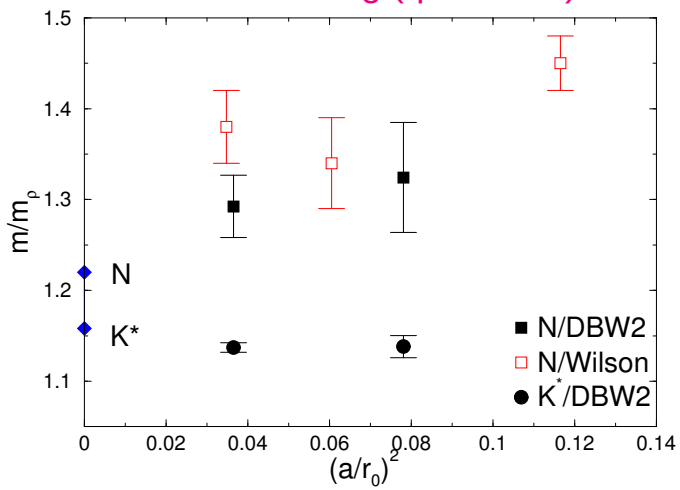
RBC Projects

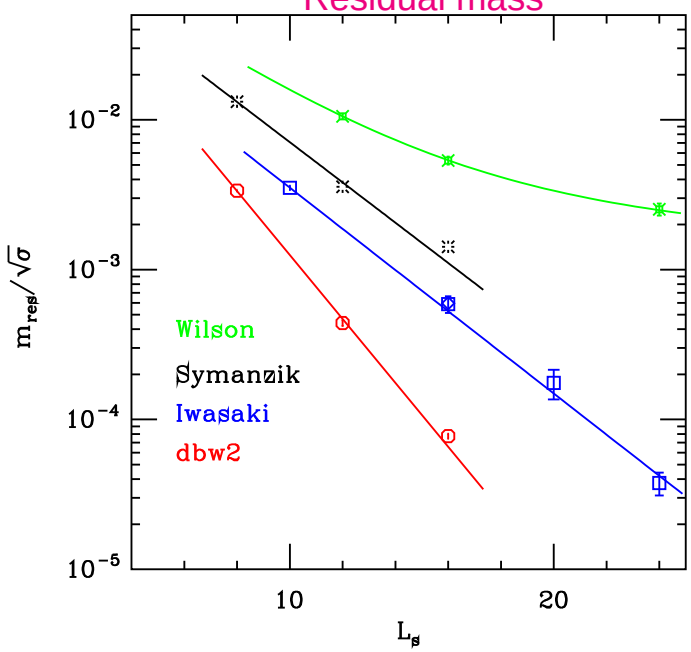
- Weak Matrix Elements
- Dynamical Domain Wall Fermions
- Nucleon Structure Functions
- Nucleon axial charge
- Proton Decay

Domain wall fermions

Scaling (quenched)



Residual mass



Why Domain Wall Fermions?

- Excellent chiral properties

  at finite lattice spacing:

  - $L_s \to \infty$: Exact chiral symmetry
  - L_s finite: Residual chiral breaking

- Have $O(a^2)$ errors
- Excellent scaling properties
- No operator improvement to $O(a^2)$ needed.
- Simpler renormalization due to chiral symmetry.

  [hep-lat/0102005, C.Dawson LAT02].
- No exceptional configurations

- Using the DBW2 gauge action the residual chiral symmetry breaking is negligible

  [KO, Y.Aoki LAT01,RBC hep-lat/0211023].

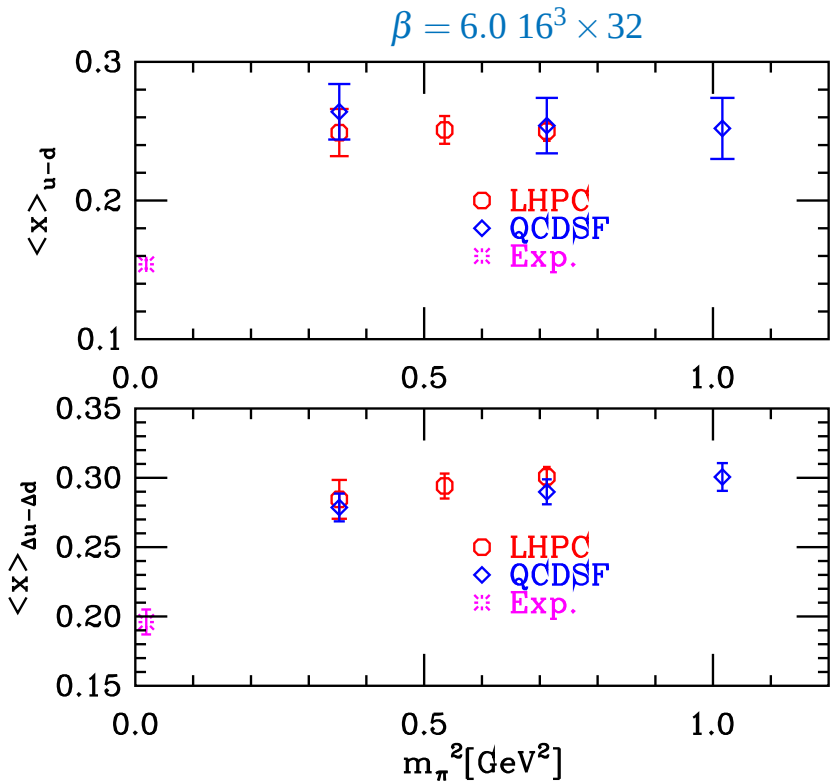
Moments of Structure Functions

$$
\begin{aligned}
2\int_0^1 dx x^{n-1} F_1(x,Q^2) &= \sum_{q=u,d} c_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n \rangle_q(\mu) + \mathscr{O}(1/Q^2), \\
\int_0^1 dx x^{n-2} F_2(x,Q^2) &= \sum_{f=u,d} c_{2,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n \rangle_q(\mu) + \mathscr{O}(1/Q^2), \\
2\int_0^1 dx x^{n} g_1(x,Q^2) &= \sum_{q=u,d} e_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n \rangle_{\Delta q}(\mu) + \mathscr{O}(1/Q^2), \\
2\int_0^1 dx x^{n} g_2(x,Q^2) &= \frac{1}{2}\frac{n}{n+1} \sum_{q=u,d} [e_{2,n}^{q}(\mu^2/Q^2, g(\mu))\, d_n^q(\mu) - \\
&- 2e_{1,n}^{q}(\mu^2/Q^2, g(\mu))\, \langle x^n \rangle_{\Delta q}(\mu)] + \mathscr{O}(1/Q^2)
\end{aligned}
$$

- c_1, c_2 and e_1, e_2 are the Wilson coefficients (perturbative),
- $\langle x^n \rangle_q(\mu)$, $\langle x^n \rangle_{\Delta q}(\mu)$ and d_n are forward nucleon matrix elements of certain local operators $\mathscr{O}$.

What happens at the chiral limit?

Existing calculations (quenched and dynamical) at relatively heavy quark masses seem to disagree with experiment.



[QCDSF: Phys.Rev.D53 1996]
[LHPC-SESAM: hep-lat/0201021]

Plausible resolution(?):

- Finite lattice spacing
- Finite volume(?)(see g_A)
- Chiral logs

$$V(m_\pi^2) = V_c \left[1 + C_\chi\, m_\pi^2\, ln \frac{m_\pi^2}{\mu_\chi^2} \right]$$

C_χ calculated in χ-PT,

depends of f_π and g_A only

Arndt&Savage

Chen&Ji,Chen&Savage

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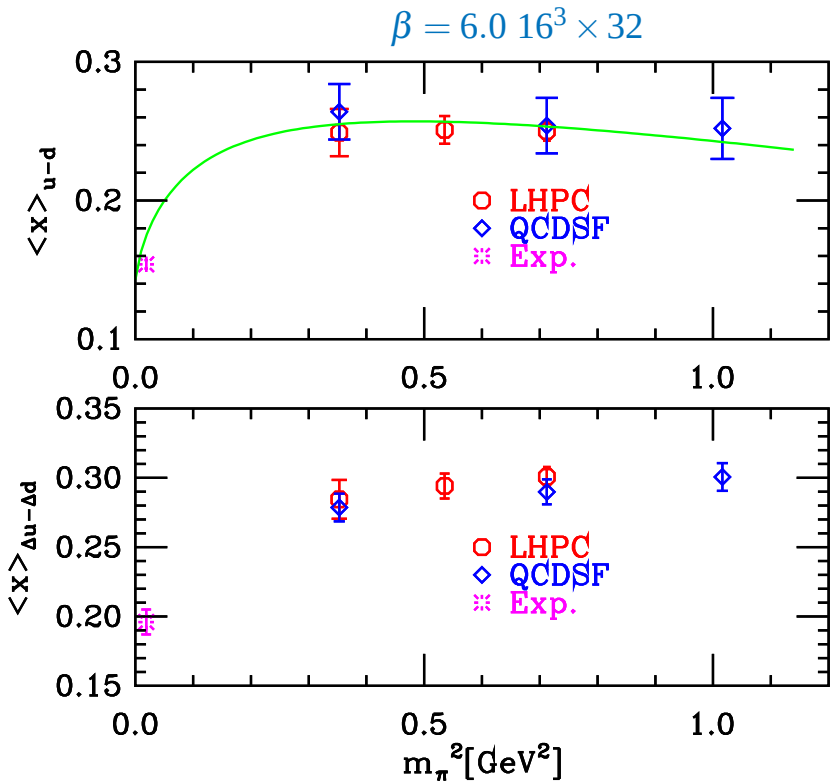

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[Detmold et.al. Phys.Rev.Lett.87 2001]

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C_χ calculated in χ-PT,

depends of f_π and g_A only

Assume:

$$V(m_\pi^2) = V_c \left[1 + C_\chi m_\pi^2 \, ln \frac{m_\pi^2}{m_\pi^2 + \mu^2} \right] + B m_\pi^2$$

μ^2 phenomenological parameter

$\mu \sim$ 550MeV.

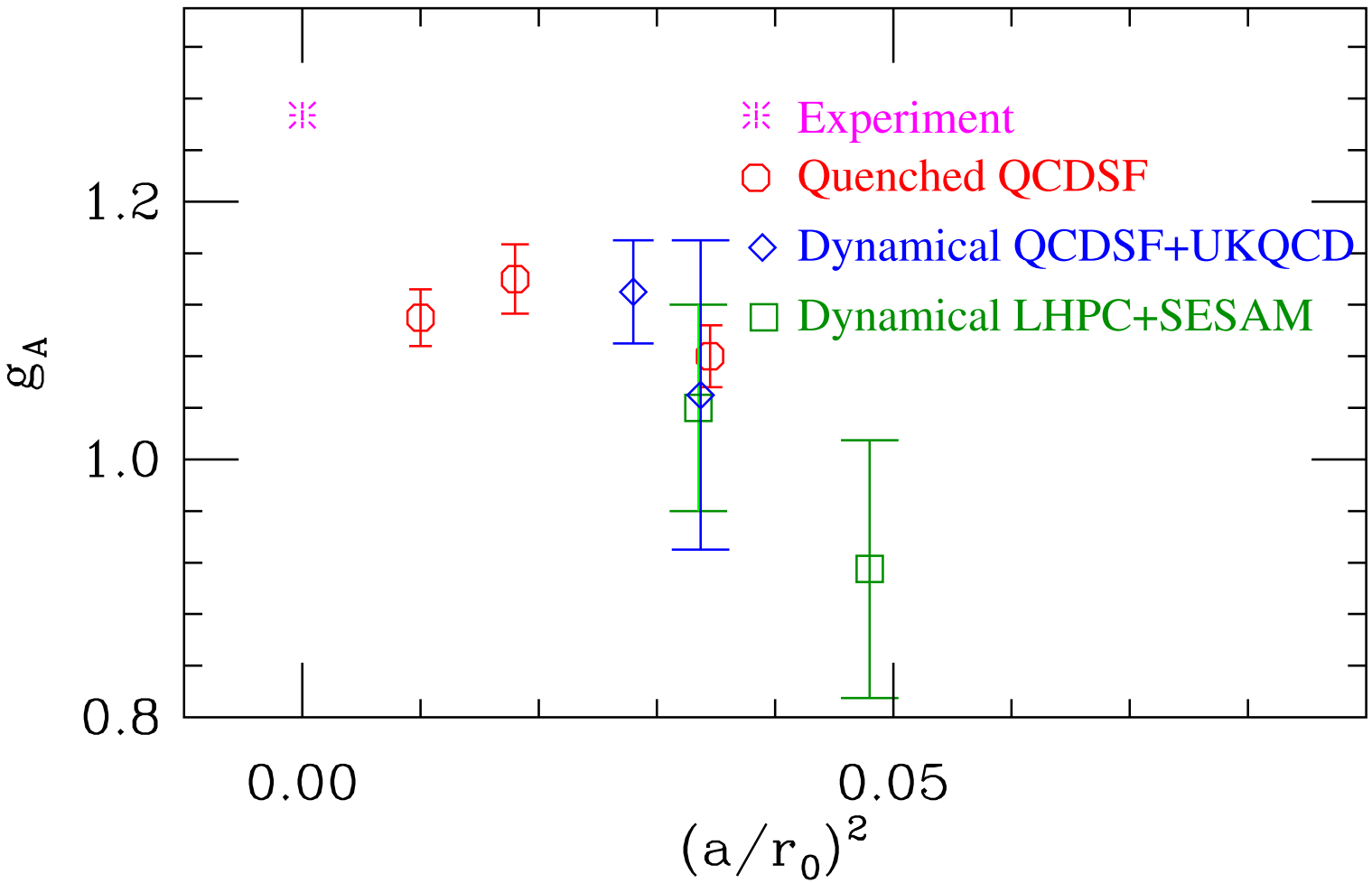
Detmold et.al.

Arndt&Savage

Chen&Ji,Chen&Savage

Axial Charge

$\langle 1 \rangle_{\Delta q}$ (g_A : Axial charge)



[Horsley, Lat02]

Simulation parameters

- Gauge Action: DBW2

$$S_g = \frac{\beta}{3}\mathrm{Re}\,\mathrm{Tr}\left[(1-8c_1)\left\langle 1-\square\right\rangle + 2c_1\left\langle 1-\square\right\rangle\right]$$

With $c_1 = -1.4067$ computed by non-perturbative RG blocking.

[Takaishi Phys.Rev. D54 (1996)]

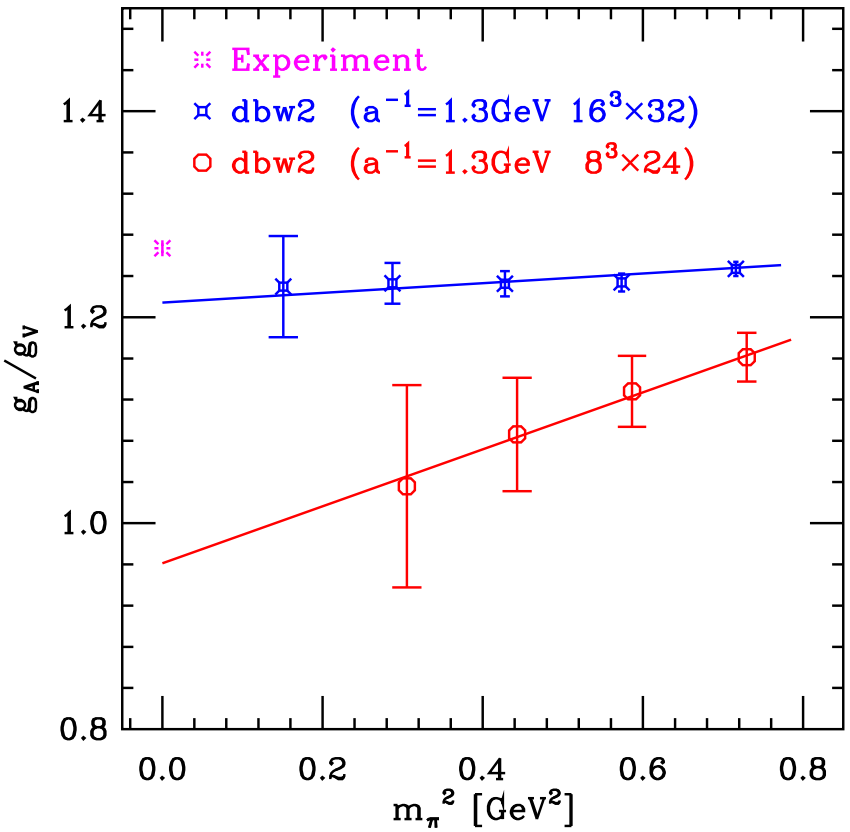
- $\beta = 0.870$ or $a^{-1} = 1.3\mathrm{GeV}$, Volume: $16^3 \times 32 \sim 2.4^3 fm^3$ box.

- Fermion Action: Domain wall fermions $L_s = 16 \rightarrow m_{\mathrm{res}} \sim .7\mathrm{MeV}$

- Statistics: 416 Lattices QCDSP 300Gflops (peak) for 4 months

- Status: PRELIMINARY!

Finite volume effect for g_A



Volumes used:

2.4fm and 1.2fm box

Clear finite volume effect

- Chiral limit (Linear fit):

$$g_A/g_V = 1.21(2)$$

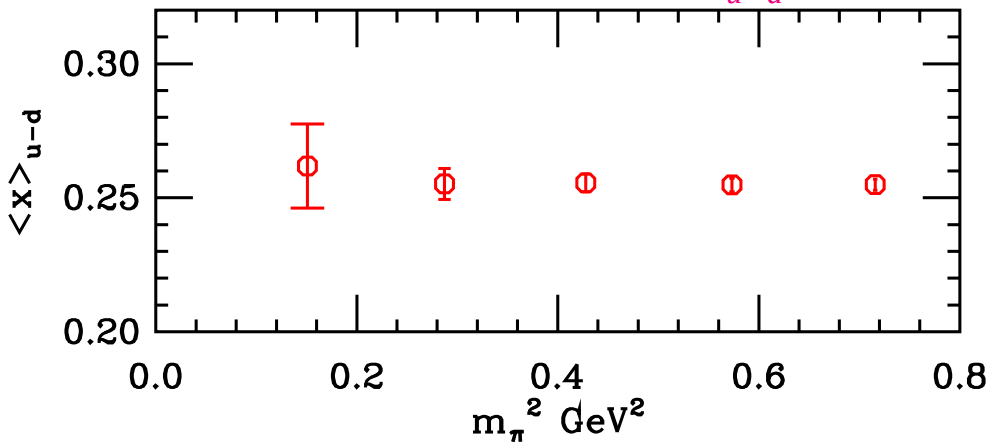
[Ohta (LAT02) hep-lat/0210006]

- Experiment:

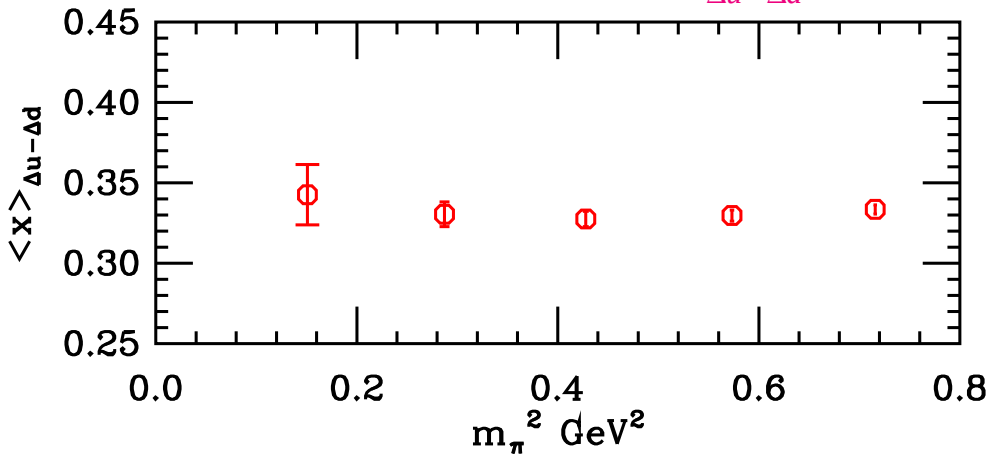
$$g_A/g_V = 1.2670(30)$$

For dwf: $Z_A = Z_V = 1/g_V$

Flavor non-singlet $< x >_{u-d}$



Flavor non-singlet $< x >_{\Delta u-\Delta d}$



Quark density

$$\mathcal{O}^{q}_{44} = \bar{q}\left[\gamma_4 \overleftrightarrow{D}_4 - \frac{1}{3}\sum_{k=1}^{3}\gamma_k \overleftrightarrow{D}_k\right] q \rightarrow \langle x \rangle_q$$

- Hypercubic group rep. $\mathbf{3}^{+}_{\mathbf{1}}$

Helicity

$$\mathcal{O}^{5q}_{34} = \frac{1}{4}\bar{q}\gamma_5\left[\gamma_3 \overleftrightarrow{D}_4 + \gamma_4 \overleftrightarrow{D}_3\right] q \rightarrow \langle x \rangle_{\Delta q}$$

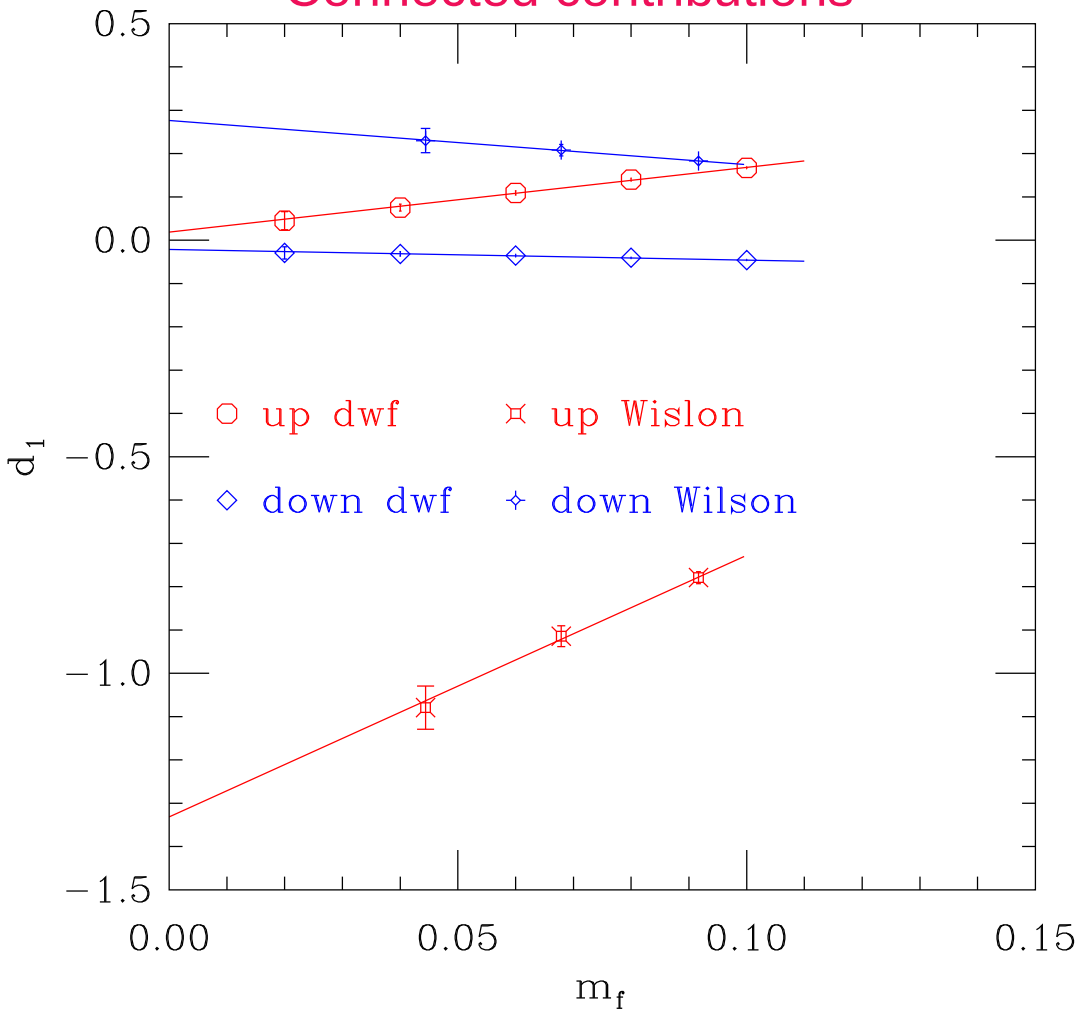
- Hypercubic group rep.: $\mathbf{6}^{-}_{\mathbf{3}}$

- Momentum: $\vec{P} = 0$
- Renormalization: Multiplicative

Note:

- Unrenormalized
- No Curvature seen so far in the chiral limit

Connected contributions



Measure:

$$\mathcal{O}_{34}^{[5]q} = \frac{1}{4}\bar{q}\gamma_5\left[\gamma_3 \overleftrightarrow{D}_4 - \gamma_4 \overleftrightarrow{D}_3\right] q \quad \rightarrow \quad d_1^q$$

- Hyper-cubic group representation: $\mathbf{6_1^+}$
- Momentum: $\vec{P} = 0$

- Renormalization:

Multiplicative (Chiral symmetry)

chiral symmetry breaking:

mixing with: $\mathcal{O}_{34}^{\sigma q} = \bar{q}\gamma_5\sigma_{34}q$

Note:

- Unrenormalized
- Disagreement with the Wilson results

  Power divergent mixing

  [LHPC-SESAM: hep-lat/0201021]
- Small at chiral limit

Dynamical Domain Wall Fermions

- Extend earlier Columbia dynamical domain wall fermion studies to weaker coupling
- Two improvements on earlier dynamical domain wall fermion studies
  - Modified pseudofermion action
  - Chronological inverter

    [Brower, Ivanenko, Levi, Orginos Nucl.Phys. B484 (1997)]
- Using DBW2 gauge action to reduce chiral symmetry breaking.

Modified Pseudofermion Action

- DWF uses a set Pauli-Villars fields to cancel off the bulk divergence.

$$\Phi^\dagger D^\dagger(m_f = 1) D(m_f = 1)\Phi$$

- Previous work used two random bosonic fields: Cancellation of fermion and Pauli-Villars "on average".
- Use one pseudo-fermion field for both fermion and Paulli-Villars

$$\frac{\det\left[D^\dagger(m_f)D(m_f)\right]}{\det\left[D^\dagger(1)D(1)\right]} = \det\left[\frac{1}{D^\dagger(1)}D^\dagger(m_f)\times D(m_f)\frac{1}{D(1)}\right]$$

- CG count reduced by $\approx 20\%$, Acceptance also increased. Overall speed-up $\approx 30\%$ (this is a pre-thermalised number, we are working on a better one).

Chronological Inverter

- In HMC we need to solve: $M^\dagger M\chi = \phi$
- Idea: Extrapolate in MD time
- Polynomial extrapolation (up to second order)

  [Gottlieb, Liu, Toussaint, Renken, Sugar Phys. Rev. D35 (1987)]

- Any extrapolation would result:

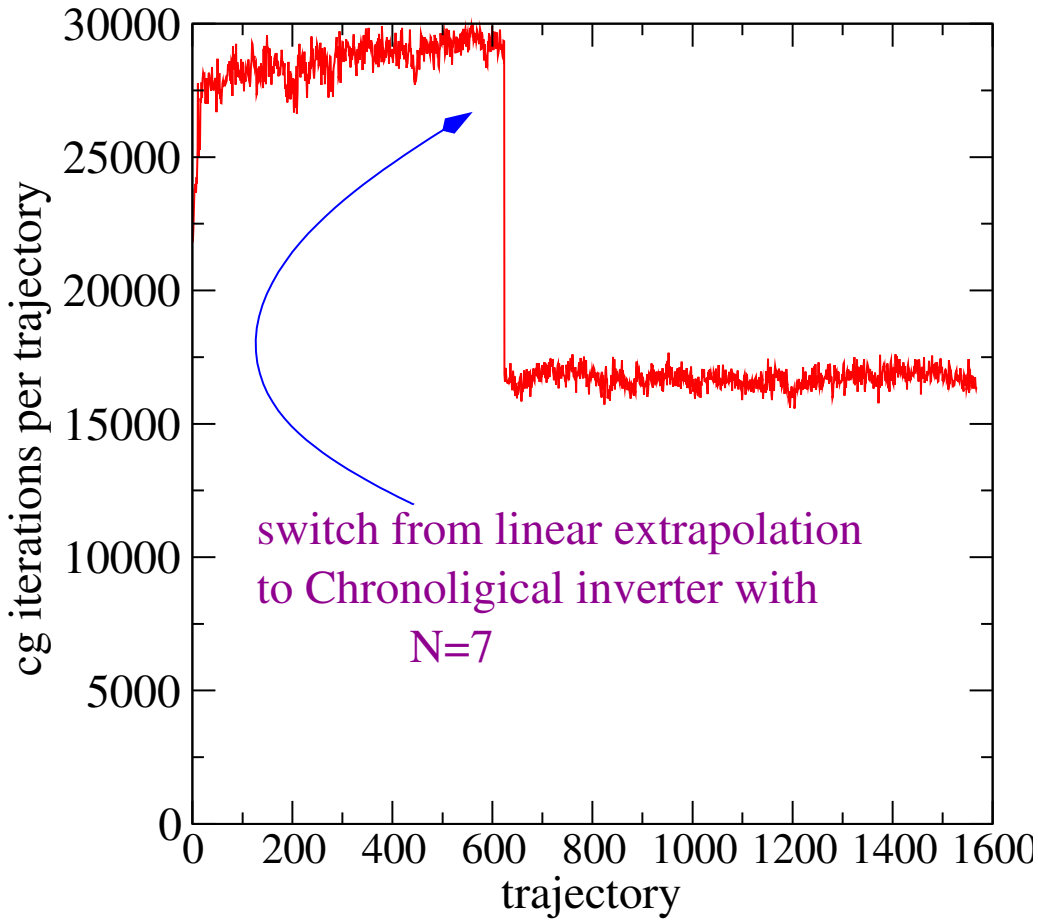
$$\chi_{guess} = \sum_{t=-1}^{-N} c_t \chi_t$$

- Find c_t so that

$$F(\chi) = \chi^\dagger M^\dagger M\chi - \phi^\dagger\chi - \chi^\dagger\phi$$

  is minimized

- In other words minimize $F(\chi)$ in the subspace of the past N solutions χ_t



- CG iterations for dynamical DWF

$$m_f = 0.020$$
$$\beta = 0.80$$
$$a^{-1} = 1.8\text{GeV}$$

- About a factor of **1.7** reduction in CG iterations after the Chronological Inverter was switched on
- Past vectors used: $N = 7$

Current Runs

- Fermion action: Domain Wall ($L_s = 12$, $M_5 = 1.8$).
- Gauge action : DBW2 ($\beta = 0.80$)
- Three evolutions for lattices of size $16^3 \times 32$:

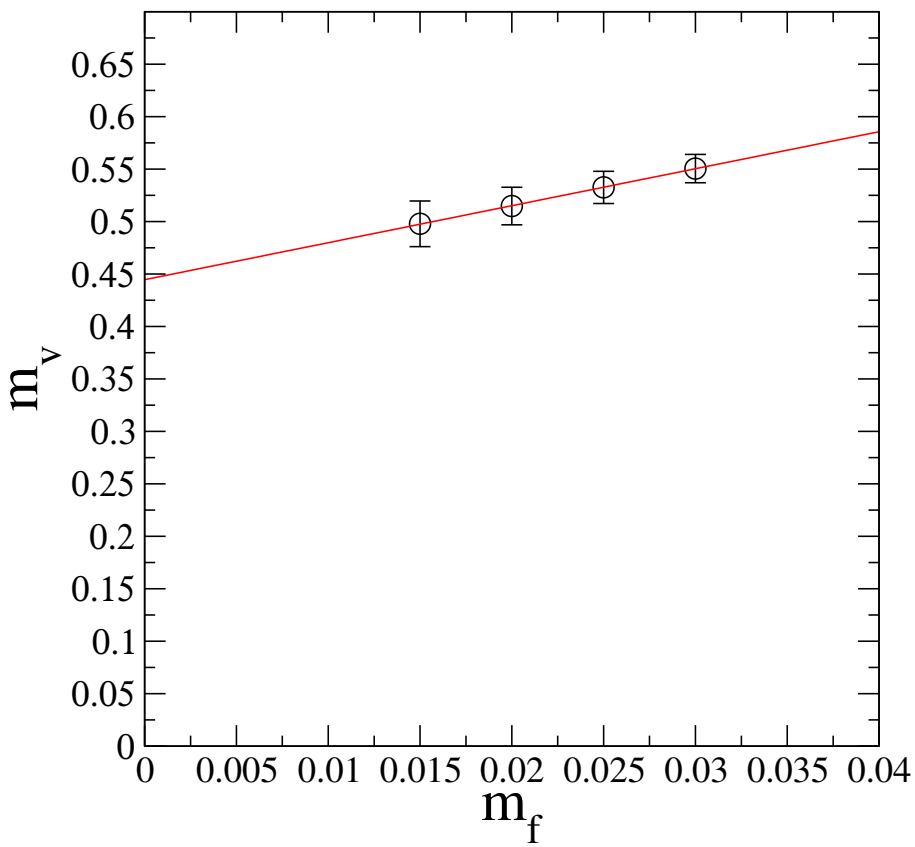
m_f	step size	acceptance	trajs / day	trajs
0.02	$1/50$	80%	24 (200GF peak)	3600
0.03	$1/50$	80%	24 (100GF peak)	2400
0.04	$1/40$	65%	34 (100GF peak)	1700

- Each trajectory travels ≈ 0.5 in “MD” time

Preliminary Results

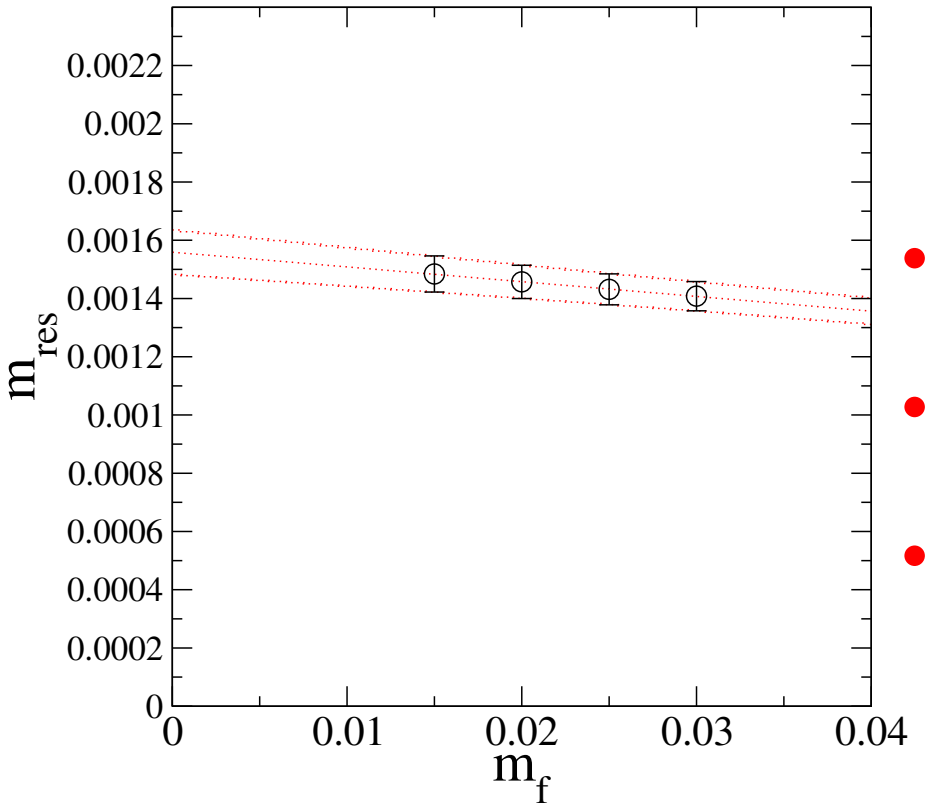
- Data shown from the $m_f = 0.02$ evolution
- 22 configurations separated by 100 HMC trajectories.
- Questions:
  - What's the scale?
  - How much explicit chiral symmetry breaking?
  - How big is $m_f = 0.02$?

Scale



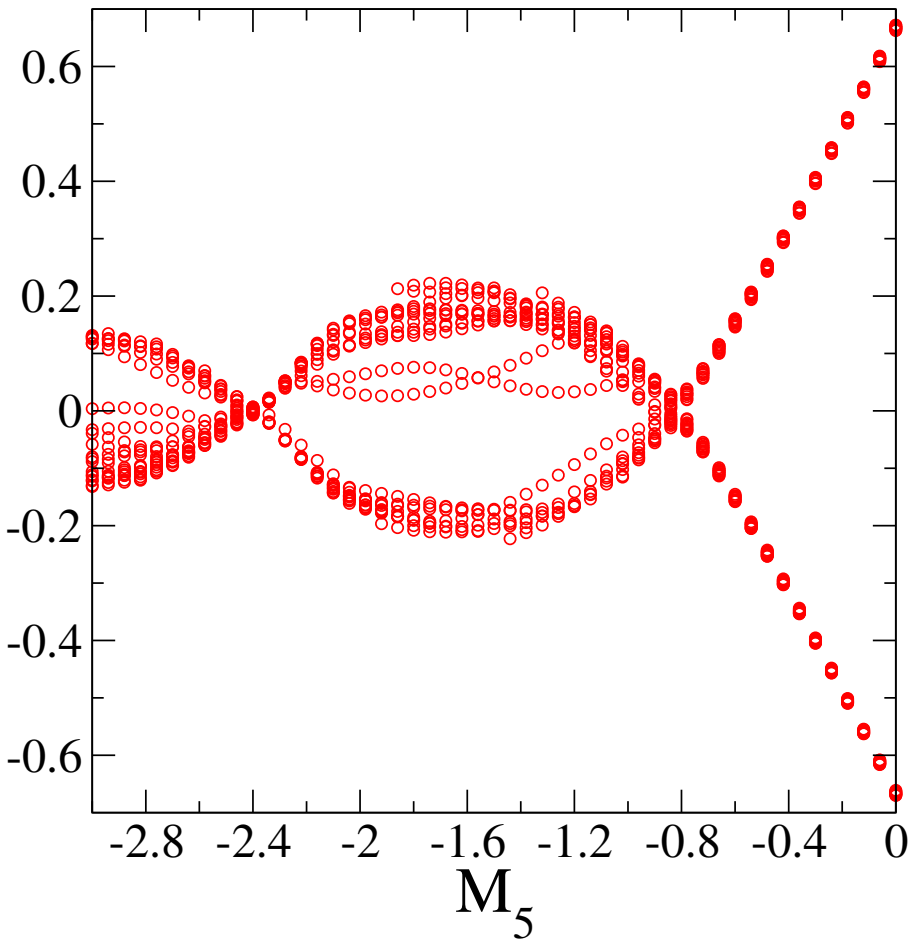
- Set scale from am_ρ
- Linear extrapolation gives:
  $a^{-1} = 1753(130)\mathrm{GeV}$

Chirality: m_{res}



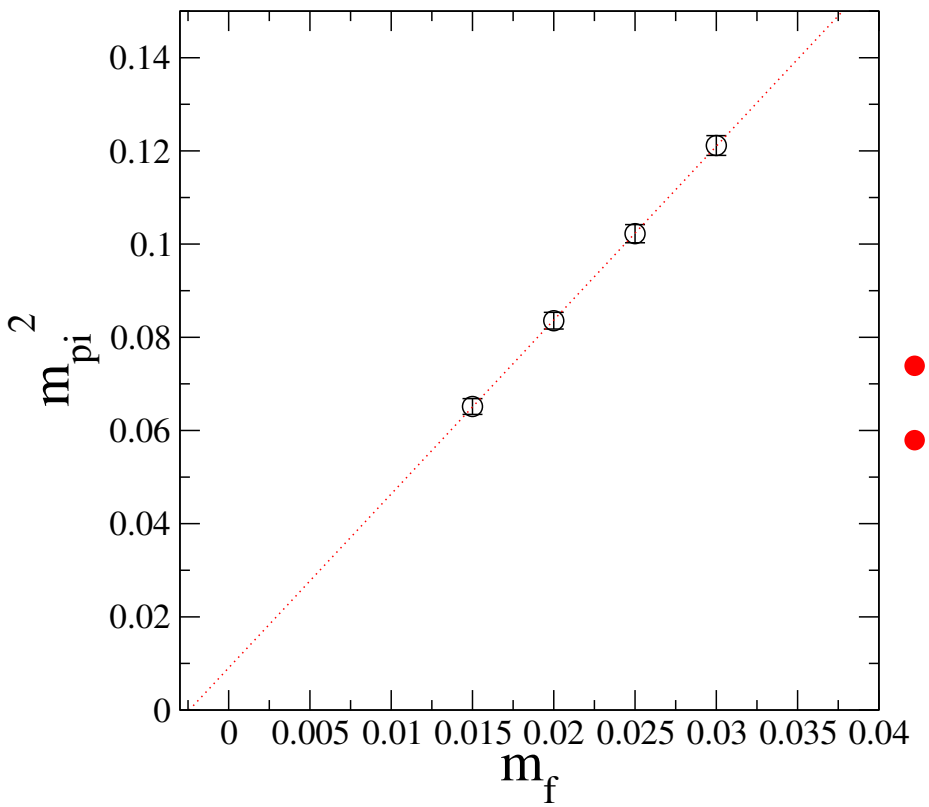
- Residual mass from Ward Identity
- Average between timeslice 6 and 16 gives $m_{\mathrm{res}} \approx 1.5 \times 10^{-3}$
- $m_{\mathrm{res}} = 3 - 5\mathrm{MeV}$

Chirality: Spectral Flow



- Spectral flow of $\gamma_5 D_W(M_5)$.
- Graph shows $\approx$ traj 800 for the $m_f = 0.03$ evolution
- Clear gap present for $M_5 = -1.8$

m_π^2



- For $m_f = 0.02\ m_\pi \approx 500\text{MeV}$.
- $m_\pi / m_\rho \approx .54$

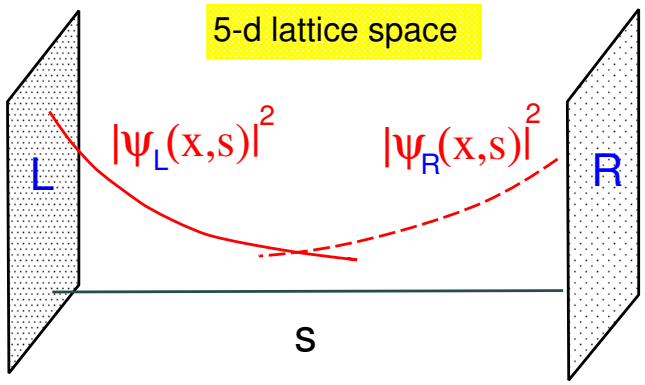
Calculation of Kaon matrix elements ($\Delta I = 1/2$ rule, B_K, ε'/ε)

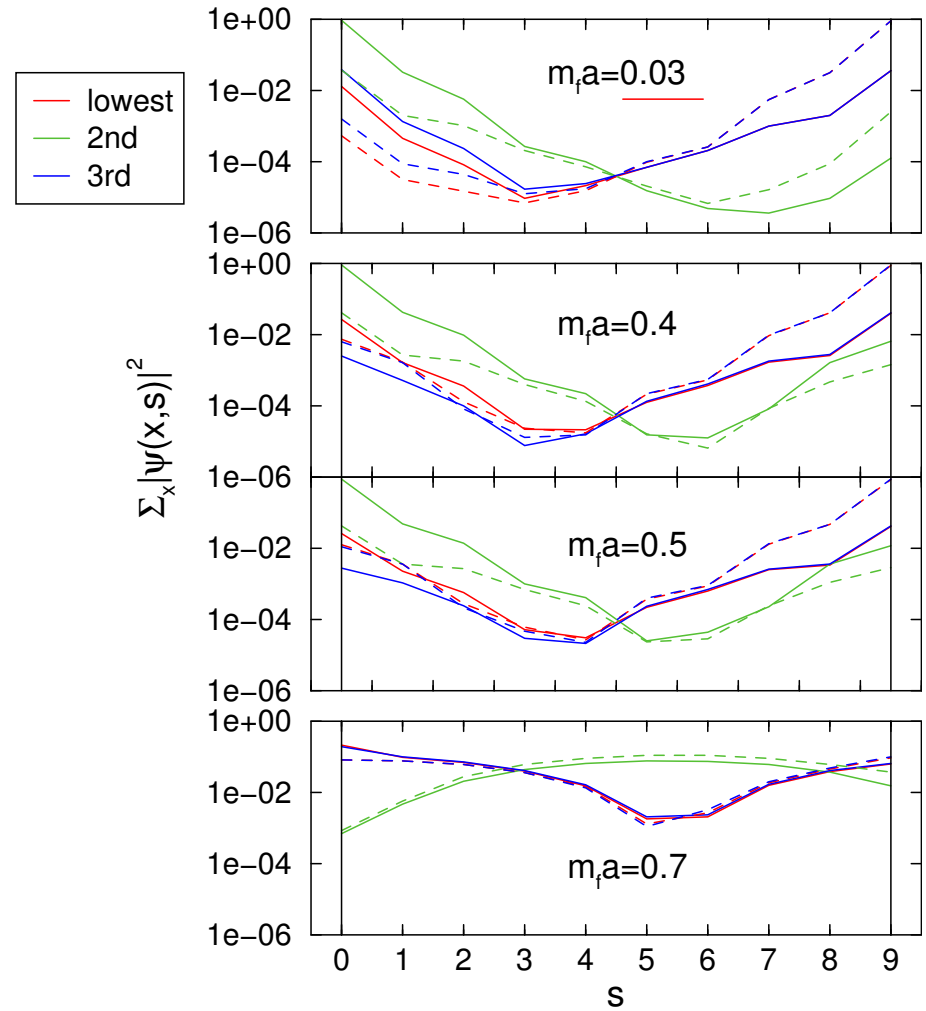
(1) $a^{-1} \approx 3$ GeV to include charm quark on the lattice

$$\left\langle \pi\pi \left| H_W^{\Delta S=1} \right| K \right\rangle = \frac{G_F}{\sqrt{2}} V_{us} V_{ud}^* \sum_i W_i(\mu) \left\langle \pi\pi \left| Q_i^{\Delta S=1}(\mu) \right| K \right\rangle + \mathcal{O}(1/m_b^2)$$

cf. CP-PACS, RBC (2001) : $a^{-1} \approx 2$ GeV $\rightarrow$ charm out $\rightarrow$ $\mathcal{O}(1/m_c^2)$, $a_s^2(m_c)$

- Localization of the chiral modes





Study fifth dimension dependence of charm quark eigenvectors

$\Psi_{L/R}$ fails to localize for large $m_f a$

$a^{-1} \simeq 2.9$ GeV $\Rightarrow$ $m_c a \approx 0.44$

DWF appear to work at m_c?

more tests under way

- Simulation parameters

Chiral symmetry on the lattice is crucial.

Domain-Wall fermion

$$\begin{aligned} \text{5-th width} &: L_s = 10, \\ \text{DW height} &: M_5 = 1.65 \quad \Rightarrow m_{\rm res} = 0.263(12)\ \text{MeV} \\ m_f a &= 0.008,\ 0.016,\ 0.024,\ 0.032,\ 0.040 \\ m_c a &= 0.08,\ 0.12,\ 0.16,\ 0.20,\ 0.30,\ 0.40,\ 0.50 \end{aligned}$$

DBW2 gauge config.

$$\begin{aligned} \text{lattice volume} &: 24^3 \times 48, \\ \text{coupling} &: \beta = 1.22 \ \Rightarrow a^{-1} \simeq 2.9\ \text{GeV} \quad (m_\rho = 770\ \text{MeV}) \end{aligned}$$

- Kaon B-parameter hep-lat/0211013

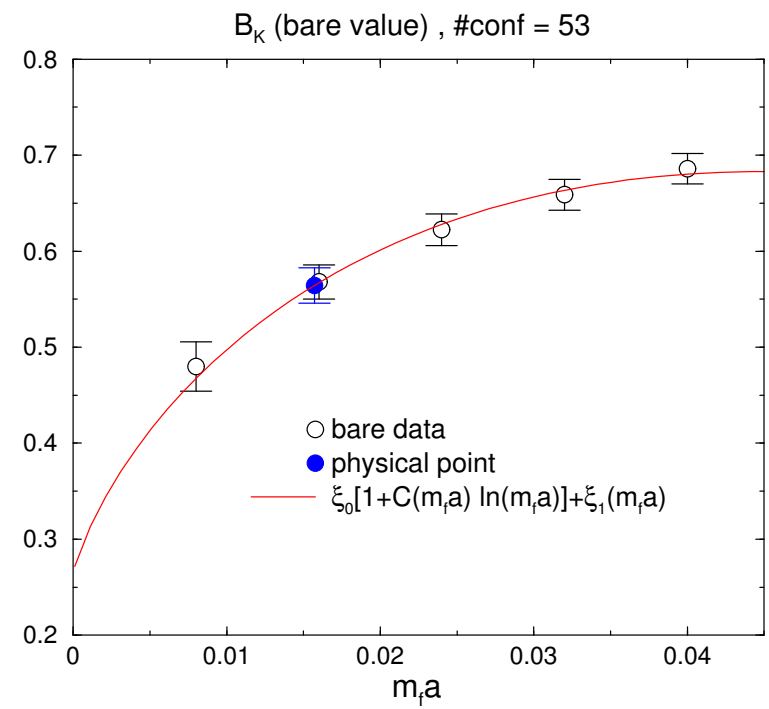
$$B_K = \frac{\langle \overline{K} | [\bar{s}\gamma_\mu(1-\gamma_5)d]^2 | K \rangle}{\frac{8}{3}\langle \overline{K} | \bar{s}\gamma_4\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_4\gamma_5 d | K \rangle}$$

fit function:

$$B_K = \xi_0[1 + C(m_K^2)\ln\left(\frac{m_K^2}{\Lambda^2}\right)] + \xi_1(m_K^2)$$

quenched ChPT : $C = -\frac{6}{(4\pi f_\pi)^2}$ [Sharpe]

with $\chi^2/\text{dof} = 0.187$



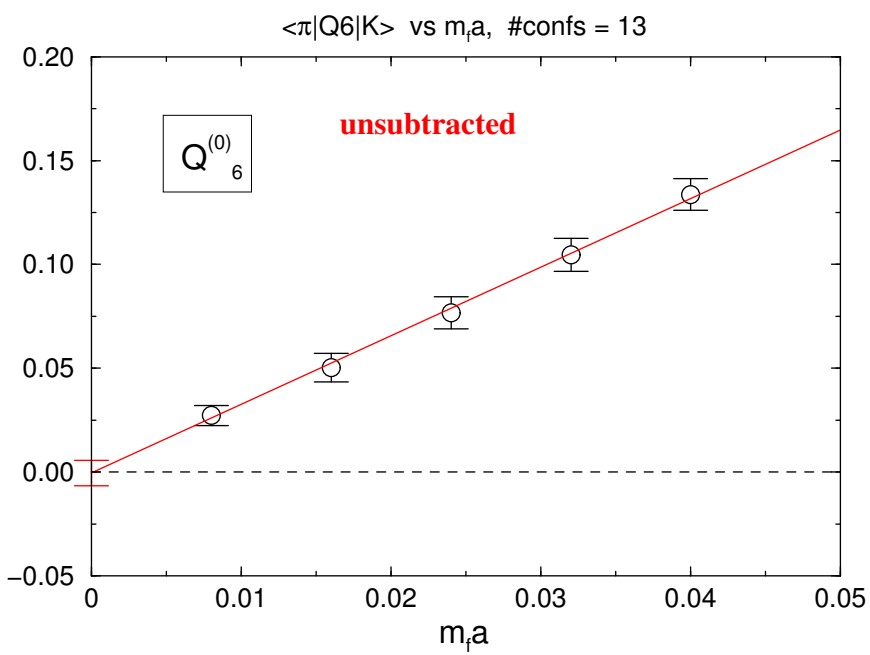
- $K \to \pi$ matrix elements

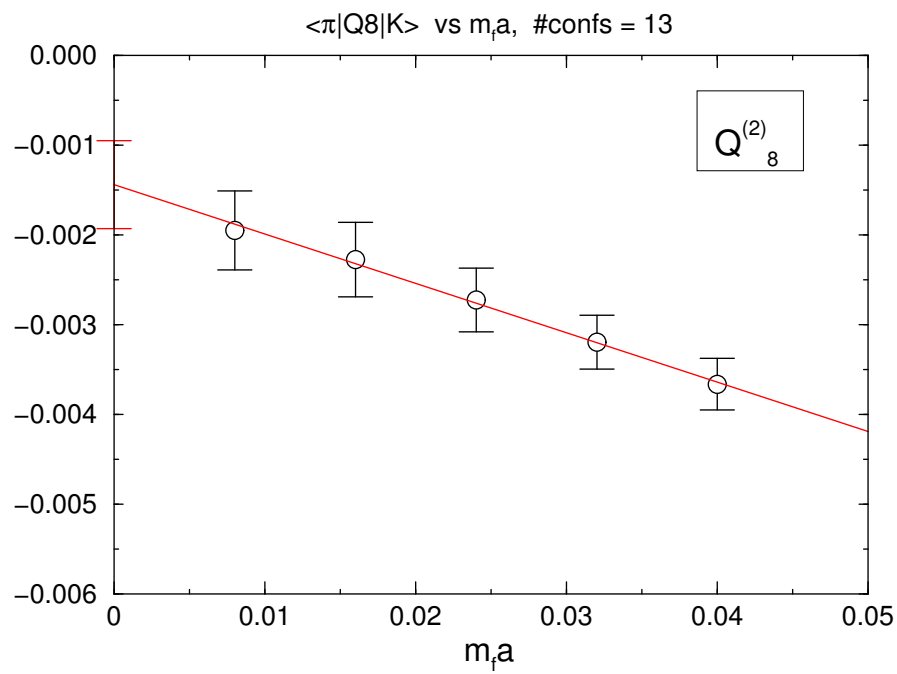
Leading order of ChPT : $\langle \pi^+ | Q_i | K^+ \rangle$ & $\langle 0 | Q_i | K^0 \rangle \Leftrightarrow \langle \pi^+\pi^-(I) | Q_i | K^0 \rangle$

$$\varepsilon'/\varepsilon \propto \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0}\left(\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} - \frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right)$$

Dominant contributions

$\mathrm{Re}A_{0,2}$	Q_2
$\mathrm{Im}A_0$	$Q_4,\ Q_6$
$\mathrm{Im}A_2$	Q_8



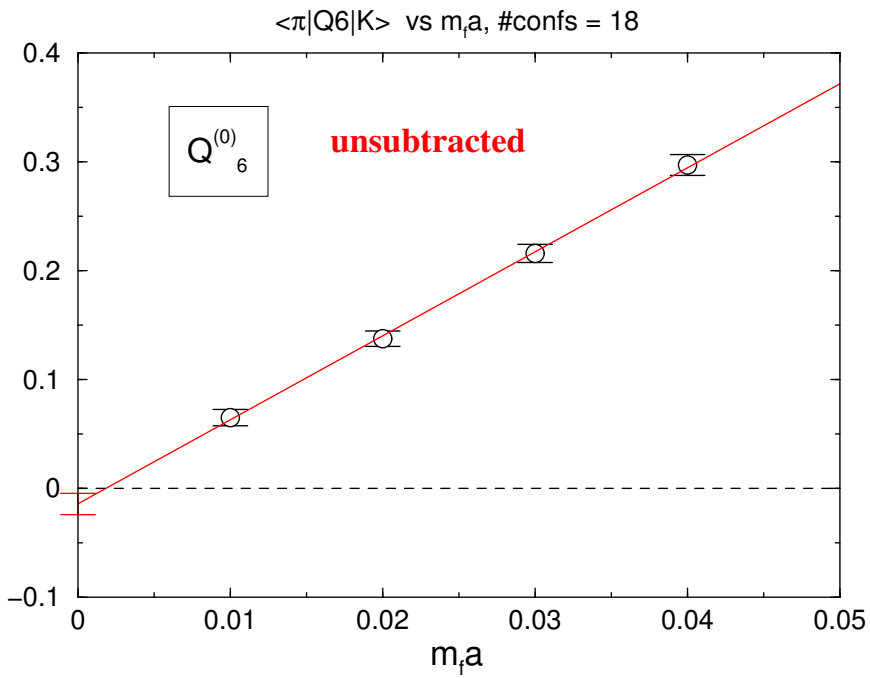


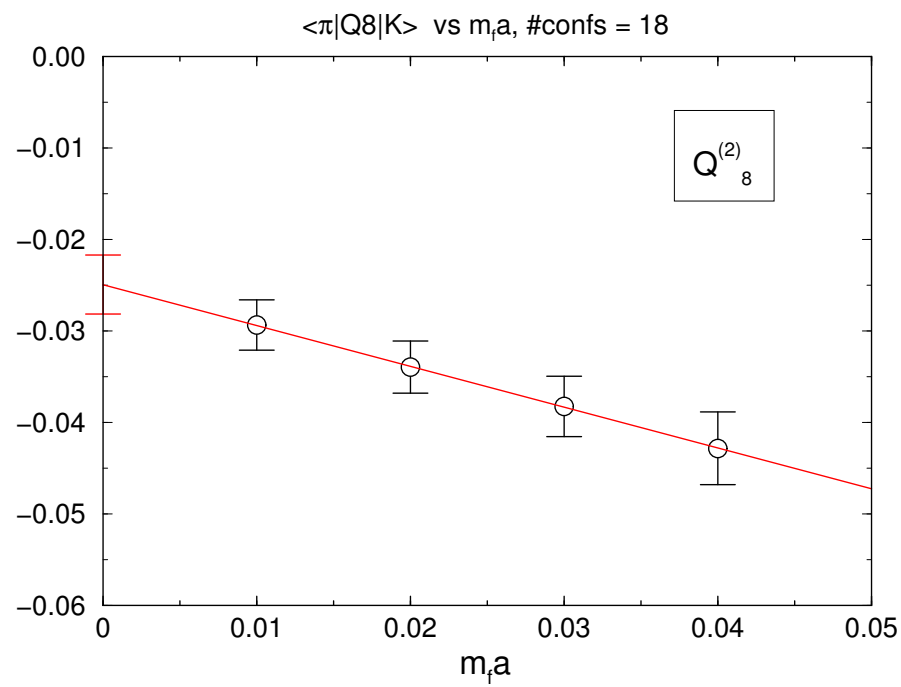
(2) Calculation on the dynamical lattices

- Simulation parameters

Domain-Wall fermion	5-th width	:	$L_s = 12,$
	DW height	:	$M_5 = 1.8$
	$m_f a$	$=$	$0.01,\ 0.02,\ 0.03,\ 0.04$
Gauge config.(DBW2 + DWF, $N_f = 2$)	lattice volume	:	$16^3 \times 32$
	sea quark mass	:	$m_{\rm sea} a = 0.02$

- $K \to \pi$ matrix elements





Hadronic Matrix Element for Proton Decay

Proton decay by dimension-six operator from (SUSY) GUT.

$$\langle \pi^0 e^+ | qqql | p \rangle \tag{1}$$

Hadronic matrix element

$$\langle \pi^0 | \varepsilon_{ijk} (u^{iT} C P_{R/L} d^j) P_L u^k | p \rangle = P_L \left[W_0(q^2) - W_q(q^2) i \not{q} \right] u_p. \tag{2}$$

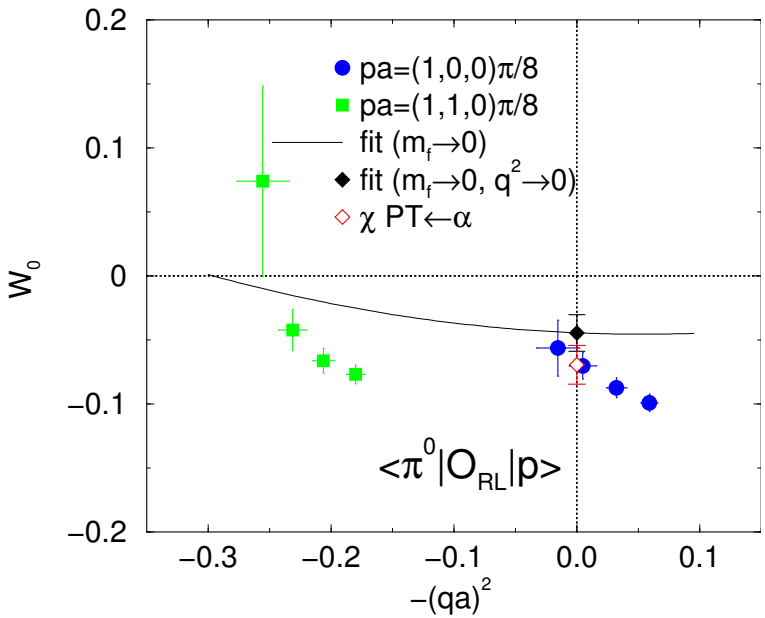
Relevant form factor W_0 (JLQCD, PRD 62 (00) 014506).

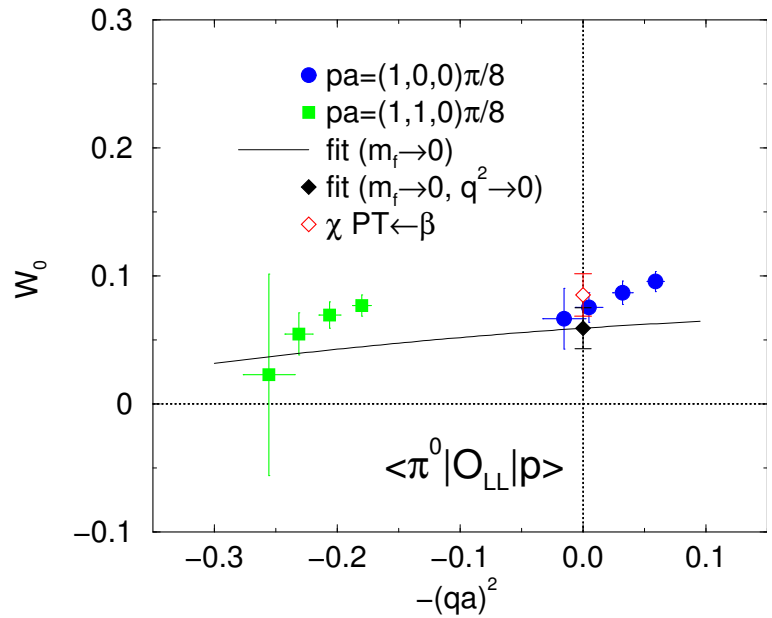
- Direct method by calculating three point function.
- Indirect method using χ PT (Claudson et al., NPB 195 (82) 297), and the low energy constants calculated on the lattice:

$$\langle 0 | \varepsilon_{ijk} (u^{iT} C P_R d^j) P_L u^k | p \rangle = \alpha P_L u_p \tag{3}$$

$$\langle 0 | \varepsilon_{ijk} (u^{iT} C P_L d^j) P_L u^k | p \rangle = \beta P_L u_p \tag{4}$$

Results for Proton Decay Matrix Element with DWF (preliminary)





- Physical kinematics point: $m_\pi \simeq m_e \simeq 0 : m_f \to 0, -q^2 \to 0$.
- No distinct difference between direct and indirect results within the error. But, $\sim 40\%$ excess of central value from indirect method is quite similar with JLQCD result.
- Need to be renormalized by non-perturbative renormalization.
- Non-degenerate mass run underway for $p \to K^+\overline{\nu}$, etc.

Summary and Outlook

- RBC projects:
  - Dynamical Domain Wall fermions
  - ε'/ε
  - Nucleon Structure Functions
  - Proton decay
- Serious effort well underway to create a library of dynamical DW (n_f=2) lattices with $a^{-1} \approx .1.7\mathrm{GeV}$, m_{sea} =0.02,0.03,0.04. Additional lattice spacings also being contemplated.
- Address the systematic errors for ε'/ε
  - 2.9 GeV quenched $\rightarrow$ four active flavors (underway)
  - Dynamical fermions at 1.7 GeV (underway)
  - $K \rightarrow \pi\pi$ a la Lellouch & Luscher with G-parity boundary conditions
  - NLO χPT