Conventions for massless spinor products

$$\langle pq\rangle = \langle p-|q+\rangle, \quad [pq] = \langle p+|q-\rangle$$

$$\langle p\pm|\gamma_\mu|p\pm\rangle = 2p_\mu$$

$$\langle p+|q+\rangle = \langle p-|q-\rangle = \langle pp\rangle = [pp] = 0$$

$$\langle pq\rangle = -\langle qp\rangle, \quad [pq] = -[qp]$$

$$2|p\pm\rangle\langle q\pm| = \tfrac{1}{2}(1\pm\gamma_5)\gamma^\mu\langle q\pm|\gamma_\mu|p\pm\rangle$$

$$\langle pq\rangle^* = -\mathrm{sign}(p\cdot q)[pq] = \mathrm{sign}(p\cdot q)[qp]$$

$$|\langle pq\rangle|^2 = \langle pq\rangle\langle pq\rangle^* = 2|p\cdot q| \equiv |s_{pq}|$$

$$\langle pq\rangle[qp] = 2p\cdot q \equiv s_{pq}$$

$$\langle p\pm|\gamma_{\mu_1}\dots\gamma_{\mu_{2n+1}}|q\pm\rangle = \langle q\mp|\gamma_{\mu_{2n+1}}\dots\gamma_{\mu_1}|p\mp\rangle$$

$$\langle p\pm|\gamma_{\mu_1}\dots\gamma_{\mu_{2n}}|q\mp\rangle = -\langle q\pm|\gamma_{\mu_{2n}}\dots\gamma_{\mu_1}|p\mp\rangle$$

$$\langle AB\rangle\langle CD\rangle = \langle AD\rangle\langle CB\rangle + \langle AC\rangle\langle BD\rangle$$

$$\langle A+|\gamma_\mu|B+\rangle\langle C-|\gamma^\mu|D-\rangle = 2[AD]\langle CB\rangle$$

$$\langle A\pm|\gamma^\mu|B\pm\rangle\,\gamma_\mu = 2\left[\,|A\mp\rangle\langle B\mp| + |B\pm\rangle\langle A\pm|\,\right]$$

For polarization with momentum k and gauge vector b

$$\varepsilon^\pm_\mu(k,b) = \pm\frac{\langle k\pm|\gamma_\mu|b\pm\rangle}{\sqrt{2}\langle b\mp|k\pm\rangle}$$

Hence we have that

$$\varepsilon^+_\mu(k,b) = \frac{\langle k+|\gamma_\mu|b+\rangle}{\sqrt{2}\langle bk\rangle}, \quad \varepsilon^-_\mu(k,b) = \frac{\langle k-|\gamma_\mu|b-\rangle}{\sqrt{2}[kb]}$$

and

$$\gamma^\mu \varepsilon^+_\mu(k,b) = \frac{\sqrt{2}[|k-\rangle\langle b-| + |b+\rangle\langle k+|]}{\langle bk\rangle}$$

$$\gamma^\mu \varepsilon^-_\mu(k,b) = \frac{\sqrt{2}[|k+\rangle\langle b+| + |b-\rangle\langle k-|]}{[kb]}$$

We can write this in a gamma matrix notation with another auxiliary vector

$$\gamma^\mu \varepsilon^+_\mu(k,b,a) = +\frac{\sqrt{2}}{\langle ka\rangle[ab]\langle bk\rangle}\{\gamma_L \hat{k}\hat{a}\hat{b} + \gamma_R \hat{b}\hat{a}\hat{k}\}$$

$$\gamma^\mu \varepsilon^-_\mu(k,b,a) = -\frac{\sqrt{2}}{[ka]\langle ab\rangle[bk]}\{\gamma_R \hat{k}\hat{a}\hat{b} + \gamma_L \hat{b}\hat{a}\hat{k}\}$$