Dynamic Likelihood Mass Analysis in l+jets

(PWA .....)

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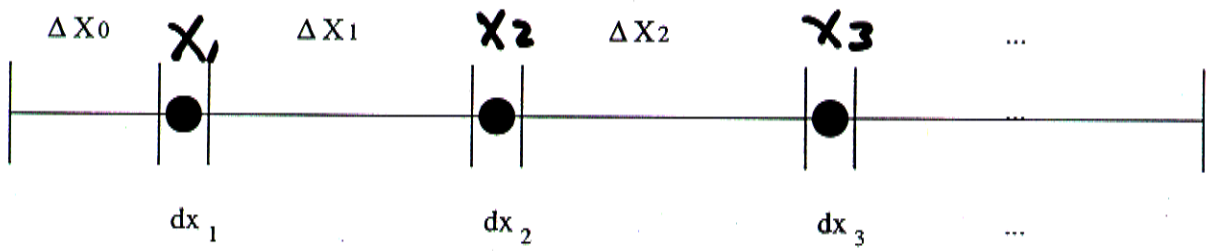


X represents all the variables needed to characterize the event

$$P(x_1 ... x_N)dx_1 ... dx_N = Prob\begin{pmatrix} 0 & \text{events} \\ \text{in} & \Delta x_0 \end{pmatrix} \times Prob\begin{pmatrix} 1 & \text{event} \\ \text{in} & dx_1 \end{pmatrix}$$
$$\times Prob\begin{pmatrix} 0 & \text{events} \\ \text{in} & \Delta x_1 \end{pmatrix} \times Prob\begin{pmatrix} 1 & \text{event} \\ \text{in} & dx_2 \end{pmatrix} \quad ....$$

Using Poisson statistics

$$Prob\begin{pmatrix} 0 & \text{events} \\ \text{in} & \Delta x_i \end{pmatrix} = e^{-\overline{n}} = e^{-N\int_{\Delta x_i}\overline{P}(x)dx}$$

and

$$Prob\begin{pmatrix} 1 & \text{event} \\ \text{in} & \Delta dx_i \end{pmatrix} = \overline{n}e^{-\overline{n}} = N\overline{P}(x_i)dx \quad e^{-N\overline{P}(x_i)dx}$$

$$P(x_1 ... x_N)dx_1 ... dx_N = \underbrace{e^{-N\int \overline{P}(x)dx} \prod_{i=1}^{N} N\overline{P}(x_i)dx_i}_{\text{Likelihood}}$$

$$L(\alpha) = e^{-N\int \overline{P}(x;\alpha)dx} \prod_{i=1}^{N} \overline{P}(x_i;\alpha)$$

Maximize L(α) to get best estimate of α

1 $$\boxed{-lnL(\alpha) = -\sum_{i=1}^{N} ln\overline{P}(x_i;\alpha) + N\int \overline{P}(x;\alpha)dx}$$

How should one deal with the acceptance?

$$\overline{P}(x;\alpha) = Acc(x) \quad P(x;\alpha)$$

measured probability — acceptance — physics probability

2 $$\boxed{-lnL(\alpha) = -\sum_{i=1}^{N} lnP(x_i;\alpha) + N\int Acc(x)P(x;\alpha)dx}$$

Should one take path 1 or path 2?

Calculation of $P(x)$

$$d^n\sigma = \frac{(2\pi)^4|M|^2}{4\sqrt{(q_1.q_2)^2 - m_1^2 m_2^2}} \quad d\Phi_n(q_1+q_2; p_1, \ldots, p_n)$$

where $d\Phi_n$ is the phase space

$$d\Phi_n(P; p_1, \ldots, p_n) = \delta^4(P - \sum_{i=1}^{n} p_i) \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 2E_i}$$

$$P(x) = \int d^n\sigma(y) f(q_1) f(q_2) W(y, x)$$

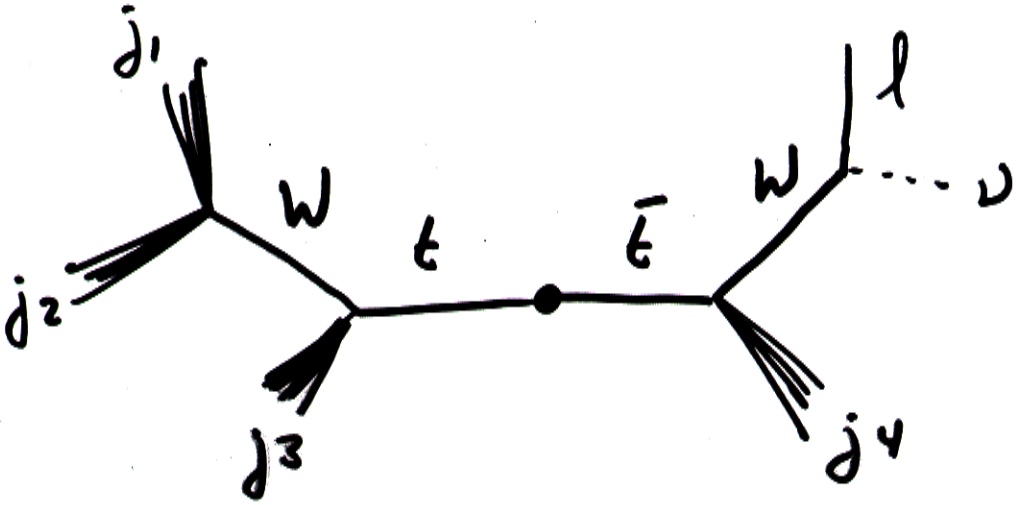
$y = \{\vec{p}_i\}$; f = structure functions

$W(y,x)$ = probability of measuring x when y was produce

$W = \begin{cases} 1 & \text{no measurement} \\ \text{"gaussian"} & \text{detector resolution matters} \\ \delta & \text{if quantity is well measured} \end{cases}$

Add M_i for interfering amplitudes, add P_i for non-interfering ones, and do a weighted sum over all permutations

Appl. to $q\bar{q} \to t\bar{t} \to lepton + jets$



$|M|^2$ = Mahlon, Parke (no $t\bar{t}$ correlations)

f = CTEQ3 (PDF)

l + jet angles = well measured
jet energies = gaussian distribution

$P_t(top) = P_t(\overline{top})$

all 12 combinations are included with equal weights

$$W(\mathrm{y},\mathrm{x}) = \prod_{i=1}^{4} \frac{1}{\sigma_i\sqrt{2\pi}} \exp \frac{(\rho_i - \overline{\rho}_i)^2}{2\sigma_i^2} \prod_{j=1}^{5} \delta^2(\Omega_j - \overline{\Omega}_j)\delta\rho_e$$

jet energies

l+ angles

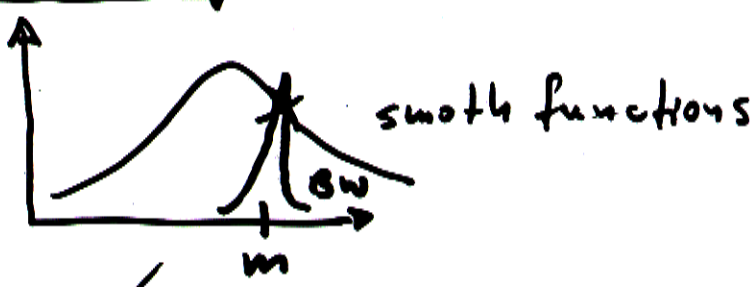
$$P(X) = \int_5 \cdots \int (4\times dE_{jets})\, dP_L^{\nu}\, (4\times BW) \times \text{smoth functions}$$

$$\hookrightarrow (4\times dM)\, dE_{j1}$$

$$\begin{pmatrix}\text{change}\\ \text{variables}\end{pmatrix} = \left(\begin{array}{l}\text{given the top and W masses (and the}\\ \text{jet angles + } \vec{P}_\ell) \text{ and } E_{j1} \text{ calculate}\\ E_{j2} \cdots E_{j4}\ P_L^{\nu}\end{array}\right)$$

there are 2 ν solutions (most of the time) both are included.

$$P(X) = \int dE_{j1} \underbrace{\int_4 \cdots \int 4\times dM\ (4\times BW)}_{} \times \text{smoth functions}$$

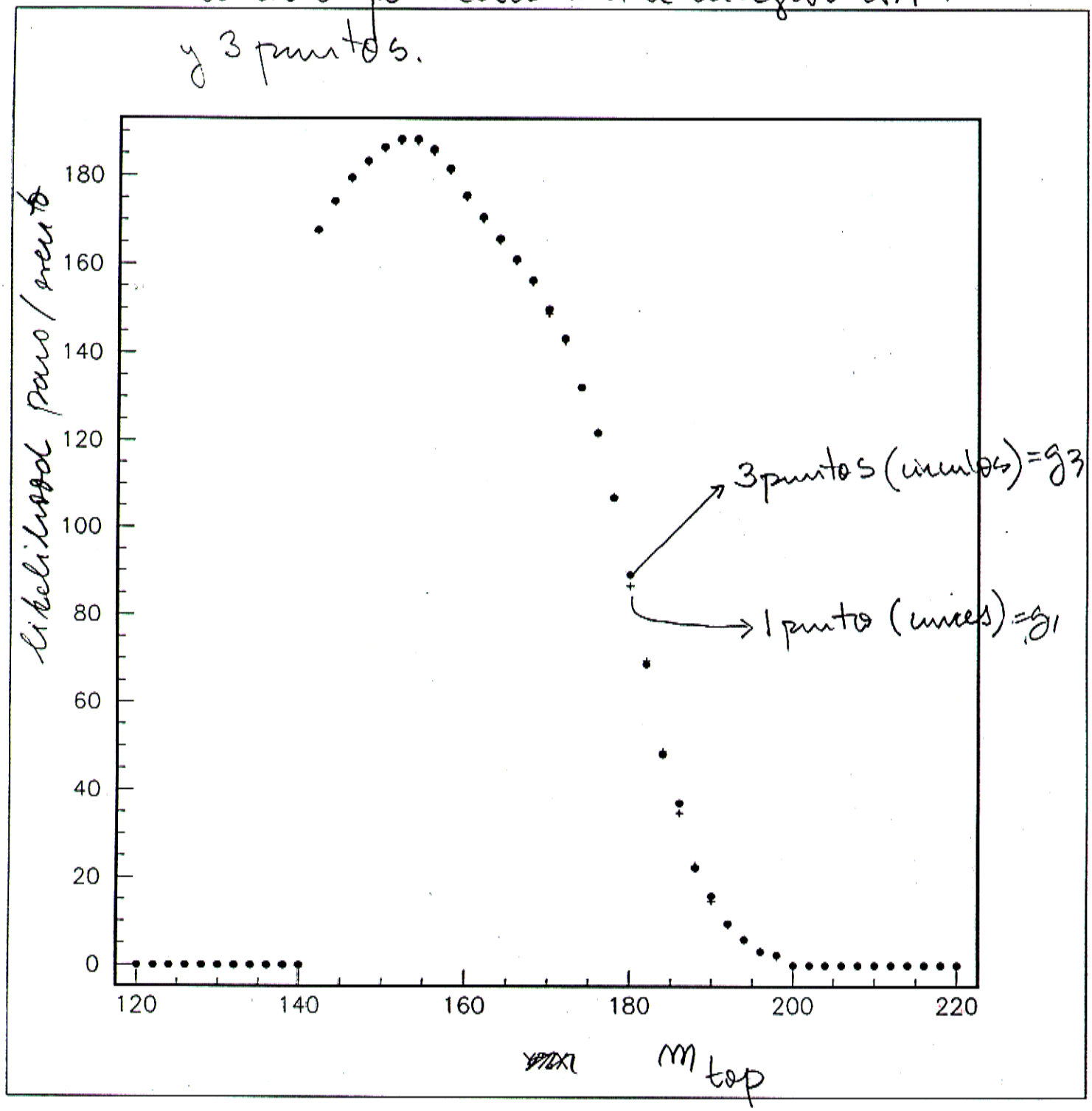


4 integrals calculated with one point each

$$P(X) = \int dE_j \text{ smoth functions (at the top + W masses)}$$

$\hookrightarrow$ calculated numerically

gaston: ya ~~no~~ tengo + el problema de la diferencia entre integrar con 1 y 3 puntos.



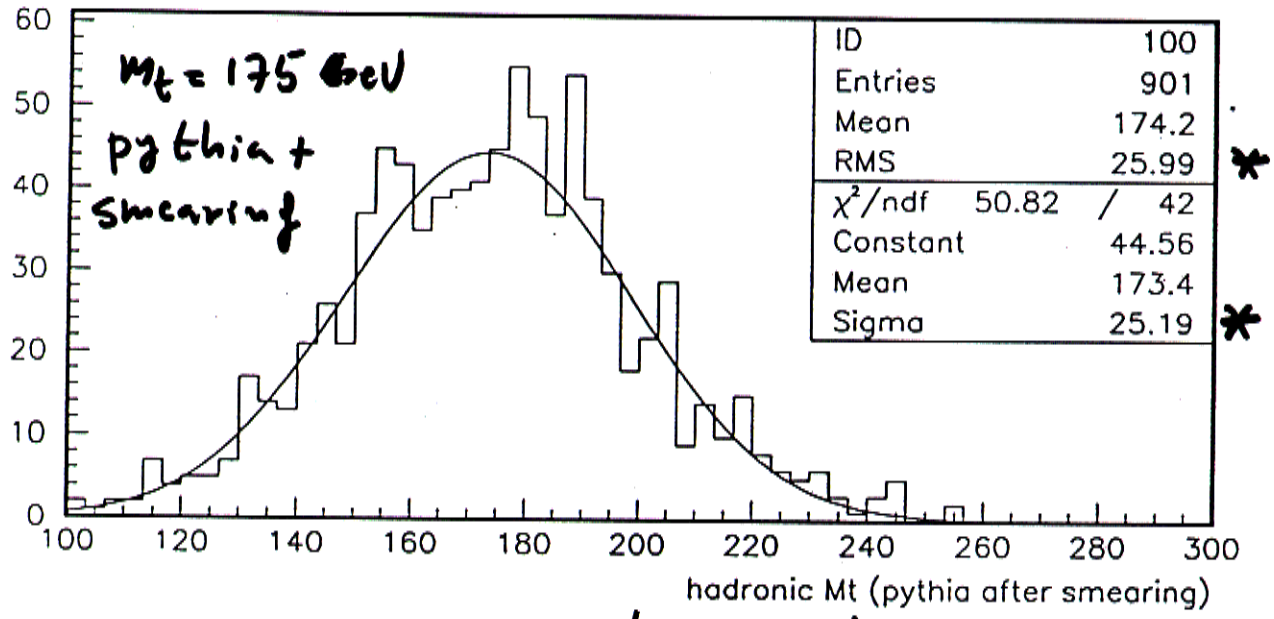
esto lo calcule hoy (10/19) con prob_4.f y el resultado esto en fort.21 (col 1 col 2 col 3 m_t g1 g3)

todo está en: /projects/795/emid-2/estrada/top2

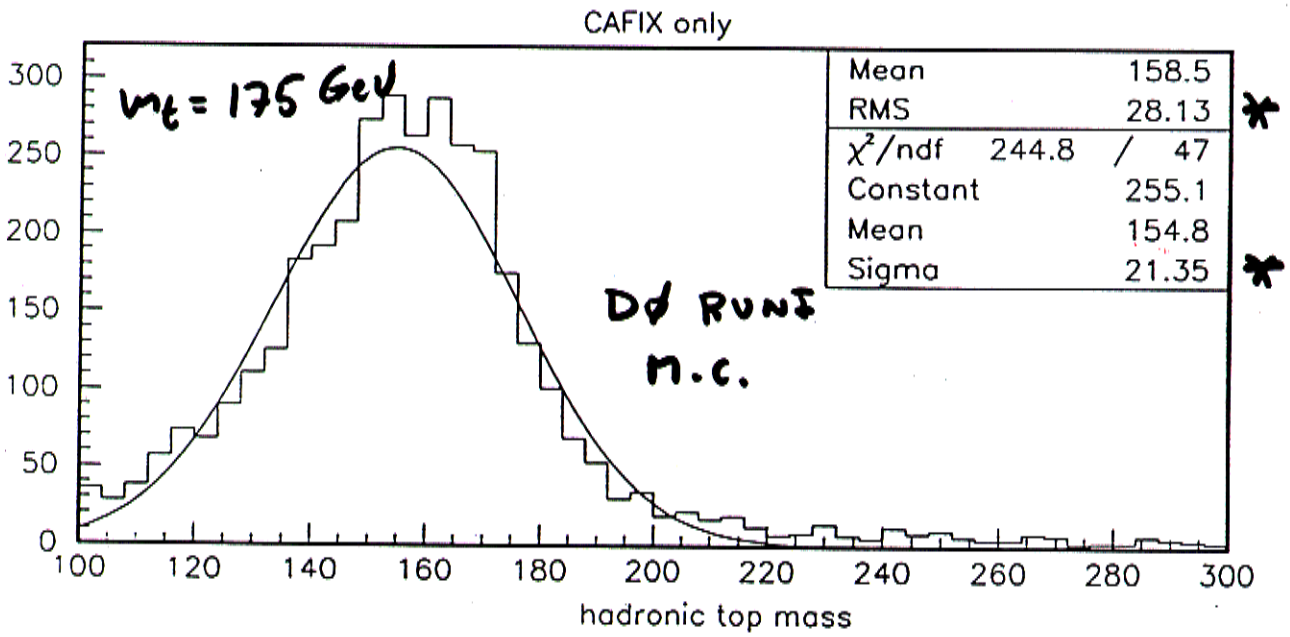
Method was tested in two samples

① Pythia events (parton level) smeared according to resolution

② DØ Run I Monte Carlo events
Herwig 5.7 (VECBOS) + CTEQ3M + DØ GEANT
+ DØ event reconstruction

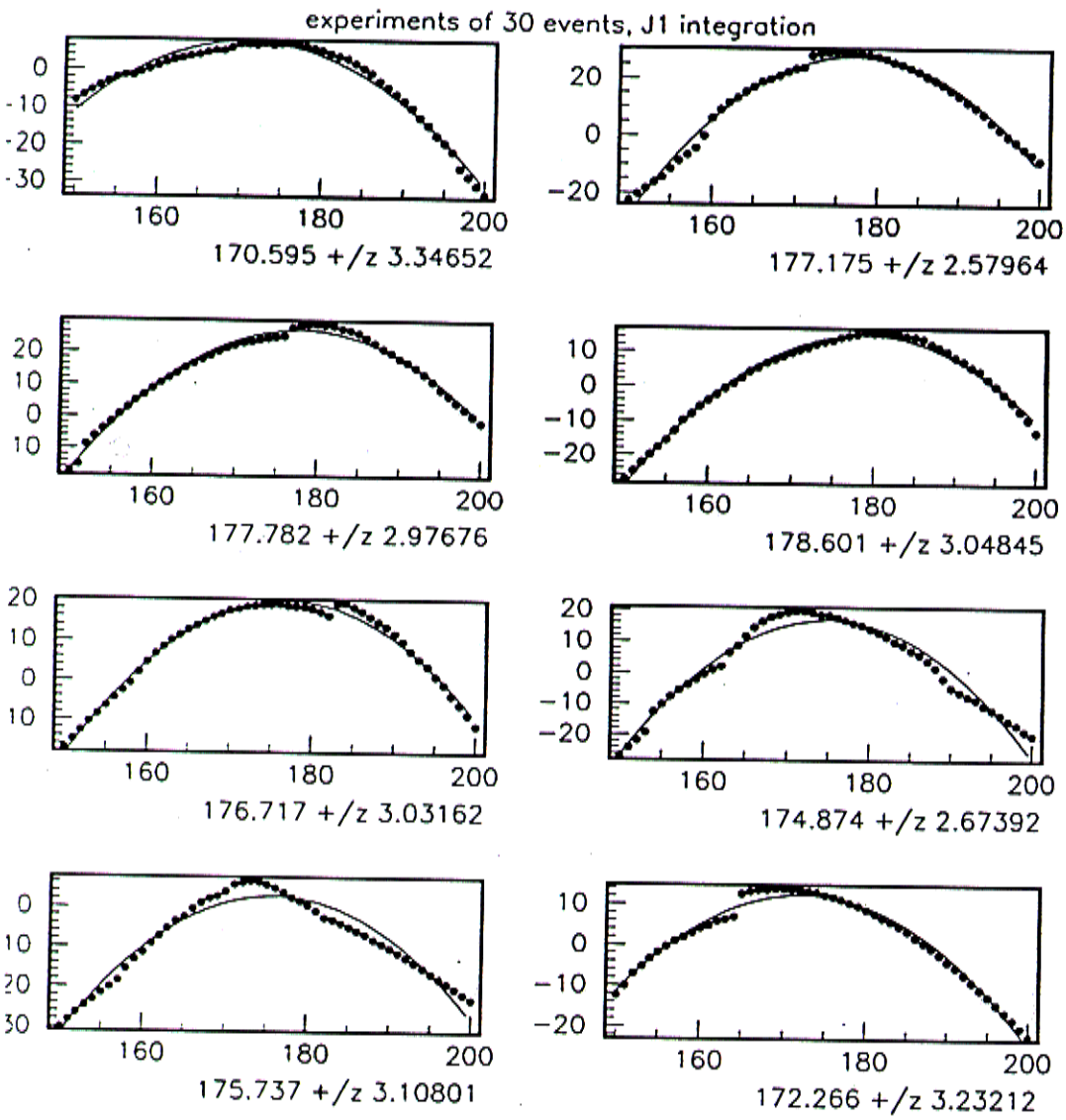


$t \to 3$ jets invariant mass



$t \to 3$ jets invariant mass

Pythia + smearing
8 experiments with 30 events each



$\sigma_{\Delta(\ln \mathcal{L}) = 0.5}$

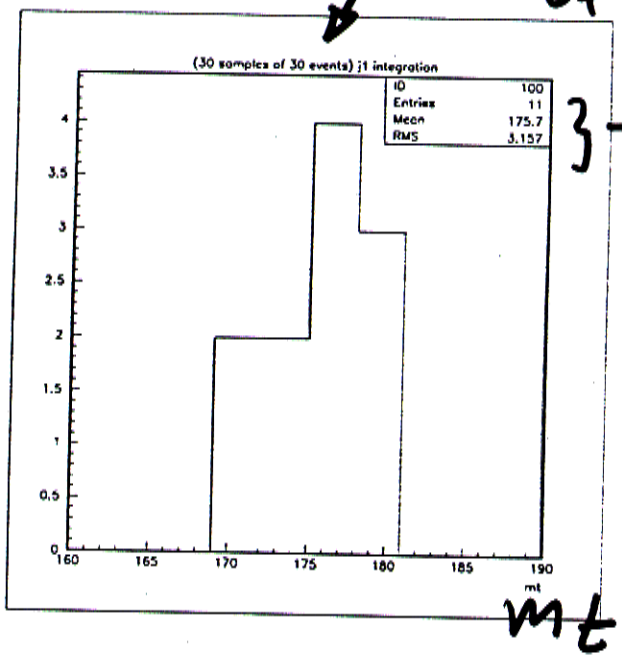
3.4
2.6
3.0
3.0
3.0
2.7
3.1
3.2

3.0

average σ

: 9: Likelihood as a function of the top mass for 8 experiments of 30 events. Events generated with pythia for a top quark mass of 175GeV and smeared accoding to n1.2.

Pythia + smearing → m_t for 11 experiments of 30 events each



$\bar{m}_t = 175.7$

rms = 3.16

$\sigma_{\Delta(\ln \mathcal{L})} = 3.0$

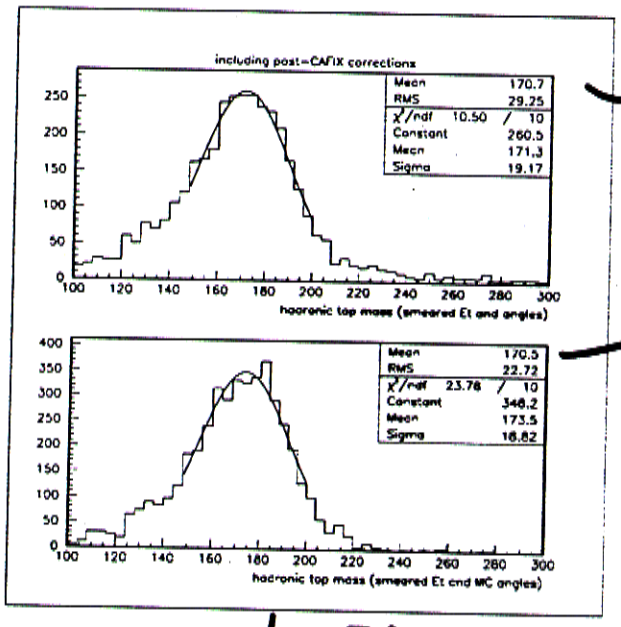
m_t

Figure 10: Distribution of most probable to mass obtained in 11 experiments of the type in Fig.9

DØ Run I M.C.

what mass should we expect?

generated with 175 GeV



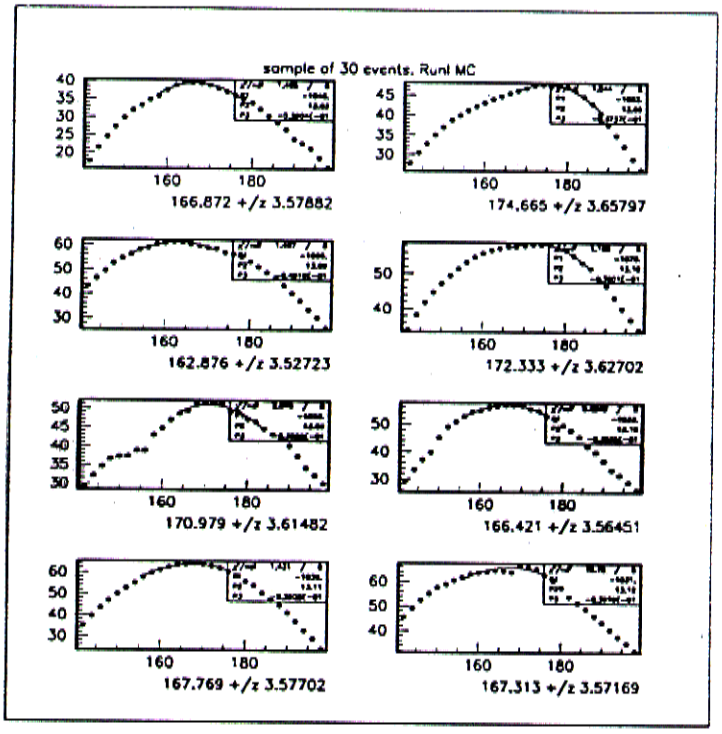
→ 170.7 GeV

→ 170.5 GeV

$t \to 3j$ invariant mas

Figure 11: Top mass for the hadronic branch after CAFIX+post-cafix corrections. The top quark was generated with a mass of $175 GeV$

DØ Run I
M. C.



$\sigma_{D(\ln \mathcal{L})} = 0.5$

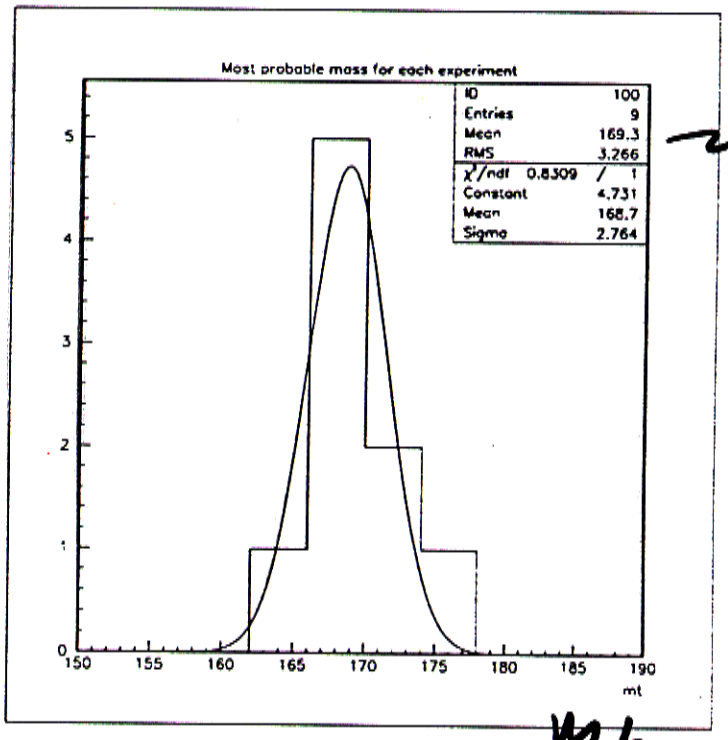
3.6
3.7
3.5
3.6
3.6
3.6
3.6
3.6

3.6

Figure 12: Likelihood as a function of the top mass for experiments of 30 MC events with full DØ simulation and a top quark mass of $175 GeV$

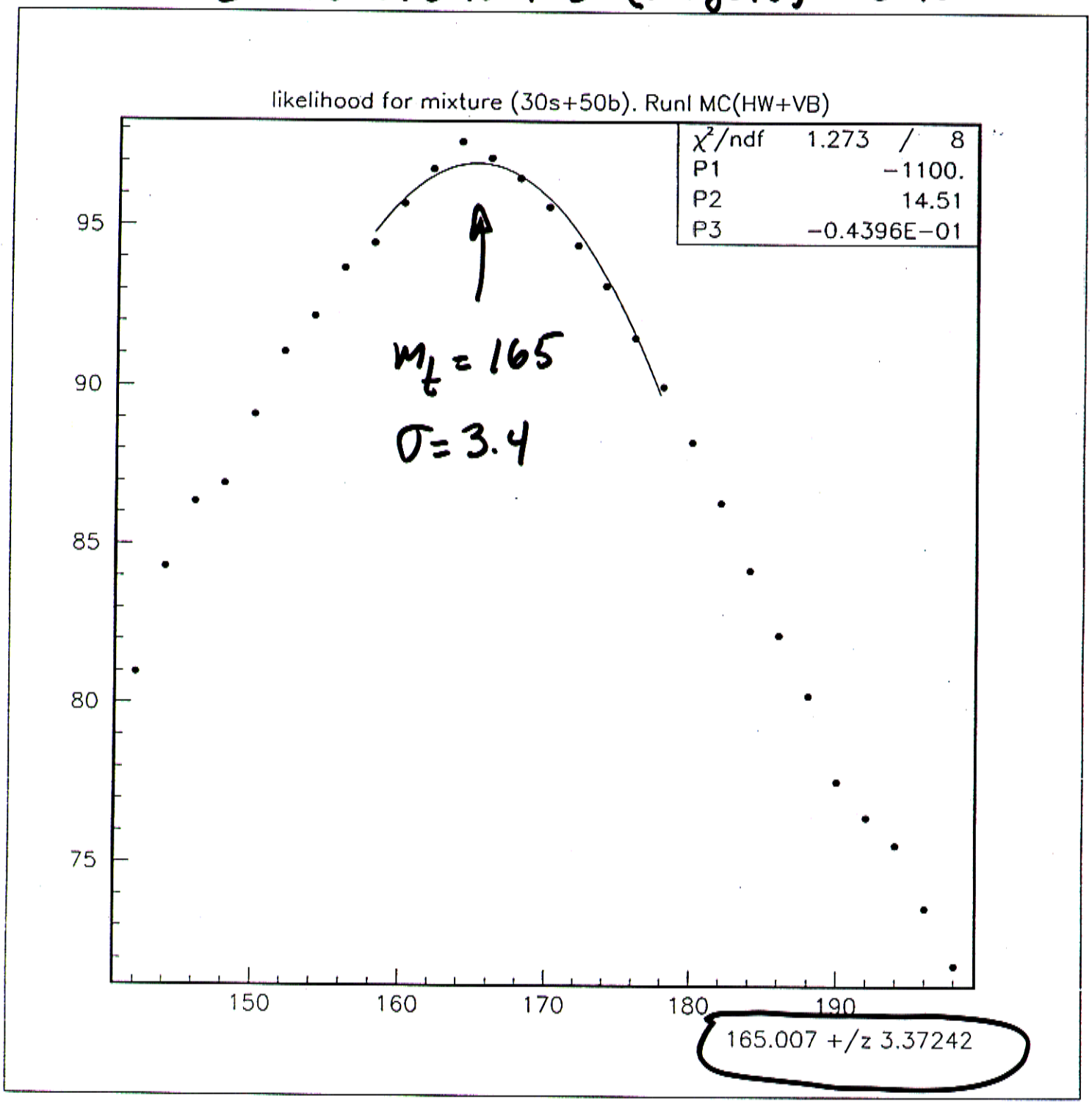
30 experiments of 30 events each

DØ
Run I M.C



$\bar{m}_t = 169.3$
$\sigma = 3.3$

m_t

DØ Run I M.C.

30 $t\bar{t}$ events + 50 (W+jets) events