Mass Fitting Techniques

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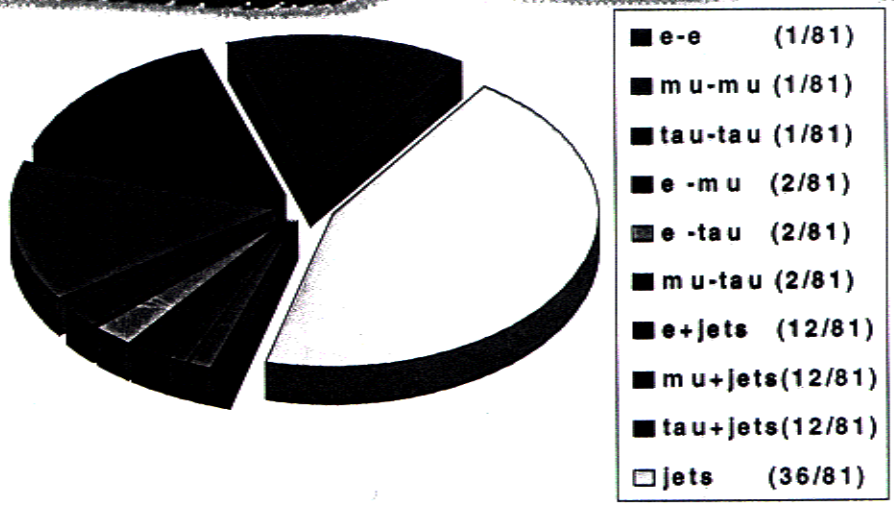
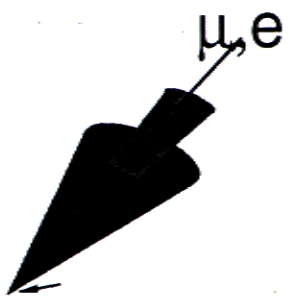
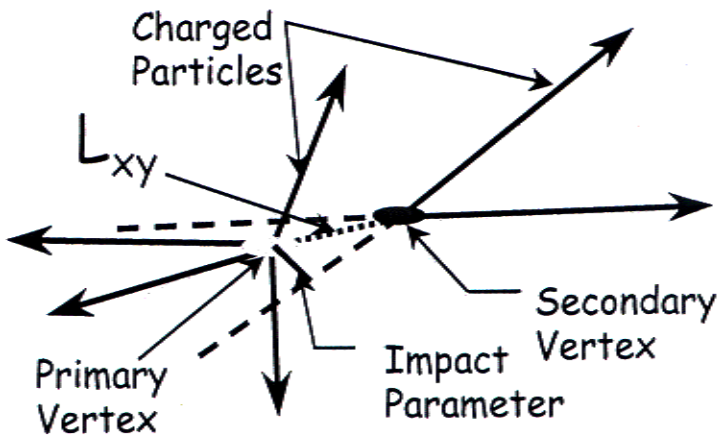
- Outline:
  - Summarize the existing techniques for top mass determination (CDF and DØ) in the dilepton, lepton+jets and multijets channels
  - Emphasizing on some "positive" and "negative" features of the techniques
- This talk doesn't cover an event selection, or problems how to create top reach data sample(s)

Top Decay (SM)

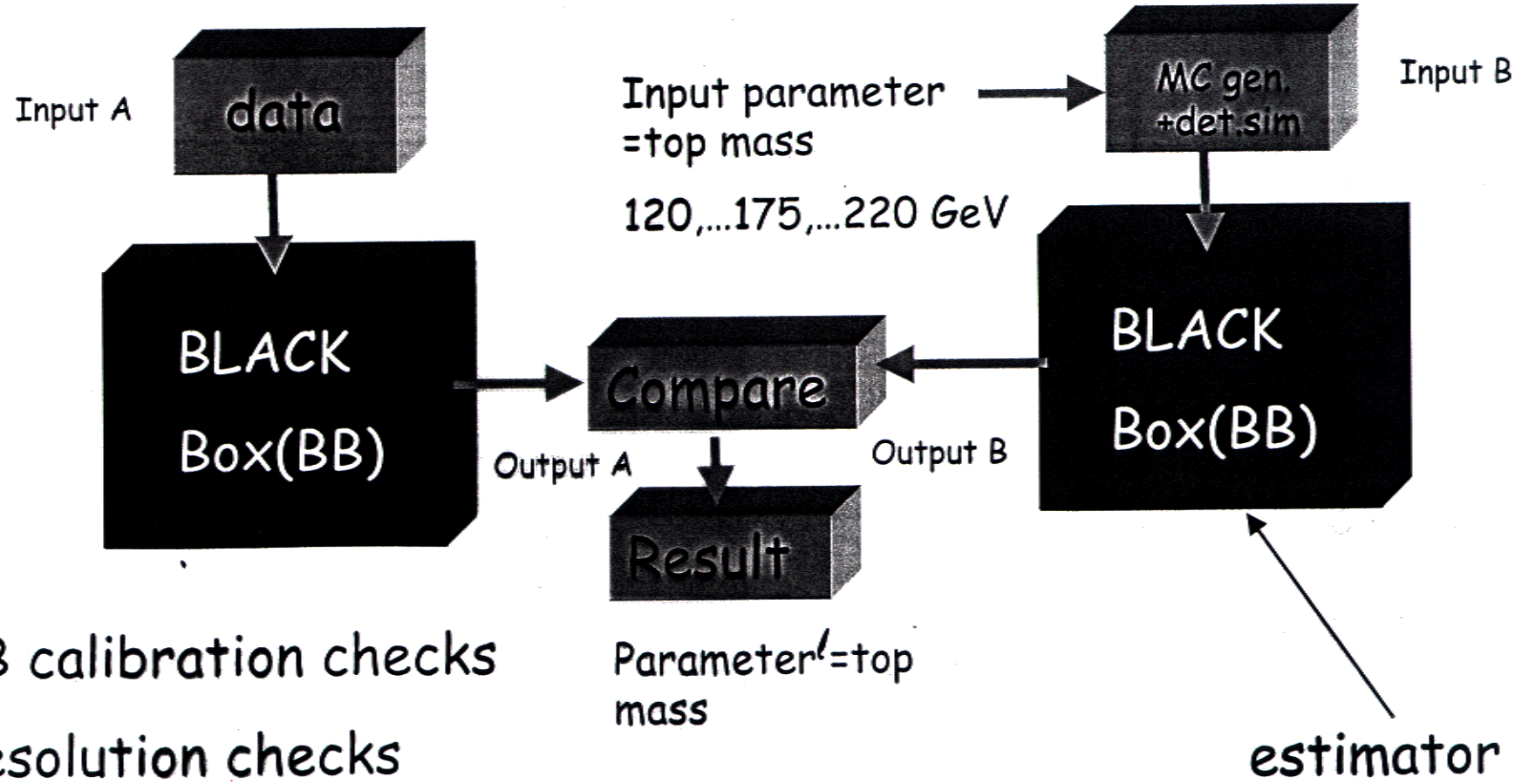
- BR(t $\rightarrow$Wb) $\cong$100 %
  - Both W's decay via W $\rightarrow$ lν
    - final state: lν lν **bb** -
      - **DILEPTON channel**
  - One W decays via W $\rightarrow$ lν
    - final state: lν qq **bb** -
      - **LEPTON+JETS channel**
  - Both W's decay via W $\rightarrow$ qq
    - final state: qq qq **bb** -
      - **ALL HADRONIC channel**
- S/B ~ 7/1, 1/6, 1/200
- Tagging algorithms
  - Low P_T e and μ from: b $\rightarrow$l, b $\rightarrow$ c $\rightarrow$ l
  - b-quark has a long lifetime
    cτ~450μm; b hadrons (P_T~50 GeV)
    travel L_{xy} ~ 3 mm before decay







A top mass analysis (diagram)



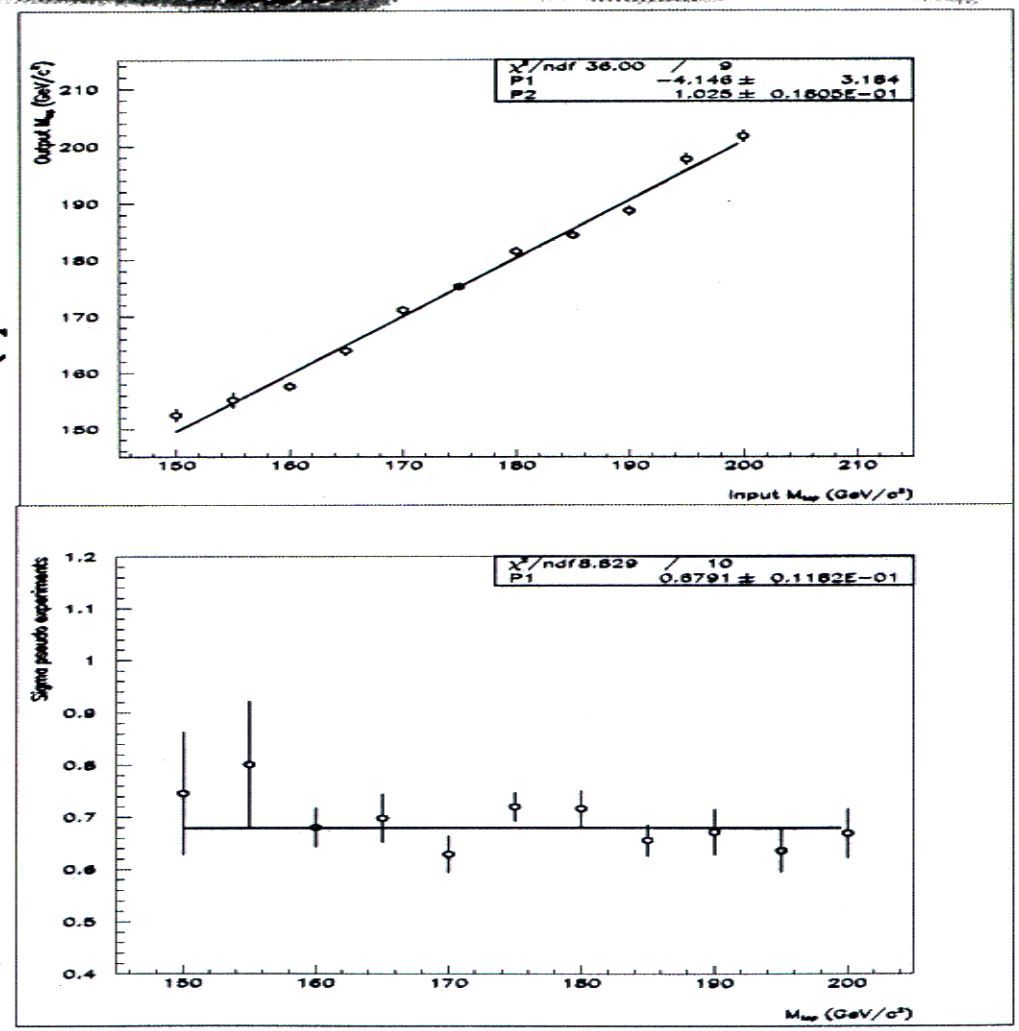
- BB calibration checks
- Resolution checks

BB calibration and resolution

- Calibration:
  - using ensemble tests (ET) or pseudo experiments (PE)
    - repeat many times (at least 1K) the reconstruction procedure supplying the MC samples for INPUT A
    - vary the input top mass
  - plot the pulls from PE

$$Pull = \frac{(m_{top}^{rec} - m_{top}^{input})^2}{(\sigma^{rec})^2}$$

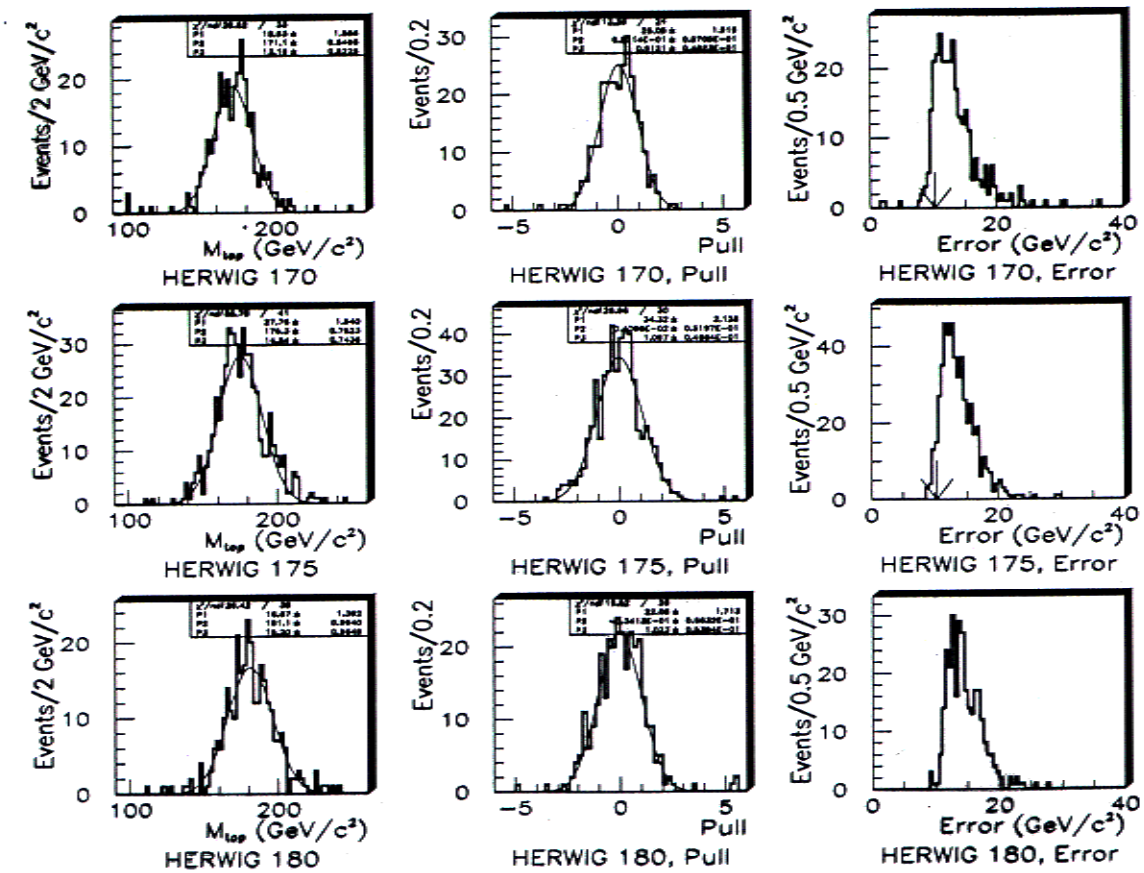
- If the BB is calibrated, pull = G(0,1)



Pulls

- Ensemble test:

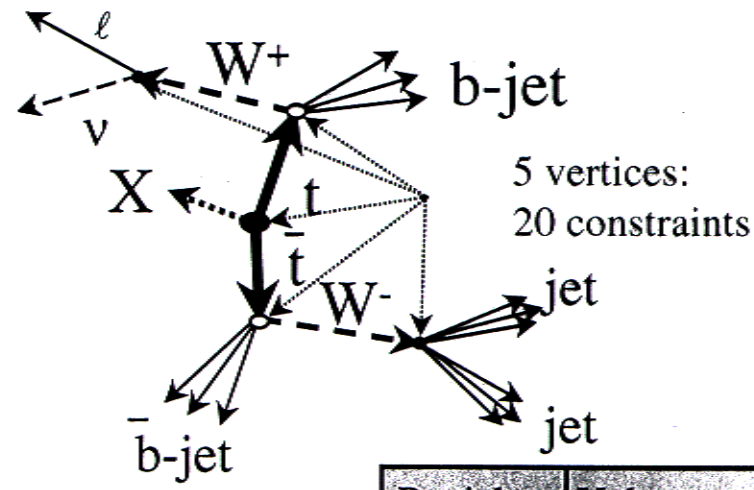
  signal (HERWIG 170, 175,180) +background



Lepton + jets

- ℓ+jets channel → 24 combinations:
  - 12 correspond to the jet parton match; every combination has two solutions for neutrino p_z momentum
- We impose: $M_{t1}=M_{t2}$, M(jet,jet) = $M(\ell,\nu) = M_W$. 2-C fit was applied. The combination with lowest –ln(L) is chosen. **Seems easy** !
- Every kinematical fitting program will work (for example SQUAW)
- Create an own likelihood function and use multifunctional MINUIT to do the job ... Example CDF $\chi 2$

$$\chi^2 = \sum_{l,jets} \frac{(\hat{P}_T - P_T)^2}{\sigma^2_{P_T}} + \sum_{i=x,y} \frac{(\hat{U}_i - U_i)^2}{\sigma^2_{U_i}} + \frac{(M_{l\nu} - M_W)^2}{\sigma^2_{M_W}} + \frac{(M_{jj} - M_W)^2}{\sigma^2_{M_W}} + \frac{(M_{l\nu j} - M_t)^2}{\sigma^2_{M_t}} + \frac{(M_{jjj} - M_t)^2}{\sigma^2_{M_t}}$$



Particles	Unknowns
t's	7
X	2
W's	6
b's	0
q's	0
l	0
ν	3
Total	18

Multijet analysis (all hadronic)

- No ν's - all jets are measured.
- Because of low S/N additional kinematical cuts have to be applied.
- **Resolution**: dominate the combinatorial effect - 90 permutations.
- Kinematic fit to individual events (5C or 3C fit).
- Permutaion with lowest χ^2 is chosen

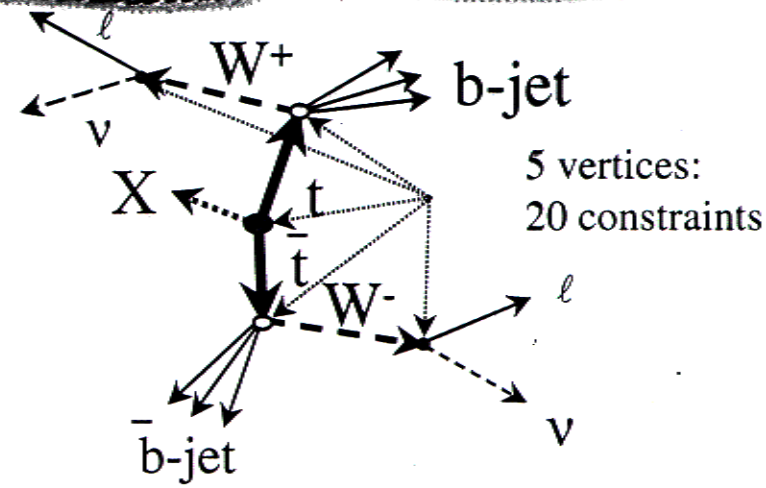
  Experimental mass distribution is compared to the tt MC plus background sample(s) (usually QCD data).

Particles	Unknowns
t's	7
X	2
W's	6
b's	0
q's	0
Total	15

$$\chi^2 = \sum_{jets} \frac{(\hat{P}_T - P_T)^2}{\sigma^2_{P_T}} + \frac{(M_{l\nu} - M_W)^2}{\sigma^2_{M_W}} + \frac{(M_{jj} - M_W)^2}{\sigma^2_{M_W}} + \frac{(M_{l\nu j} - M_t)^2}{\sigma^2_{M_t}} + \frac{(M_{jjj} - M_t)^2}{\sigma^2_{M_t}}$$

Dilepton mass measurement

- Dilepton events - under constrained kinematics
- Additional constraint is needed
- Let's assume that we know one of the kinematical parameter - 0 CF. We can solve the kinematics !
- If we integrate (or sum) over the parameter phase space and sum over pairing combinations - the result is one number.
- Next step: compare to the MC prediction



Particles	Unknowns
t's	7
X	2
W's	6
b's	0
l1	0
l2	0
ν1	3
ν2	3
Total	21

Matrix-Element Weighting

- Extension of ideas proposed by
  R.H. Dalitz and G.R. Goldstein, Phys. Rev. D **45**, 1531 (1992)
- Use the weight

$$W(m_t) = A(m_t) f(x) f(\bar{x}) p(E^*(\ell) | m_t) p(E^*(\bar{\ell}) | m_t)$$

  - $f(x)$: Structure function (PDF) for valence quarks
  - $p(E^*(\ell) | m_t)$: Probability distribution for the energy of the charged lepton in the top rest frame
  - $A(m_t)$: Normalization factor
- Sum over both possible pairings of *b*-jets with leptons

How to calculate $W(m_t)$?

- Additional constrain is assuming value for the top quark, or 0 CF
- Solve for top and tbar momenta
  - two quadratic equations for ν momenta or 0, 2, 4 solutions
- In addition we have two ways to pair leptons and jets
- Final weight is average over all solution weights

CDF and DØ use this method

Neutrino Weighting (νWT)

- Authors of the method: M.Strovink, E. Varnes, DØ
- Assume a top mass (m_t) and a rapidity for each neutrino (η_1, η_2)
- The event can then be solved without using the measured missing $\not{E}_T$
- Assign the weight based on the agreement between the calculated neutrino p_T ($\boldsymbol{p}_T(\nu\nu)$) and the measured missing $\not{E}_T$:

$$W(m_t, \eta_1, \eta_2) = \sum_{\text{Solutions}} \prod_{k=x,y} \exp\left[-\frac{(E_k - p_k(\nu_1\nu_2))^2}{2\sigma(E_k)^2} \right]$$

- Integrate over the neutrino rapidity phase space:

$$W(m_t) = \int d\eta_1 d\eta_2 p(\eta_1 \mid m_t) p(\eta_2 \mid m_t) W(m_t, \eta_1, \eta_2)$$

- Approximate prior probabilities $p(\eta_i \mid m_t)$ by gaussians with widths taken from top Monte Carlo
- In additional average the weight over
  - ν momentum solution
  - jet/lepton paring

(νWT) cont.

- Detector resolution:

  Vary values of observables by experimental resolution and repeat weight calculation (MC integration)

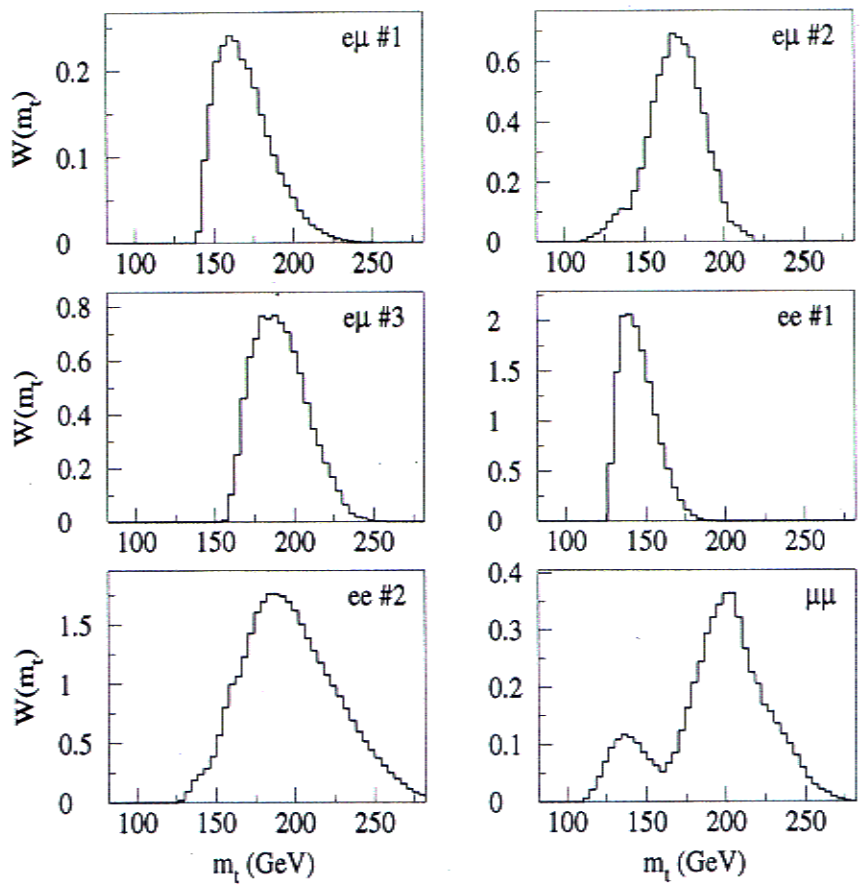
- Gluon radiation:

  Repeat weight calculation for all possible assignments of 3 leading jets

  Calculate weighted average over all jet permutation

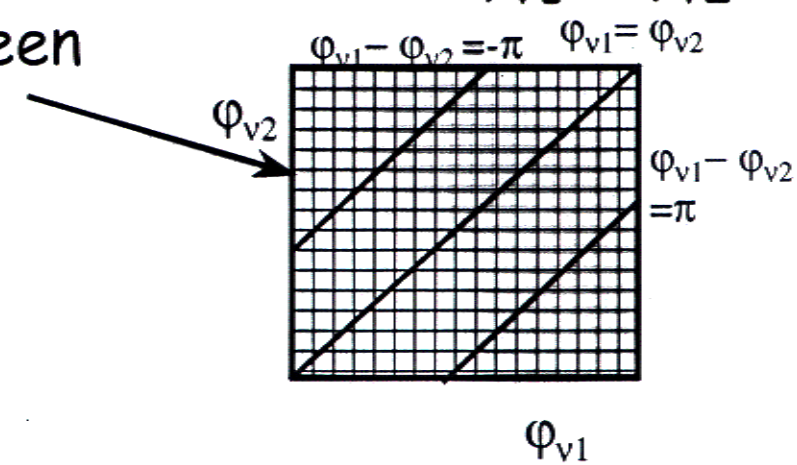
- CDF uses this method for the official dilepton mass analysis

- Both methods depend on MC !



Minuit based method

- The method exploits the CDF l+jets top mass determination procedure.
- If we know some kinematic characteristics of the event: we can constrain the fit. After this we have to vary these characteristics and weight the solutions.
- How to constrain the fit? We selected $\varphi_{\nu 1}$, $\varphi_{\nu 2}$
  - They are homogeneous between $0 \div 2\pi$ (easy to integrate in $\varphi_{\nu 1}$, $\varphi_{\nu 2}$ plane).
  - The distribution is MC independent!!!



Minuit based method (cont.)

- We have a linear system:

$$p_T^{\nu 1} \cos \varphi_{\nu 1} + p_T^{\nu 2} \cos \varphi_{\nu 2} = MET_x$$
$$p_T^{\nu 1} \sin \varphi_{\nu 1} + p_T^{\nu 2} \sin \varphi_{\nu 2} = MET_y$$

- Problem: the system has no solution in case of $\varphi_{\nu 1} - \varphi_{\nu 2} = \kappa \pi$, $p_{x,y}^{\nu}$ are $\sim 1/\sin(\varphi_{\nu 1} - \varphi_{\nu 2})$
- When we fix $\varphi_{\nu 1}$, $\varphi_{\nu 2}$ we have 1 CF.
- The χ^2 is similar as in the l+jets case:

χ^2 = **Resolution + Constrains, where**

$$\text{Resolution} = \sum_{i=1,2} \frac{(P_T^i - \tilde{P}_T^i)^2}{\sigma_i^2} + \sum_{j=1,..E_T>8\,GeV} \frac{(P_T^i - \tilde{P}_T^i)^2}{\sigma_j^2}$$

$$+ \sum_{i=x,y} \frac{(UE^i - U\tilde{E}^i)^2}{UE^{i2}}$$

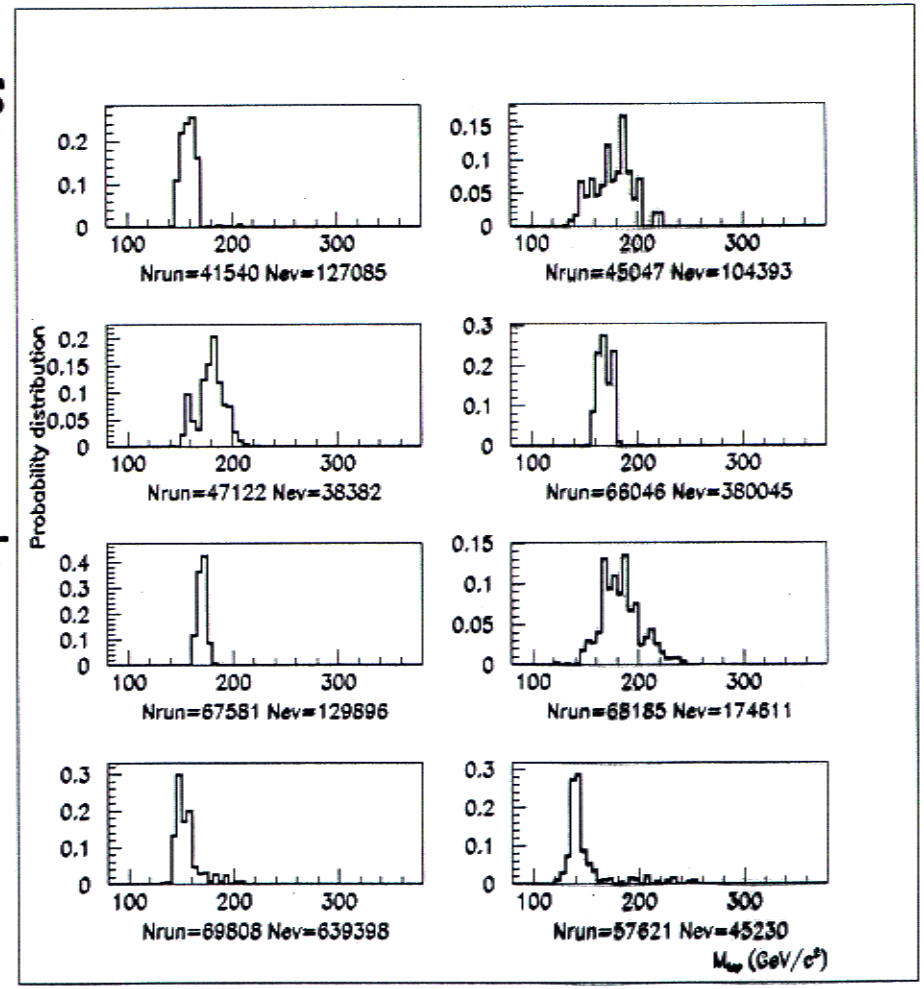
Constrains = $(m_W, m_t = m_{\bar{t}}$, with $\sigma_{m_W} = 2.1$, $\sigma_{m_t} = 2.8\,GeV/c^2$.

Minuit based method (cont.)

- The solutions are 8 for every point, or if we have $n_{\nu 1} \times n_{\nu 2}$ points in $\varphi_{\nu 1}$, $\varphi_{\nu 2}$ plane, it will be corresponded to $8 \times n_{\nu 1} \times n_{\nu 2}$ minimization per event. It was used $n_{\nu 1} = n_{\nu 2} = 12$, (1152 per event).
- The output for every event is: $\chi^2_{i,j,k}$ and $M^{top}_{i,j,k}$, i=1,12, j=1,12, k=1,8.
- The minimal $\chi^2_{i,j,k}$ solution for every point in the $\varphi_{\nu 1}$, $\varphi_{\nu 2}$ plane was selected.
- $M^{top}_{i,j}$ (for minimal $\chi^2_{i,j}$ solution) is weighted with $\exp(-\chi^2_{i,j}/2)/W$, where $W = \sum_i \sum_j \exp(-\chi^2_{i,j}/2)$.

Why this method?

- This method does not depend on low level MC distributions
- Event kinematic fitting uses an algorithm which is very close to the lepton+jet procedure - proved
- We do not apply any cuts. The templates are the simple sum of p.d.f. for every event



Top mass result

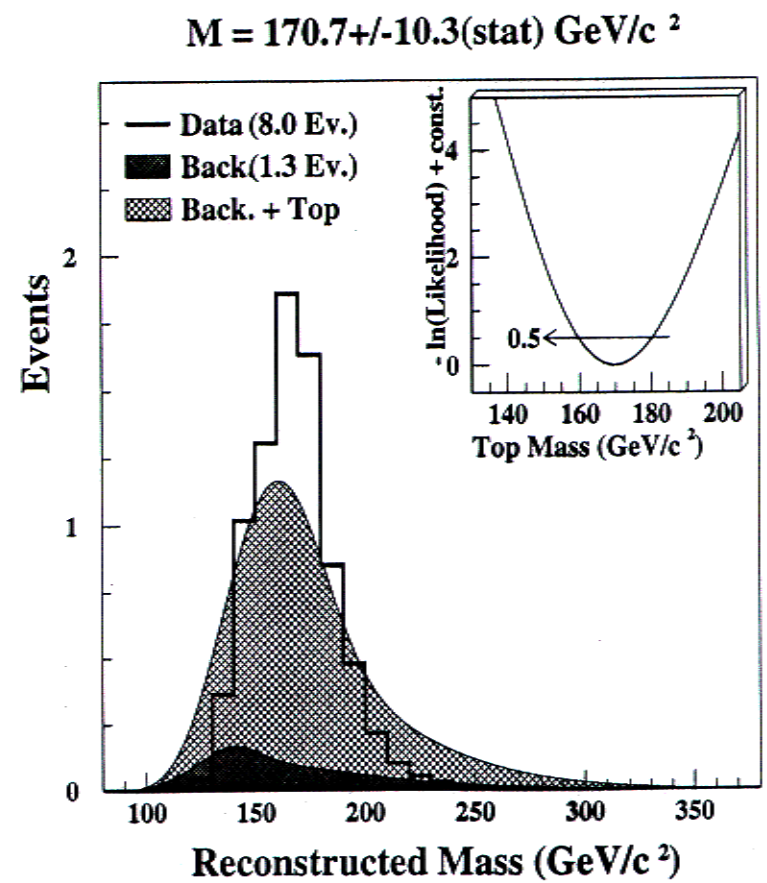
- The result is:

$$M_{top} = 170.7^{+10.6}_{-10.0}\ \mathrm{GeV}/c^2 \text{ constrained fit}$$

$$M_{top} = 170.7^{+9.6}_{-9.2}\ \mathrm{GeV}/c^2 \text{ non-constrained fit}$$

$$n_{bgr} = 0.0^{+1.14}_{-0.00} \text{ events}$$

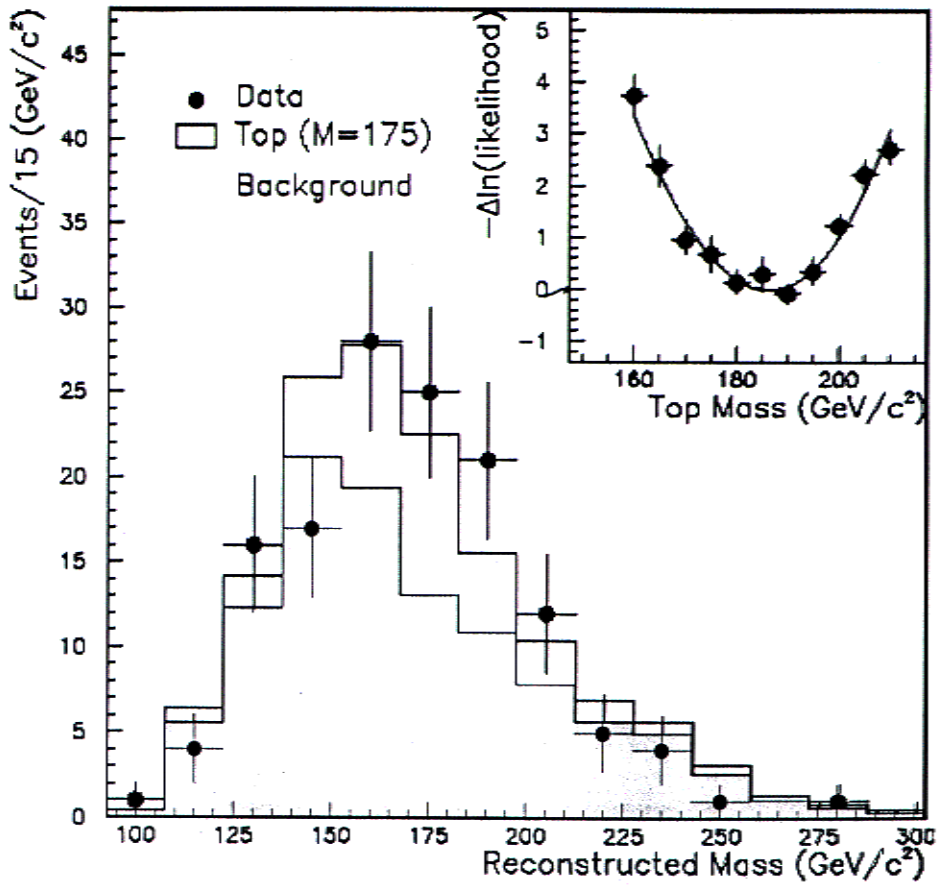
CDF preliminary



Data and MC comparison

- What exactly is the box Compare?

- Depending on the BB Outputs 1 and 2 we could distinguish two general cases
  - Output1 and output2 are histograms. In this case, the binned LH will work for the comparator. Problems:
    - how to incorporate the finite MC statistics in the final result
    - what is the optimal size of the histogram bin
    - how to determinate the top mass and the its uncertainty



Parameterized MC PDFs (parameterized templates)

- Let's assume for a moment that we know the analytical output from BB2. The comparator has a form

$$L_{shape} = \prod_{i=1}^{N_{events}} ((1-x_b) f_s(M_i, M_{top}, \vec{\alpha}) - x_b f_b(M_i, \vec{\beta}))$$

where: M_{top} is only unconstrained parameter, $f_{s,b}$ are the signal and background PDFs, x_b is a background fraction, M_i are reconstructed masses for N events

- How to find analytical form of PDFs $f_{s,b}$?
  - One solution is to use the technique of Probability Density Estimation (L. Holmström *et al.*, Comp. Phys. Comm. **88**, 195 (1995)) The idea is to insert a n-dimensional (n=2) Gaussian kernel at the position of each MC point.

Parameterized MC PDFs, cont.

- The CDF analyses, a following parameterization of the Output2 is used

$$f(M, M_{top}, \vec{\alpha}) = \{p_6 N_1 \varphi_1(M, p_{1,2,3}) + (1 - p_6) N_2 \varphi_2(M, p_{4,5})\}$$

$$N_1 = p_3^{(1+p_2)} \Gamma^{-1}(1 + p_2) \quad \varphi_1 = (M - p_1)^2 e^{-p_3(M - p_1)}$$

$$N_2 = (\sqrt{2\pi}\, p_5)^{-1} \quad \varphi_2 = e^{-\frac{1}{2}(\frac{M - p_4}{p_5})^2}$$

$$\int_{-\infty}^{\infty} N_1 \varphi_1(M) = \int_{-\infty}^{\infty} N_2 \varphi_2(M) = 1 \quad \forall \vec{p}$$

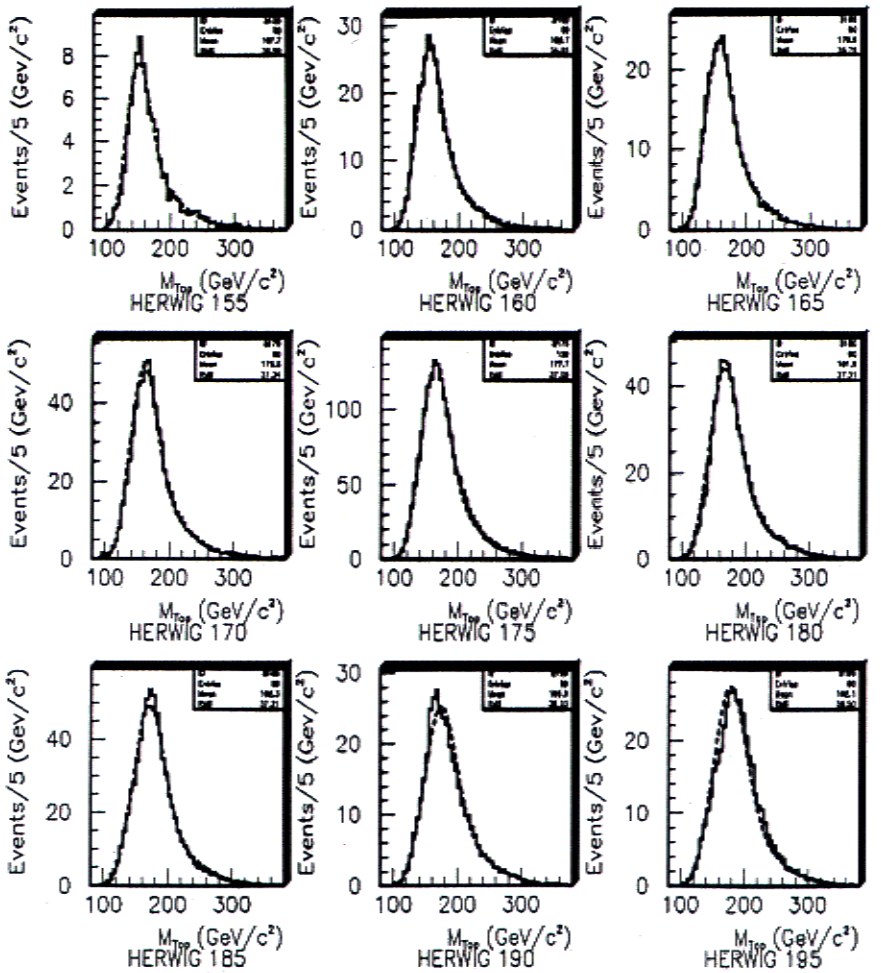
and each of $6\ p_i$ *is assumed to depend linearly on top mass*

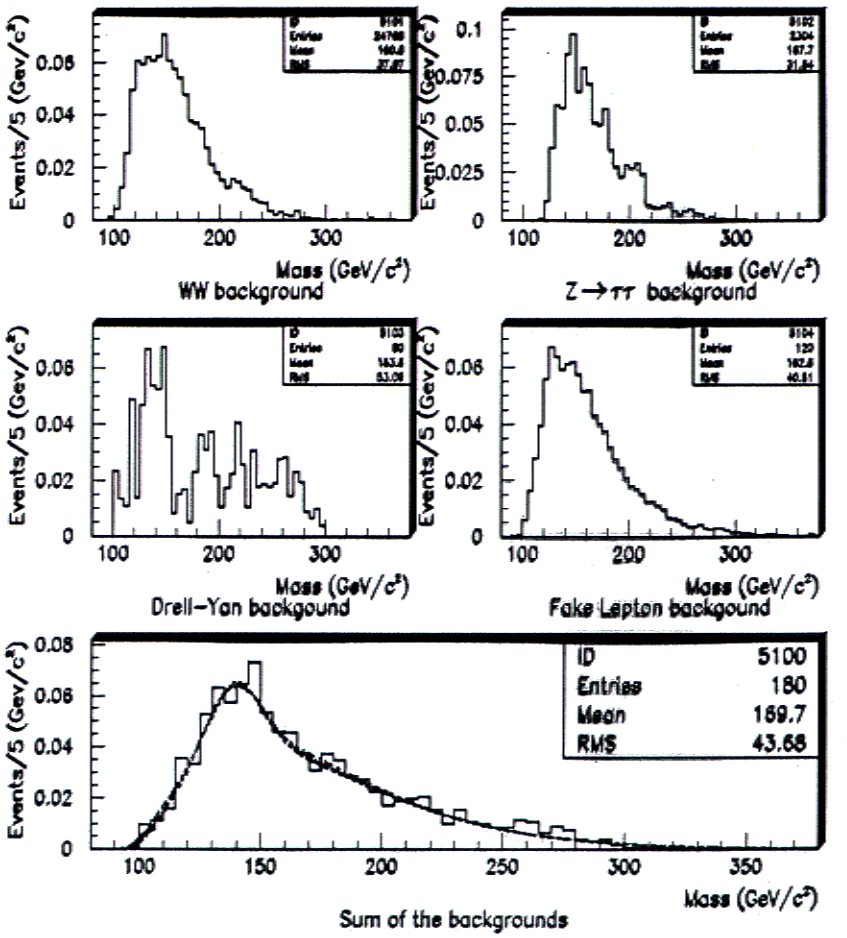
$$p_i = \alpha_i + \alpha_{i+6}(M_{top} - 175)$$

- For background PDF we do not assume linear dependence on the parameter M_{top}

Results

CDF preliminary





Parameterized MC PDFs, Lh form

- Or the final form of our comparator is

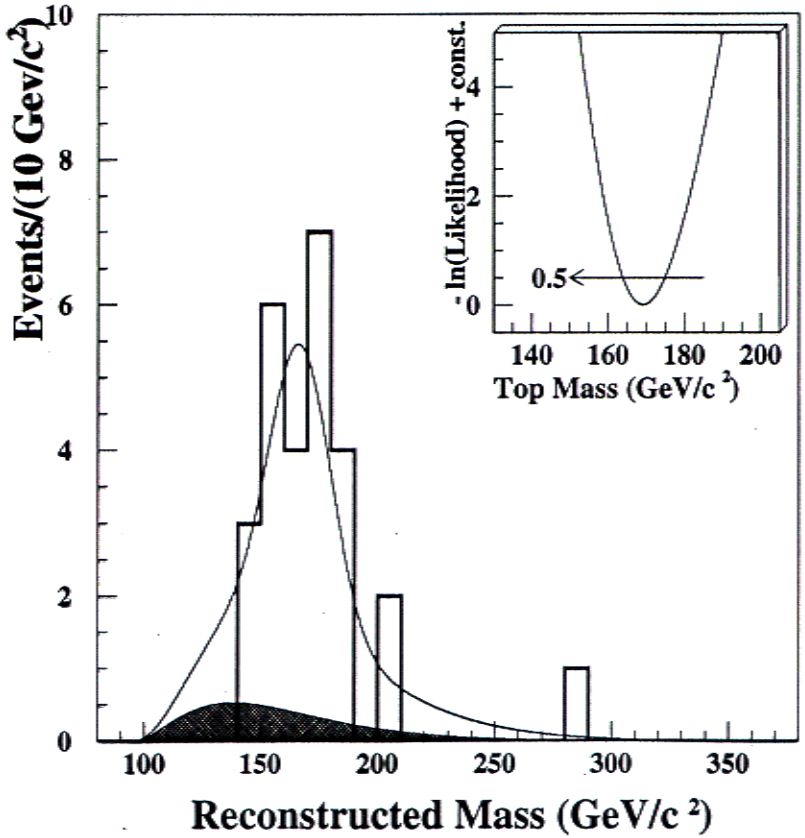
$$L_{shape} = \prod_{i=1}^{N_{events}} ((1-x_b) f_s(M_i, M_{top}, \vec{\alpha}) - x_b f_b(M_i, \vec{\beta})),$$

$$L_{parameters} = \exp\left\{-\frac{1}{2}\left[(\vec{\alpha}-\vec{\alpha}_0)^T U^{-1}(\vec{\alpha}-\vec{\alpha}_0) + (\vec{\beta}-\vec{\beta}_0)^T V^{-1}(\vec{\beta}-\vec{\beta}_0)\right]\right\}$$

$$L_{background} = P(x_b), \quad L = L_{shape} \times L_{parameters} \times L_{background}$$

- Where the vectors α and β are the parameters, constrained to the result of the parameterization via their covariance matrices U and V.
- And that's it ...

And the result ...



Summary

- The existing techniques for top mass determination (CDF and DØ) in the dilepton, lepton+jets and multijets channels are summarized
- A CDF dilepton top mass estimator, similar to lepton + jets estimator, is introduced. It was proved to work.
- A method of the "parameterized templates" used in the CDF top mass analyses is discussed