On Asymmetry Observables In $b \to c\tau\nu$

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Based on : 1905.03311
With : David Shih
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1810.06597, 2004.XXXXX
With : Matthew Buckley, Jorge Camalich, Anna Hallin, David Shih, Susanne Westhoff

Talk Presented @ Fermilab

March 26, 2020

Outline

- Overview of $R_{D^{(*)}}$
- $F^L_{D^*}$ and $R_{J/\psi}$
- A New Asymmetry Observable

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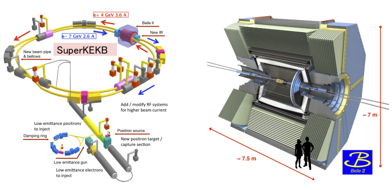

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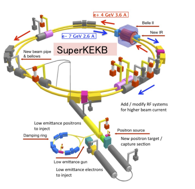

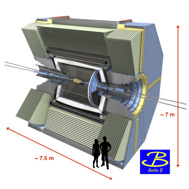

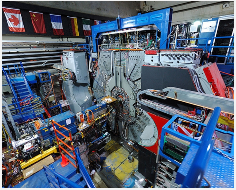

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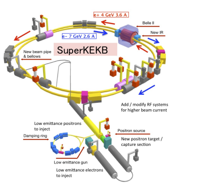

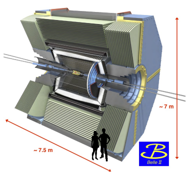

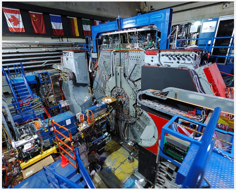

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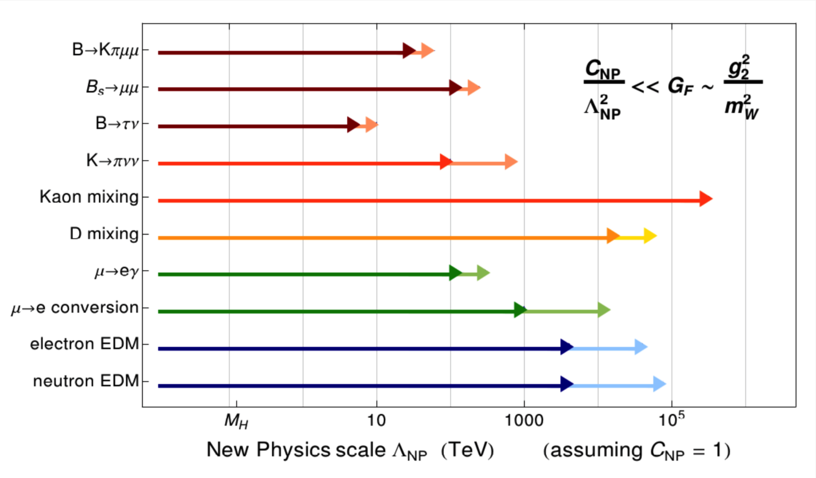
Flavor physics precision measurements can unveil the structure of NP in higher energies.

Probing Higher Energies



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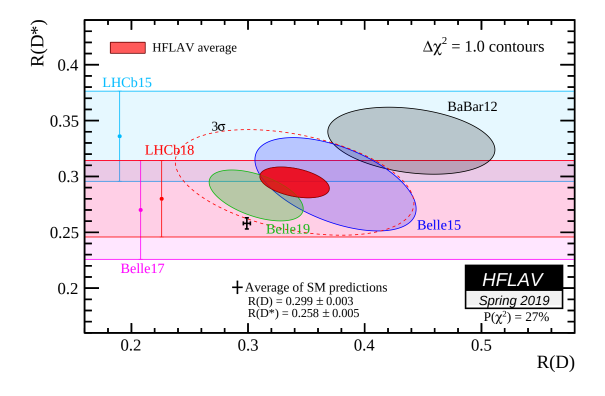
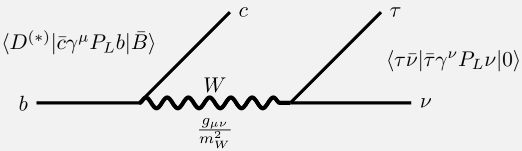
$$R_D^{obs} = 0.340 \pm 0.028, \qquad R_{D^*}^{obs} = 0.295 \pm 0.013.$$

Experimental Results



The Theory



$$\langle D^{(*)}\tau\nu|\left(\bar{c}\gamma^{\mu}P_L b\right)\left(\bar{\tau}\gamma^{\nu}P_L\nu\right)|\bar{B}\rangle$$

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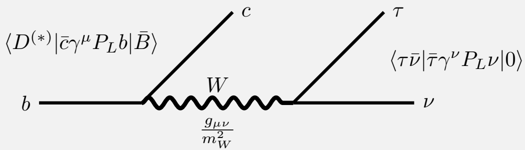

$$\langle D^{(*)}\tau\nu|\left(\bar{c}\gamma^\mu P_L b\right)\left(\bar{\tau}\gamma^\nu P_L \nu\right)|\bar{B}\rangle$$

- The most general dim-6 effective Hamiltonian:

$$\mathcal{H}_{\rm eff} = \frac{4 G_F V_{cb}}{\sqrt{2}} \sum_{\substack{X=S,V,T \\ M,N=L,R}} C^X_{MN} \mathcal{O}^X_{MN},$$

$$\begin{aligned}
\mathcal{O}^S_{MN} &\equiv (\bar{c}P_M b)(\bar{\tau}P_N\nu), \\
\mathcal{O}^V_{MN} &\equiv (\bar{c}\gamma^\mu P_M b)(\bar{\tau}\gamma_\mu P_N\nu), \\
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\end{aligned}$$

for $M, N = R$ or L ($\mathrm{SM} : C^V_{LL} = 1$).

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- Do these observables prefer any of $R_{D^{(*)}}$ solutions?

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- They all give rise to very small deviation from SM prediction for $F^L_{D^*}$ and $R_{J/\psi}$.

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- Is there any combination of the dim-6 operators that can explain these observed values?
- What is the maximum attainable $F^L_{D^*}$ or $R_{J/\psi}$ in the space of all WCs?[1905.03311]

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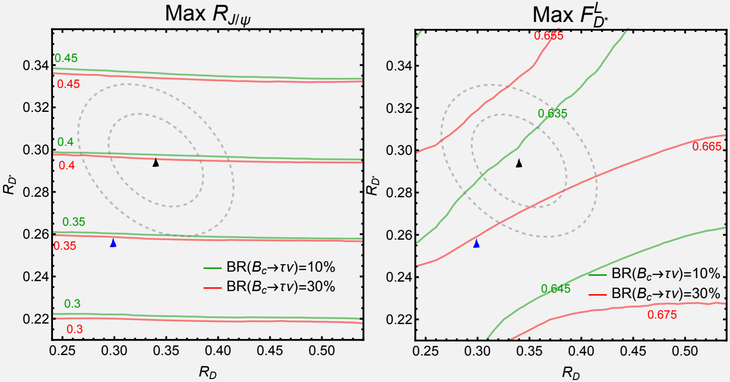
- Three further constraints : R_D, R_{D^*}, $Br(B_c \to \tau\nu)$. Two remaining degrees of freedom to maximize $F^L_{D^*}$ or $R_{J/\psi}$ over.

Global Maximums



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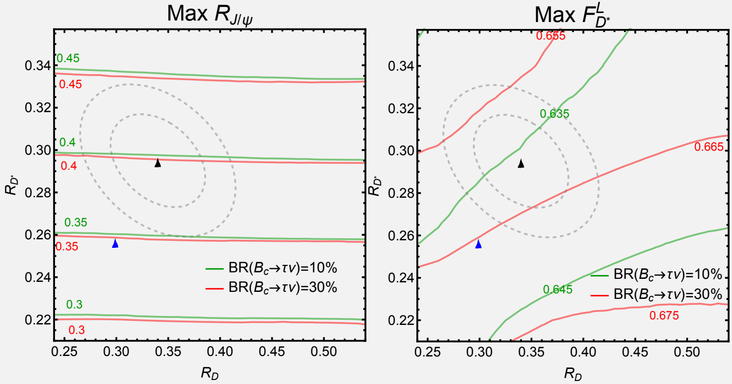

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- Some other asymmetry observables may help.

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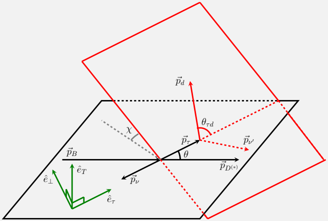
Discerning Different Solutions



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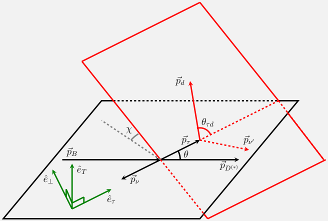

$$\mathcal{A}_{FB}^{(*)} = \frac{1}{\Gamma^{(*)}} \left(-\int_{\theta=0}^{\theta=\pi/2} + \int_{\theta=\pi/2}^{\theta=\pi} \right) d\theta \frac{d\Gamma^{(*)}}{d\theta}$$

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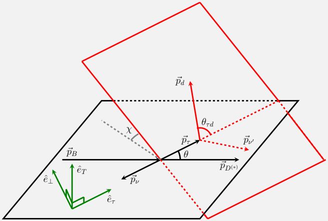

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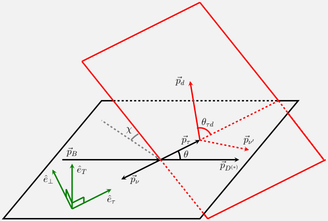

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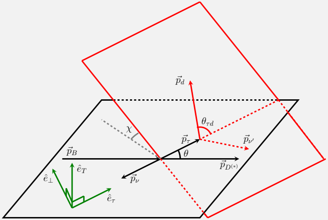
Observable	$\mathcal{A}_{FB}$	$\mathcal{A}^*_{FB}$	$\mathcal{P}_L$	$\mathcal{P}^*_L$	$\mathcal{P}_\perp$	$\mathcal{P}^*_\perp$	$\mathcal{P}_T$	$\mathcal{P}^*_T$
SM value	-0.360	0.063	0.325	-0.497	-0.842	-0.499	0	0

Discerning Different Solutions



$$\mathcal{A}_{FB}^{(*)} = \frac{1}{\Gamma^{(*)}} \left(-\int_{\theta=0}^{\theta=\pi/2} + \int_{\theta=\pi/2}^{\theta=\pi} \right) d\theta \frac{d\Gamma^{(*)}}{d\theta}, \quad \mathcal{P}_{\hat{e}}^{(*)} = \frac{\Gamma_{+\hat{e}}^{(*)} - \Gamma_{-\hat{e}}^{(*)}}{\Gamma_{+\hat{e}}^{(*)} + \Gamma_{-\hat{e}}^{(*)}}.$$

Observable	$\mathcal{A}_{FB}$	$\mathcal{A}_{FB}^*$	$\mathcal{P}_L$	$\mathcal{P}_L^*$	$\mathcal{P}_\perp$	$\mathcal{P}_\perp^*$	$\mathcal{P}_T$	$\mathcal{P}_T^*$
SM value	-0.360	0.063	0.325	-0.497	-0.842	-0.499	0	0

With enough precision, these observables can discern different models/operators used for $R_{D^{(*)}}$ anomalies [1810.06597].

Another Asymmetry[2004.XXXXX]

Integrating over the phase space of $B \to D^{(*)}\tau\nu$:

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- [1702.02773] : The proposal made in a slightly different language for $B \to D$ decay.
- In its Fisher information analysis, [1702.02773] is missing a term related to $sign(\cos\theta_d)$. Including that improves the precision.

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THANK YOU!

Back up

- Details of Different B Factories
- Other Flavor Anomalies
- Uncertainties
- Fiertz Transformations
- The Running of Different WCs
- Calculation Steps, FFs
- Numerical Equations and Individual Operator Contributions

- $Br(B_c \to \tau\nu)$ and $b \to s\nu\nu$ Constraints
- F^L_{D*} and $\mathcal{P}^*_\tau$ Measurement
- $R_{J/\psi}$ Calculations in the SM
- F^L_{D*} and Other WCs
- Generating C^V_{RL}
- How about the q^2-Distributions?
- Why Real WCs
- More on Fisher Information

Belle

- Asymmetric e^+e^- beam at center of mass energy of $\Upsilon(4S)$. Located at KEK facility near Tokyo. 2000s.
- $\sigma(e^+e^- \to B\bar{B}) \sim nb$, $\sim 1.25\mathrm{ab}^{-1}$. 800×10^6 $B\bar{B}$ pairs.
- Precise measurement of CKM entries and the unitarity triangle angles, Observation of CPV in neutral B-mesons, $R_{D^{(*)}}$ and $R_{K^{(*)}}$, observation of exotic states like X(3872), ...
- First measurement of $B \to D^{(*)}\tau\nu$ in 2007.
- The measurement is done in various channels.
- Channels with similar final state for signal/bkg used to cancel the efficiency uncertainties.
- Rely on the SM q^2-distribution to extract some of the uncertainties, e.g. the efficiency uncertainties.

Babar

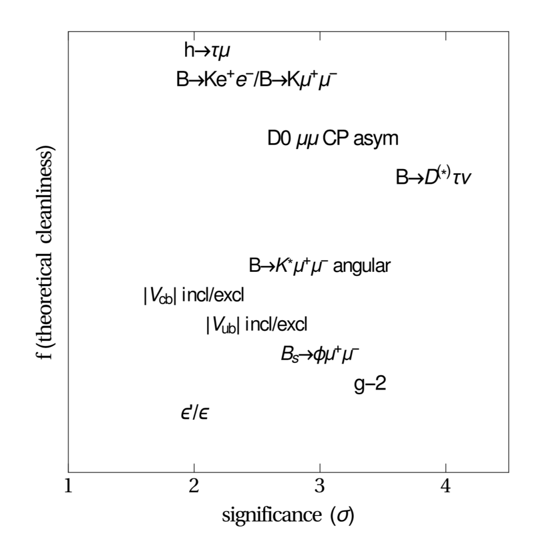
- Asymmetric e^+e^- beam at center of mass energy of $\Upsilon(4S)$. Located at SLAC. 2000s.
- $\sigma(e^+e^- \to B\bar{B}) \sim nb$, $\sim 0.5\mathrm{ab}^{-1}$. 400×10^6 $B\bar{B}$ pairs.
- Similar physics achievements as Belle.
- First measurement of $B \to D^{(*)}\tau\nu$ in 2007-2008.
- First time observation of significant fluctuation : 2012.

Decay	N_{sig}	N_{norm}	$\varepsilon_{\mathrm{sig}}/\varepsilon_{\mathrm{norm}}$	$\mathcal{R}(D^{(*)})$	$\mathcal{B}(B \to D^{(*)}\tau\nu)$ (%)
$B^- \to D^0\tau^-\overline{\nu}_\tau$	314 ± 60	1995 ± 55	0.367 ± 0.011	$0.429 \pm 0.082 \pm 0.052$	$0.99 \pm 0.19 \pm 0.12 \pm 0.04$
$B^- \to D^{*0}\tau^-\overline{\nu}_\tau$	639 ± 62	8766 ± 104	0.227 ± 0.004	$0.322 \pm 0.032 \pm 0.022$	$1.71 \pm 0.17 \pm 0.11 \pm 0.06$
$\overline{B}^0 \to D^+\tau^-\overline{\nu}_\tau$	177 ± 31	986 ± 35	0.384 ± 0.014	$0.469 \pm 0.084 \pm 0.053$	$1.01 \pm 0.18 \pm 0.11 \pm 0.04$
$\overline{B}^0 \to D^{*+}\tau^-\overline{\nu}_\tau$	245 ± 27	3186 ± 61	0.217 ± 0.005	$0.355 \pm 0.039 \pm 0.021$	$1.74 \pm 0.19 \pm 0.10 \pm 0.06$
$\overline{B} \to D\tau^-\overline{\nu}_\tau$	489 ± 63	2981 ± 65	0.372 ± 0.010	$0.440 \pm 0.058 \pm 0.042$	$1.02 \pm 0.13 \pm 0.10 \pm 0.04$
$\overline{B} \to D^*\tau^-\overline{\nu}_\tau$	888 ± 63	11953 ± 122	0.224 ± 0.004	$0.332 \pm 0.024 \pm 0.018$	$1.76 \pm 0.13 \pm 0.10 \pm 0.06$

LHCb

- *pp* collider located at CERN.
- $\sigma(e^+e^- \to B\bar{B}) \sim \mu b$, $\sim \mathcal{O}(1)\mathrm{fb}^{-1}$. 10^{10} $B\bar{B}$ pairs.
- CPV studies, heavier B-mesons, exotic states, $R_{J/\psi}$, ...
- First time observation of significant fluctuation : 2012.

Other Anomalies



$R_{D^{(*)}} + R_{K^{(*)}}$

Model	$R_{K^{(*)}}$	$R_{D^{(*)}}$	$R_{K^{(*)}}$ & $R_{D^{(*)}}$
S_1	✗*	✓	✗*
R_2	✗*	✓	✗
$\widetilde{R}_2$	✗	✗	✗
S_3	✓	✗	✗
U_1	✓	✓	✓
U_3	✓	✗	✗

Uncertainties

BaBar@Hadronic(τ→l)

Source of uncertainty	$\mathcal{R}(D)$ (%)	$\mathcal{R}(D^*)$ (%)
Additive uncertainties		
PDFs		
MC statistics	4.4	2.0
$B \to D^{(*)}(\tau^-/\ell^-)\bar{\nu}$ FFs	0.2	0.2
$D^{**} \to D^{(*)}(\pi^0/\pi^\pm)$	0.7	0.5
$\mathcal{B}(\bar{B} \to D^{**}\ell^-\bar{\nu}_\ell)$	0.8	0.3
$\mathcal{B}(\bar{B} \to D^{**}\tau^-\bar{\nu}_\tau)$	1.8	1.7
$D^{**} \to D^{(*)}\pi\pi$	2.1	2.6
Cross-feed constraints		
MC statistics	2.4	1.5
$f_{D^{**}}$	5.0	2.0
Feed-up/feed-down	1.3	0.4
Isospin constraints	1.2	0.3
Fixed backgrounds		
MC statistics	3.1	1.5
Efficiency corrections	3.9	2.3
Multiplicative uncertainties		
MC statistics	1.8	1.2
$B \to D^{(*)}(\tau^-/\ell^-)\bar{\nu}$ FFs	1.6	0.4
Lepton PID	0.6	0.6
$\pi^0/\pi^\pm$ from $D^* \to D\pi$	0.1	0.1
Detection/Reconstruction	0.7	0.7
$\mathcal{B}(\tau^- \to \ell^-\bar{\nu}_\ell\nu_\tau)$	0.2	0.2
Total syst. uncertainty	9.6	5.5
Total stat. uncertainty	13.1	7.1
Total uncertainty	16.2	9.0

Belle@Semileptonic(τ→l)

Sources	$\mathcal{R}(D^*)$ [%] $\ell^{\rm sig} = e, \mu$
MC size for each PDF shape	2.2
PDF shape of the normalization in $\cos\theta_{B\text{-}D^*\ell}$	$^{+1.1}_{-0.0}$
PDF shape of $B \to D^{**}\ell\nu_\ell$	$^{+1.0}_{-1.7}$
PDF shape and yields of fake $D^{(*)}$	1.4
PDF shape and yields of $B \to X_c D^*$	1.1
Reconstruction efficiency ratio $\varepsilon_{\rm norm}/\varepsilon_{\rm sig}$	1.2
Modeling of semileptonic decay	0.2
$\mathcal{B}(\tau^- \to \ell^-\bar{\nu}_\ell\nu_\tau)$	0.2
Total systematic uncertainty	$^{+3.4}_{-3.5}$

Scales with MC statistics

Scales with DATA statistics

Theory/External

Irreducible
Requires additional studies

Belle@Hadronic(τ→h)

Source	$R(D^*)$	P_τ
Hadronic B composition	$^{+7.8\%}_{-6.9\%}$	$^{+0.14}_{-0.11}$
MC statistics for each PDF shape	$^{+3.5\%}_{-2.8\%}$	$^{+0.13}_{-0.11}$
Fake D^* PDF shape	3.0%	0.010
Fake D^* yield	1.7%	0.016
$\bar{B} \to D^{**}\ell^-\bar{\nu}_\ell$	2.1%	0.051
$\bar{B} \to D^{**}\tau^-\bar{\nu}_\tau$	1.1%	0.003
$\bar{B} \to D^*\ell^-\bar{\nu}_\ell$	2.4%	0.008
τ daughter and ℓ^- efficiency	2.1%	0.018
MC statistics for efficiency calculation	1.0%	0.018
EvtGen decay model	$^{+0.8\%}_{-0.0\%}$	$^{+0.016}_{-0.000}$
Fit bias	0.3%	0.008
$\mathcal{B}(\tau^- \to \pi^-\nu_\tau)$ and $\mathcal{B}(\tau^- \to \rho^-\nu_\tau)$	0.3%	0.002
P_τ correction function	0.1%	0.018
Common sources		
Tagging efficiency correction	1.4%	0.014
D^* reconstruction	1.3%	0.007
D sub-decay branching fractions	0.7%	0.005
Number of $B\bar{B}$	0.4%	0.005
Total systematic uncertainty	$^{+10.4\%}_{-9.5\%}$	$^{+0.20}_{-0.17}$

Three Classes of Solutions

Three broad classes of heavy mediators, i.e. minimal solutions:

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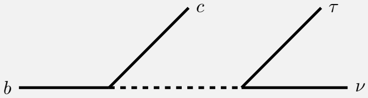
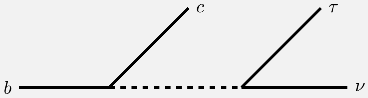


(a, b)

Three Classes of Solutions

Three broad classes of heavy mediators, i.e. minimal solutions:

- (*a*) Colorless scalar, e.g. heavy higgs.
- (*b*) A heavy colorless vector : W'.
- (*c*) Leptoquarks (LQs).



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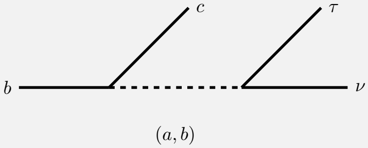

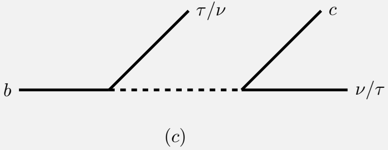

The Viable Minimal Models

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Mediator	Operator Combination	Viability
Colorless Scalars	$\mathcal{O}^S_{XL}$	✗ $(Br\,(B_c \to \tau\nu))$
W'^{μ} (LH fermions)	$\mathcal{O}^V_{LL}$	✗ (collider bounds)
S_1 LQ $(\bar{3}, 1, 1/3)$ (LH fermions)	$\mathcal{O}^S_{LL} - x\mathcal{O}^T_{LL}$, $\mathcal{O}^V_{LL}$	✓
U_1^{μ} LQ $(3, 1, 2/3)$ (LH fermions)	$\mathcal{O}^S_{RL}$, $\mathcal{O}^V_{LL}$	✓
R_2 LQ $(3, 2, 7/6)$	$\mathcal{O}^S_{LL} + x\mathcal{O}^T_{LL}$	✓
S_3 LQ $(\bar{3}, 3, 1/3)$	$\mathcal{O}^V_{LL}$	✗ $(b \to s\nu\nu)$
U_3^{μ} LQ $(3, 3, 2/3)$	$\mathcal{O}^V_{LL}$	✗ $(b \to s\nu\nu)$
V_2^{μ} LQ $(\bar{3}, 2, 5/6)$	$\mathcal{O}^S_{RL}$	✗ ($R_{D^{(*)}}$ value)

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S_1 LQ $(\bar{3}, 1, 1/3)$ (LH fermions)	$\mathcal{O}^S_{LL} - x\mathcal{O}^T_{LL}$, $\mathcal{O}^V_{LL}$	✓
U_1^{μ} LQ $(3, 1, 2/3)$ (LH fermions)	$\mathcal{O}^S_{RL}$, $\mathcal{O}^V_{LL}$	✓
R_2 LQ $(3, 2, 7/6)$	$\mathcal{O}^S_{LL} + x\mathcal{O}^T_{LL}$	✓
S_3 LQ $(\bar{3}, 3, 1/3)$	$\mathcal{O}^V_{LL}$	✗ $(b \to s\nu\nu)$
U_3^{μ} LQ $(3, 3, 2/3)$	$\mathcal{O}^V_{LL}$	✗ $(b \to s\nu\nu)$
V_2^{μ} LQ $(\bar{3}, 2, 5/6)$	$\mathcal{O}^S_{RL}$	✗ ($R_{D^{(*)}}$ value)
Colorless Scalars	$\mathcal{O}^S_{XR}$	✗ $(Br(B_c \to \tau\nu))$
W'^{μ} (RH fermions)	$\mathcal{O}^V_{RR}$	✓
$\tilde{R}_2$ LQ $(3, 2, 1/6)$	$\mathcal{O}^S_{RR} + x\mathcal{O}^T_{RR}$	✗ $(b \to s\nu\nu)$
S_1 LQ $(\bar{3}, 1, 1/3)$ (RH fermions)	$\mathcal{O}^V_{RR}$, $\mathcal{O}^S_{RR} - x\mathcal{O}^T_{RR}$	✓
U_1^{μ} LQ $(3, 1, 2/3)$ (RH fermions)	$\mathcal{O}^S_{LR}$, $\mathcal{O}^V_{RR}$	✓

All Operators

	Operator	Fierz identity	Allowed Current	$\delta\mathcal{L}_{\text{int}}$
$\mathcal{O}_{V_L}$	$(\bar{c}\gamma_\mu P_L b)(\bar{\tau}\gamma^\mu P_L \nu)$		$(\mathbf{1},\mathbf{3})_0$	$(g_q \bar{q}_L \boldsymbol{\tau}\gamma^\mu q_L + g_\ell \bar{\ell}_L \boldsymbol{\tau}\gamma^\mu \ell_L) W'_\mu$
$\mathcal{O}_{V_R}$	$(\bar{c}\gamma_\mu P_R b)(\bar{\tau}\gamma^\mu P_L \nu)$			
$\mathcal{O}_{S_R}$	$(\bar{c}P_R b)(\bar{\tau}P_L \nu)$		$\rangle(\mathbf{1},\mathbf{2})_{1/2}$	$(\lambda_d \bar{q}_L d_R \phi + \lambda_u \bar{q}_L u_R i\tau_2 \phi^\dagger + \lambda_\ell \bar{\ell}_L e_R \phi)$
$\mathcal{O}_{S_L}$	$(\bar{c}P_L b)(\bar{\tau}P_L \nu)$			
$\mathcal{O}_T$	$(\bar{c}\sigma^{\mu\nu}P_L b)(\bar{\tau}\sigma_{\mu\nu}P_L \nu)$			
$\mathcal{O}'_{V_L}$	$(\bar{\tau}\gamma_\mu P_L b)(\bar{c}\gamma^\mu P_L \nu)$	$\longleftrightarrow \mathcal{O}_{V_L} \langle$	$(\mathbf{3},\mathbf{3})_{2/3}$	$\lambda \bar{q}_L \boldsymbol{\tau}\gamma_\mu \ell_L \boldsymbol{U}^\mu$
$\mathcal{O}'_{V_R}$	$(\bar{\tau}\gamma_\mu P_R b)(\bar{c}\gamma^\mu P_L \nu)$	$\longleftrightarrow -2\mathcal{O}_{S_R}$	$\rangle(\mathbf{3},\mathbf{1})_{2/3}$	$(\lambda \bar{q}_L \gamma_\mu \ell_L + \tilde{\lambda}\bar{d}_R \gamma_\mu e_R) U^\mu$
$\mathcal{O}'_{S_R}$	$(\bar{\tau}P_R b)(\bar{c}P_L \nu)$	$\longleftrightarrow -\frac{1}{2}\mathcal{O}_{V_R}$		
$\mathcal{O}'_{S_L}$	$(\bar{\tau}P_L b)(\bar{c}P_L \nu)$	$\longleftrightarrow -\frac{1}{2}\mathcal{O}_{S_L} - \frac{1}{8}\mathcal{O}_T$	$(\mathbf{3},\mathbf{2})_{7/6}$	$(\lambda \bar{u}_R \ell_L + \tilde{\lambda}\bar{q}_L i\tau_2 e_R) R$
$\mathcal{O}'_T$	$(\bar{\tau}\sigma^{\mu\nu}P_L b)(\bar{c}\sigma_{\mu\nu}P_L \nu)$	$\longleftrightarrow -6\mathcal{O}_{S_L} + \frac{1}{2}\mathcal{O}_T$		
$\mathcal{O}''_{V_L}$	$(\bar{\tau}\gamma_\mu P_L c^c)(\bar{b}^c\gamma^\mu P_L \nu)$	$\longleftrightarrow -\mathcal{O}_{V_R}$		
$\mathcal{O}''_{V_R}$	$(\bar{\tau}\gamma_\mu P_R c^c)(\bar{b}^c\gamma^\mu P_L \nu)$	$\longleftrightarrow -2\mathcal{O}_{S_R}$	$(\bar{\mathbf{3}},\mathbf{2})_{5/3}$	$(\lambda \bar{d}^c_R \gamma_\mu \ell_L + \tilde{\lambda}\bar{q}^c_L \gamma_\mu e_R) V^\mu$
$\mathcal{O}''_{S_R}$	$(\bar{\tau}P_R c^c)(\bar{b}^c P_L \nu)$	$\longleftrightarrow \frac{1}{2}\mathcal{O}_{V_L} \langle$	$(\bar{\mathbf{3}},\mathbf{3})_{1/3}$	$\lambda \bar{q}^c_L i\tau_2 \boldsymbol{\tau}\ell_L \boldsymbol{S}$
$\mathcal{O}''_{S_L}$	$(\bar{\tau}P_L c^c)(\bar{b}^c P_L \nu)$	$\longleftrightarrow -\frac{1}{2}\mathcal{O}_{S_L} + \frac{1}{8}\mathcal{O}_T$	$\rangle(\bar{\mathbf{3}},\mathbf{1})_{1/3}$	$(\lambda \bar{q}^c_L i\tau_2 \ell_L + \tilde{\lambda}\bar{u}^c_R e_R) S$
$\mathcal{O}''_T$	$(\bar{\tau}\sigma^{\mu\nu}P_L c^c)(\bar{b}^c\sigma_{\mu\nu}P_L \nu)$	$\longleftrightarrow -6\mathcal{O}_{S_L} - \frac{1}{2}\mathcal{O}_T$		

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- There is also running and mixing between $C^S_{LL} - C^T_{LL}$ above the EWSB scale.

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- There is also running and mixing between $C^S_{LL} - C^T_{LL}$ above the EWSB scale.
- All in all, $C^S_{LL}(\Lambda_{NP}) = \pm 4 C^T_{LL}(\Lambda_{NP}) \Rightarrow C^S_{LL}(m_b) \approx \pm 8 C^T_{LL}(m_b)$.

Form Factors

$$\langle D|\,\bar{c}\,b\,|\overline{B}\rangle = \sqrt{m_B m_D}\,h_S\,(w+1)\,,$$
$$\langle D|\,\bar{c}\gamma^5 b\,|\overline{B}\rangle = \langle D|\,\bar{c}\gamma^\mu\gamma^5 b\,|\overline{B}\rangle = 0\,,$$
$$\langle D|\,\bar{c}\gamma^\mu b\,|\overline{B}\rangle = \sqrt{m_B m_D}\,\big[h_+(v+v')^\mu + h_-(v-v')^\mu\big],$$
$$\langle D|\,\bar{c}\sigma^{\mu\nu} b\,|\overline{B}\rangle = i\sqrt{m_B m_D}\,\big[h_T\,(v'^\mu v^\nu - v'^\nu v^\mu)\big],$$

$$\langle D^*|\,\bar{c}b\,|\overline{B}\rangle = 0\,,$$
$$\langle D^*|\,\bar{c}\gamma^5 b\,|\overline{B}\rangle = -\sqrt{m_B m_{D^*}}\,h_P\,(\epsilon^*\cdot v)\,,$$
$$\langle D^*|\,\bar{c}\gamma^\mu b\,|\overline{B}\rangle = i\sqrt{m_B m_{D^*}}\,h_V\,\varepsilon^{\mu\nu\alpha\beta}\,\epsilon^*_\nu v'_\alpha v_\beta\,,$$
$$\langle D^*|\,\bar{c}\gamma^\mu\gamma^5 b\,|\overline{B}\rangle = \sqrt{m_B m_{D^*}}\,\big[h_{A_1}(w+1)\epsilon^{*\mu} - h_{A_2}(\epsilon^*\cdot v)v^\mu - h_{A_3}(\epsilon^*\cdot v)v'^\mu\big],$$
$$\langle D^*|\,\bar{c}\sigma^{\mu\nu} b\,|\overline{B}\rangle = -\sqrt{m_B m_{D^*}}\,\varepsilon^{\mu\nu\alpha\beta}\big[h_{T_1}\epsilon^*_\alpha(v+v')_\beta + h_{T_2}\epsilon^*_\alpha(v-v')_\beta + h_{T_3}(\epsilon^*\cdot v)v_\alpha v'_\beta\big].$$

$$h_- = h_{A_2} = h_{T_2} = h_{T_3} = 0\,,$$
$$h_+ = h_V = h_{A_1} = h_{A_3} = h_S = h_P = h_T = h_{T_1} = \xi\,.$$

$$i\partial_\mu\,(\bar{c}\gamma^\mu b) = (m_b - m_c)\,\bar{c}b\,,$$
$$i\partial_\mu\,(\bar{c}\gamma^\mu\gamma^5 b) = -\,(m_b + m_c)\,\bar{c}\gamma^5 b,$$

$$\partial_\mu\,(\bar{c}\sigma^{\mu\nu} b) \;=\; -\,(m_b + m_c)\,\bar{c}\gamma^\nu b - (i\partial^\nu \bar{c})\,b + \bar{c}\,(i\partial^\nu b)$$

Leptonic/Hadronic Functions

$$H^{\lambda_M}_{V_1,\lambda}(q^2) = \epsilon^*_\mu(\lambda)\langle M(p_M,\epsilon(\lambda_M))\,|\bar{c}\gamma^\mu(1-\gamma^5)b|\bar{B}(p_B)\rangle\,,$$
$$H^{\lambda_M}_{V_2,\lambda}(q^2) = \epsilon^*_\mu(\lambda)\langle M(p_M,\epsilon(\lambda_M))\,|\bar{c}\gamma^\mu(1+\gamma^5)b|\bar{B}(p_B)\rangle\,,$$
$$H^{\lambda_M}_{S_1}(q^2) = \langle M(p_M,\epsilon(\lambda_M))\,|\bar{c}(1+\gamma^5)b|\bar{B}(p_B)\rangle\,,$$
$$H^{\lambda_M}_{S_2}(q^2) = \langle M(p_M,\epsilon(\lambda_M))\,|\bar{c}(1-\gamma^5)b|\bar{B}(p_B)\rangle\,,$$
$$H^{\lambda_M}_{\lambda\lambda'}(q^2) = i\epsilon^*_\mu(\lambda)\epsilon^*_\nu(\lambda')\langle M(p_M,\epsilon(\lambda_M))\,|\bar{c}\sigma^{\mu\nu}(1-\gamma^5)b|\bar{B}(p_B)\rangle,$$

$$L^{\lambda_\tau}_{\lambda,l}(q^2,\cos\theta_\tau) = \epsilon_\mu(\lambda)\langle\tau(p_\tau,\lambda_\tau)\bar{\nu}_l(p_\nu)|\bar{\tau}\gamma^\mu(1-\gamma_5)\nu_l|0\rangle\,,$$
$$L^{\lambda_\tau}_{l}(q^2,\cos\theta_\tau) = \langle\tau(p_\tau,\lambda_\tau)\bar{\nu}_l(p_\nu)|\bar{\tau}(1-\gamma_5)\nu_l|0\rangle\,,$$
$$L^{\lambda_\tau}_{\lambda\lambda',l}(q^2,\cos\theta_\tau) = -i\epsilon_\mu(\lambda)\epsilon_\nu(\lambda')\langle\tau(p_\tau,\lambda_\tau)\bar{\nu}_l(p_\nu)|\bar{\tau}\sigma^{\mu\nu}(1-\gamma_5)\nu_l|0\rangle\,,$$

Numerical Equations

$$
\begin{aligned}
R_D \quad &\approx \quad R_D^{SM} \times \Big\{ \left(|C^V_{LL} + C^V_{RL}|^2 + |C^V_{RR} + C^V_{LR}|^2 \right) \\
&+ \quad 1.35 \left(|C^S_{RL} + C^S_{LL}|^2 + |C^S_{LR} + C^S_{RR}|^2 \right) + 0.70 \left(|C^T_{LL}|^2 + |C^T_{RR}|^2 \right) \\
&+ \quad 1.72 \mathcal{R}e \left[(C^V_{LL} + C^V_{RL})(C^S_{RL} + C^S_{LL})^* + (C^V_{RR} + C^V_{LR})(C^S_{LR} + C^S_{RR})^* \right] \\
&+ \quad 1.00 \mathcal{R}e \left[(C^V_{LL} + C^V_{RL})(C^T_{LL})^* + (C^V_{LR} + C^V_{RR})(C^T_{RR})^* \right] \Big\},
\end{aligned}
$$

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&+ \quad 1.35 \left(|C^S_{RL} + C^S_{LL}|^2 + |C^S_{LR} + C^S_{RR}|^2 \right) + 0.70 \left(|C^T_{LL}|^2 + |C^T_{RR}|^2 \right) \\
&+ \quad 1.72 \mathcal{R}e \left[(C^V_{LL} + C^V_{RL})(C^S_{RL} + C^S_{LL})^* + (C^V_{RR} + C^V_{LR})(C^S_{LR} + C^S_{RR})^* \right] \\
&+ \quad \left. 1.00 \mathcal{R}e \left[(C^V_{LL} + C^V_{RL})(C^T_{LL})^* + (C^V_{LR} + C^V_{RR})(C^T_{RR})^* \right] \right\},
\end{aligned}
$$

$$
\begin{aligned}
R_{D^*} \quad &\approx \quad R_{D^*}^{SM} \times \left\{ \left(|C^V_{LL}|^2 + |C^V_{RL}|^2 + |C^V_{LR}|^2 + |C^V_{RR}|^2 \right) \right. \\
&+ \quad 0.04 \left(|C^S_{RL} - C^S_{LL}|^2 + |C^S_{LR} - C^S_{RR}|^2 \right) \\
&+ \quad 12.11 \left(|C^T_{LL}|^2 + |C^T_{RR}|^2 \right) - 1.78 \mathcal{R}e \left[(C^V_{LL})(C^V_{RL})^* + C^V_{RR}(C^V_{LR})^* \right] \\
&+ \quad 5.71 \mathcal{R}e \left[C^V_{RL}(C^T_{LL})^* + C^V_{LR}(C^T_{RR})^* \right] \\
&- \quad 4.15 \mathcal{R}e \left[(C^V_{LL})(C^T_{LL})^* + C^V_{RR}(C^T_{RR})^* \right] \\
&+ \quad \left. 0.12 \mathcal{R}e \left[(C^V_{LL} - C^V_{RL})(C^S_{RL} - C^S_{LL})^* + (C^V_{RR} - C^V_{LR})(C^S_{LR} - C^S_{RR})^* \right] \right\}.
\end{aligned}
$$

Numerical Equations

$$
\begin{aligned}
\mathcal{A}_{FB} &\approx \frac{1}{R_D}\Big\{-0.11\left(\left|1+C^V_{LL}+C^V_{RL}\right|^2+\left|C^V_{RR}+C^V_{LR}\right|^2\right)\\
&- 0.35\mathcal{R}e\left[(C^S_{LL}+C^S_{RL})(C^T_{LL})^*+(C^S_{RR}+C^S_{LR})^*(C^T_{RR})\right]\\
&- 0.24\mathcal{R}e\left[(1+C^V_{LL}+C^V_{RL})(C^T_{LL})^*+(C^V_{RR}+C^V_{LR})^*(C^T_{RR})\right]\\
&- 0.15\mathcal{R}e\left[(1+C^V_{LL}+C^V_{RL})(C^S_{LL}+C^S_{RL})^*+(C^V_{RR}+C^V_{LR})^*(C^S_{RR}+C^S_{LR})\right]\\
\mathcal{A}^*_{FB} &\approx \frac{1}{R_{D^*}}\Big\{-0.813\left(\left|C^T_{LL}\right|^2+\left|C^T_{RR}\right|^2\right)\\
&+ 0.016\left(\left|1+C^V_{LL}\right|^2+\left|C^V_{RR}\right|^2\right)-0.082\left(\left|C^V_{RL}\right|^2+\left|C^V_{LR}\right|^2\right)\\
&+ 0.066\mathcal{R}e\left[C^V_{RL}(1+C^V_{LL})^*+(C^V_{LR})^*C^V_{RR}\right]\\
&+ 0.095\mathcal{R}e\left[(C^S_{RL}-C^S_{LL})(C^T_{LL})^*+(C^S_{LR}-C^S_{RR})^*C^T_{RR}\right]\\
&+ 0.395\mathcal{R}e\left[(1+C^V_{LL}-C^V_{RL})(C^T_{LL})^*+(C^V_{RR}-C^V_{LR})^*(C^T_{RR})\right]\\
&+ 0.023\mathcal{R}e\left[(C^S_{LL}-C^S_{RL})(1+C^V_{LL}-C^V_{RL})^*+(C^S_{RR}-C^S_{LR})^*(C^V_{RR}-C^V_{LR})\right.\\
&- 0.142\mathcal{R}e\left[(C^T_{LL})(1+C^V_{LL}+C^V_{RL})^*+(C^T_{RR})^*(C^V_{RR}+C^V_{LR})\right]\Big\},
\end{aligned}
$$

Numerical Equations

$$
\begin{aligned}
\mathcal{P}_\tau &\approx \frac{1}{R_D}\Big\{0.402\left(\left|C^S_{LL}+C^S_{RL}\right|^2-\left|C^S_{RR}+C^S_{LR}\right|^2\right)\\
&+\ 0.013\left[\left|C^T_{LL}\right|^2-\left|C^T_{RR}\right|^2\right]+0.097\left[\left|1+C^V_{LL}+C^V_{RL}\right|^2-\left|C^V_{RR}+C^V_{LR}\right|^2\right]\\
&+\ 0.512\mathcal{R}e\left[(1+C^V_{LL}+C^V_{RL})(C^S_{LL}+C^S_{RL})^*-(C^V_{RR}+C^V_{LR})^*(C^S_{RR}+C^S_{LR})\right]\\
&-\ 0.099\mathcal{R}e\left[(1+C^V_{LL}+C^V_{RL})(C^T_{LL})^*-(C^V_{RR}+C^V_{LR})^*(C^T_{RR})\right]\Big\}\\
\mathcal{P}^*_\tau &\approx \frac{1}{R_{D^*}}\Big\{-0.127\left(\left|1+C^V_{LL}\right|^2+\left|C^V_{RL}\right|^2-\left|C^V_{RR}\right|^2-\left|C^V_{LR}\right|^2\right)\\
&+\ 0.011\left(\left|C^S_{LL}-C^S_{RL}\right|^2-\left|C^S_{RR}-C^S_{LR}\right|^2\right)+0.172\left(\left|C^T_{LL}\right|^2-\left|C^T_{RR}\right|^2\right)\\
&+\ 0.031\mathcal{R}e\Big[\left(1+C^V_{LL}-C^V_{RL}\right)\left(C^S_{RL}-C^S_{LL}\right)^*-\left(C^V_{RR}-C^V_{LR}\right)^*\left(C^S_{LR}-C^S_{RR}\right.\\
&+\ 0.350\mathcal{R}e\left[\left(1+C^V_{LL}\right)(C^T_{LL})^*-\left(C^V_{RR}\right)^*(C^T_{RR})\right]\\
&-\ 0.481\mathcal{R}e\left[(C^V_{RL})(C^T_{LL})^*-(C^V_{LR})^*(C^T_{RR})\right]\\
&+\ 0.216\mathcal{R}e\left[(1+C^V_{LL})(C^V_{RL})^*-(C^V_{RR})^*(C^V_{LR})\right]\Big\}.
\end{aligned}
$$

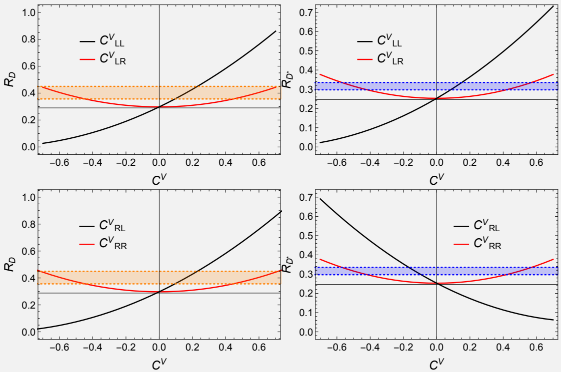
Numerical Equations

$$\begin{aligned}
\mathcal{P}_\perp &\approx \frac{1}{R_D}\mathcal{R}e\Big\{-0.350\left[\left(C^T_{LL}\right)\left(C^S_{LL}+C^S_{RL}\right)^* - \left(C^T_{RR}\right)^*\left(C^S_{RR}+C^S_{LR}\right)\right] \\
&- 0.357\left[\left(1+C^V_{LL}+C^V_{RL}\right)\left(C^S_{LL}+C^S_{RL}\right)^* - \left(C^V_{RR}+C^V_{LR}\right)^*\left(C^S_{RR}+C^S_{LR}\right)\right] \\
&- 0.247\left[(1+C^V_{LL}+C^V_{RL})^*(C^T_{LL}) - (C^V_{RR}+C^V_{LR})(C^T_{RR})^*\right] \\
&- 0.250\left[\left|1+C^V_{LL}+C^V_{RL}\right|^2 - \left|C^V_{RR}+C^V_{LR}\right|^2\right]\Big\} \\
\mathcal{P}^*_\perp &\approx \frac{1}{R_{D^*}}\mathcal{R}e\Big\{\left(C^S_{RR}-C^S_{LR}\right)\left[0.099C^T_{RR} - 0.054\left(C^V_{RR}-C^V_{LR}\right)\right]^* \\
&- \left(C^S_{LL}-C^S_{RL}\right)^*\left[0.099C^T_{LL} - 0.054\left(1+C^V_{LL}-C^V_{RL}\right)\right] \\
&+ \left(C^T_{RR}\right)\left[0.146C^V_{RR} - 0.478C^V_{LR} - 1.855C^T_{RR}\right]^* \\
&- \left(C^T_{LL}\right)^*\left[0.146(1+C^V_{LL}) - 0.478C^V_{RL} - 1.855C^T_{LL}\right] \\
&+ \left(C^V_{LR}\right)\left[-0.081C^T_{RR} + 0.025C^V_{LR} - 0.075C^V_{RR}\right]^* \\
&- \left(C^V_{RL}\right)^*\left[-0.081C^T_{LL} + 0.025C^V_{RL} - 0.075(1+C^V_{LL})\right] \\
&+ \left(C^V_{RR}\right)\left[-0.071C^T_{RR} - 0.075C^V_{LR} + 0.126C^V_{RR}\right]^*
\end{aligned}$$

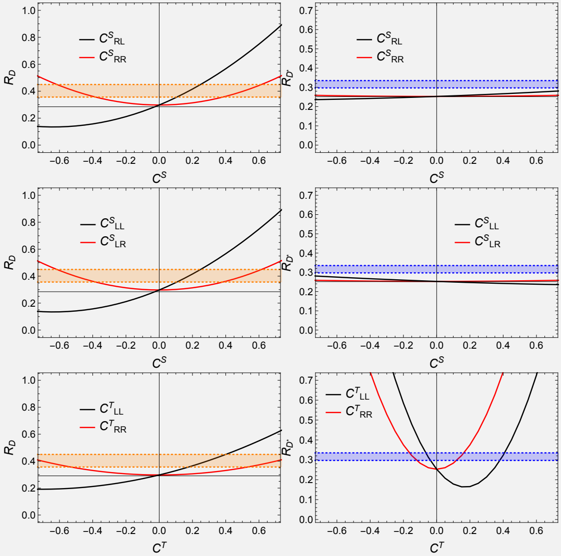
Numerical Equations

$$
\begin{aligned}
\mathcal{P}_T &\approx \frac{1}{R_D}\mathcal{I}m\Big\{-0.350\Big[(C^T_{LL})\left(C^S_{LL}+C^S_{RL}\right)^* - (C^T_{RR})^*\left(C^S_{RR}+C^S_{LR}\right)\Big] \\
&- 0.357\Big[\left(1+C^V_{LL}+C^V_{RL}\right)\left(C^S_{LL}+C^S_{RL}\right)^* - \left(C^V_{RR}+C^V_{LR}\right)^*\left(C^S_{RR}+C^S_{LR}\right)\Big] \\
&- 0.247\left[(1+C^V_{LL}+C^V_{RL})^*(C^T_{LL}) - (C^V_{RR}+C^V_{LR})(C^T_{RR})^*\right]\Big\} \\
\mathcal{P}^*_T &\approx \frac{1}{R_{D^*}}\mathcal{I}m\Big\{\left(C^S_{RR}-C^S_{LR}\right)\left[0.099\,C^T_{RR} - 0.054\left(C^V_{RR}-C^V_{LR}\right)\right]^* \\
&- \left(C^S_{LL}-C^S_{RL}\right)^*\left[0.099\,C^T_{LL} - 0.054\left(1+C^V_{LL}-C^V_{RL}\right)\right] \\
&+ (C^T_{RR})\left[0.146\,C^V_{RR} - 0.478\,C^V_{LR}\right]^* - (C^T_{LL})^*\left[0.146(1+C^V_{LL}) - 0.478\,C^V_{RL}\right] \\
&- (C^V_{LR})\left[0.081\,C^T_{RR}\right]^* + (C^V_{RL})^*\left[0.081\,C^T_{LL}\right] \\
&- (C^V_{RR})\left[0.071\,C^T_{RR}\right]^* + (1+C^V_{LL})^*\left[0.071\,C^T_{LL}\right]\Big\}
\end{aligned}
$$

The Theory of $R_{D^{(*)}}$



The Theory of $R_{D(*)}$



Constrain I : $Br(B_c \to \tau\nu)$

- Other processes can limit these large coefficients; in particular $Br(B_c \to \tau\nu)$. In SM : $Br(B_c \to \tau\nu) \approx 2.3\%$

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$$
\begin{aligned}
\frac{Br(B_c \to \tau\nu)}{Br(B_c \to \tau\nu)|_{\mathrm{SM}}} &= \left|1 + (C^V_{LL} - C^V_{RL}) + \frac{m^2_{B_c}}{m_\tau(m_b + m_c)}\,(C^S_{RL} - C^S_{LL})\right|^2 \\
&+ \left|(C^V_{RR} - C^V_{LR}) + \frac{m^2_{B_c}}{m_\tau(m_b + m_c)}\,(C^S_{LR} - C^S_{RR})\right|^2 .
\end{aligned}
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- Enhanced contribution from the scalar operators (same combination appearing in R_{D^*}).

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- Enhanced contribution from the scalar operators (same combination appearing in R_{D^*}).
- $Br(B_c \to \tau\nu) \leqslant 10\%$ from the $B_u \to \tau\nu$ at Z peak at LEP.

Constrain II : $b \to s\nu\nu$

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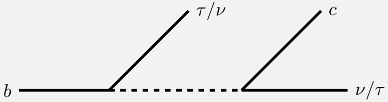
$$\mathcal{O}^V_{LL} = (\bar{c}_L \gamma^\mu b_L)(\bar{\tau}_L \gamma_\mu \nu_L),$$
$$\mathcal{O}^S_{RR} = (\bar{c}_L b_R)(\bar{\tau}_L \nu_R),$$

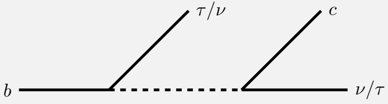


(c)

Constrain II : $b \to s\nu\nu$

Some of the mediators generating the C^V_{LL} or the $C^S_{RR} + xC^T_{RR}$ can generate $b \to s\nu\nu$ with the same couplings.

$$\begin{aligned} \mathcal{O}^V_{LL} &= (\bar{c}_L \gamma^\mu b_L)(\bar{\tau}_L \gamma_\mu \nu_L), \\ \mathcal{O}^S_{RR} &= (\bar{c}_L b_R)(\bar{\tau}_L \nu_R), \end{aligned}$$



(c)

These are neutral current constraints so will put severe bounds on the affected models.

Constrain II : $b \to s\nu\nu$

$$
\begin{aligned}
BR\,(B \to X_s\nu\nu) &\leqslant 6.4 \times 10^{-4}, \\
BR\,(B \to K\nu\nu) &\leqslant 1.6 \times 10^{-5}, \\
BR\,(B \to K^*\nu\nu) &\leqslant 2.7 \times 10^{-5}.
\end{aligned}
$$

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\end{aligned}
$$

$$
\begin{aligned}
\mathcal{H}_{\rm eff} &= -2\sqrt{2} G_F V_{tb} V^*_{ts} \frac{\alpha}{4\pi} \left[C^\nu_L \left(\bar{s}\gamma^\mu(1-\gamma^5)b\right) \left(\bar{\nu}\gamma_\mu(1-\gamma^5)\nu\right) \right. \\
&+ \left. C^\nu_R \left(\bar{s}\gamma^\mu(1+\gamma^5)b\right) \left(\bar{\nu}\gamma_\mu(1-\gamma^5)\nu\right) \right],
\end{aligned}
$$

$$
\epsilon \equiv \frac{\sqrt{|C^\nu_L|^2 + |C^\nu_R|^2}}{|(C^\nu_L)^{SM}|}, \quad \eta \equiv -\frac{\mathcal{R}e\left(C^\nu_L C^{\nu *}_R\right)}{|C^\nu_L|^2 + |C^\nu_R|^2}.
$$

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$$\epsilon \equiv \frac{\sqrt{|C^\nu_L|^2 + |C^\nu_R|^2}}{|(C^\nu_L)^{SM}|}, \quad \eta \equiv -\frac{\mathcal{R}e\,(C^\nu_L C^{\nu *}_R)}{|C^\nu_L|^2 + |C^\nu_R|^2}.$$

$$\begin{aligned} BR\,(B \to K\nu\nu) &= 4.5 \times 10^{-6}(1-2\eta)\epsilon^2, \\ BR\,(B \to K^*\nu\nu) &= 6.8 \times 10^{-6}(1+1.31\eta)\epsilon^2, \\ BR\,(B \to X_s\nu\nu) &= 2.7 \times 10^{-5}(1+0.09\eta)\epsilon^2. \end{aligned}$$

Constrain II : $b \to s\nu\nu$

$$
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\end{aligned}
$$

$$
\begin{aligned}
\mathcal{H}_{\text{eff}} &= -2\sqrt{2}G_F V_{tb}V_{ts}^* \frac{\alpha}{4\pi}\left[C_L^\nu\left(\bar{s}\gamma^\mu(1-\gamma^5)b\right)\left(\bar{\nu}\gamma_\mu(1-\gamma^5)\nu\right)\right. \\
&+ \left. C_R^\nu\left(\bar{s}\gamma^\mu(1+\gamma^5)b\right)\left(\bar{\nu}\gamma_\mu(1-\gamma^5)\nu\right)\right],
\end{aligned}
$$

$$
\epsilon \equiv \frac{\sqrt{|C_L^\nu|^2 + |C_R^\nu|^2}}{|(C_L^\nu)^{SM}|}, \quad \eta \equiv -\frac{\mathcal{R}e\left(C_L^\nu C_R^{\nu *}\right)}{|C_L^\nu|^2 + |C_R^\nu|^2}.
$$

$$
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BR\left(B \to K\nu\nu\right) &= 4.5 \times 10^{-6}(1-2\eta)\epsilon^2, \\
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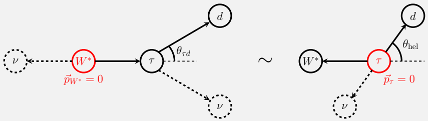
BR\left(B \to X_s\nu\nu\right) &= 2.7 \times 10^{-5}(1+0.09\eta)\epsilon^2.
\end{aligned}
$$

$C^V_{LL} \leqslant 0.006$, $\quad C^S_{RR} \leqslant 0.01$.

$\mathcal{P}_\tau$ Measurement

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_{\rm hel}} = \frac{1}{2}\left(1+\alpha_d \mathcal{P}^*_\tau \cos\theta_{\rm hel}\right)$$



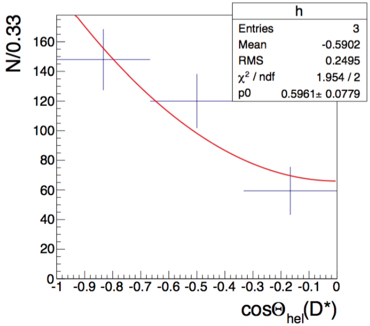
$$\cos\theta_{\tau d} = \frac{2E_\tau E_d - m_\tau^2 - m_d^2}{2|\vec{p}_\tau||\vec{p}_d|} \quad q^2 - \text{frame}$$

$$|\vec{p}_\tau| = \frac{q^2 - m_\tau^2}{2\sqrt{q^2}} \quad q^2 - \text{frame}$$

$$|\vec{p^\tau_d}|\cos\theta_{\rm hel} = -\gamma\frac{|\vec{p}_\tau|}{E_\tau}E_d + \gamma|\vec{p}_d|\cos\theta_{\tau d} \quad \tau - \text{frame}$$

F^L_{D*} Measurement

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_{\rm hel}(D^*)} = \frac{3}{4}[2F_L^{D^*}\cos^2(\theta_{\rm hel}(D^*)) + (1-F_L^{D^*})\sin^2(\theta_{\rm hel}(D^*))]$$



Number of events in:
I bin: 151±21
II bin: 125±19
III bin: 55±15
- signal yields corrected for accaptance variations

Dominant systematics:
- MC statistics (AR shape and peaking backgroud) $= \pm 0.03$

Different Calculations for $R_{J/\psi}$ in the SM

Table 1. Model predictions of $R(J/\psi)$ classified by method, which are abbreviated as: constituent quark model (CQM), relativistic quark model (RCQM), QCD sum rules (QCDSR), nonrelativistic quark model (NRQM), nonrelativistic QCD (NRQCD), and perturbative QCD calculations (pQCD).

Model	R_{theory}	Year
CQM [19]	0.28	1998
QCDSR [20]	$0.25^{+0.09}_{-0.09}$	1999
RCQM [21]	0.26	2000
QCDSR [22]	0.25	2003
RCQM [23]	0.24	2006
NRQM [24]	$0.27^{+0.02}_{-0}$	2006
NRQCD [25]	$0.07^{+0.06}_{-0.04}$	2013
pQCD [26]	$0.29^{+0.09}_{-0.09}$	2013
pQCD [27]	$0.30^{+0.11}_{-0.08}$	2016
pQCD [28]	$0.29^{+0.07}_{-0.07}$	2017
CQM [29]	0.24	2017
pQCD [30]	$0.283^{+0.048}_{-0.048}$	2017
CQM [31]	$0.24^{+0.07}_{-0.07}$	2018
RCQM [32]	0.24	2018
Range	0–0.48	–

Explaining $F^L_{D^*}$

R_D	R_{D^*}	$Br(B_c \to \tau\nu)$	C^V_{RL}	$F^L_{D^*}$	C^S_{RL}	C^S_{LL}	C^V_{LL}	C^T_{LL}	$R_{J/\psi}$
0.400	0.300	0.1	-0.3	0.510	0.330	0.152	1.012	0.092	0.340
0.400	0.300	0.1	-0.5	0.532	0.481	0.321	0.890	0.118	0.347
0.400	0.300	0.1	-0.7	0.552	0.614	0.471	0.764	0.143	0.355
0.400	0.300	0.1	-1	0.580	0.785	0.665	0.567	0.180	0.365

Explaining $F^L_{D^*}$

R_D	R_{D^*}	$Br(B_c \to \tau\nu)$	C^V_{RL}	$F^L_{D^*}$	C^S_{RL}	C^S_{LL}	C^V_{LL}	C^T_{LL}	$R_{J/\psi}$
0.400	0.300	0.1	-0.3	0.510	0.330	0.152	1.012	0.092	0.340
0.400	0.300	0.1	-0.5	0.532	0.481	0.321	0.890	0.118	0.347
0.400	0.300	0.1	-0.7	0.552	0.614	0.471	0.764	0.143	0.355
0.400	0.300	0.1	-1	0.580	0.785	0.665	0.567	0.180	0.365

- We need at least all the operators with a given neutrino chirality to explain $R_{D(*)}$ and $F^L_{D^*}$ together.

Explaining $F^L_{D^*}$

R_D	R_{D^*}	$Br(B_c \to \tau\nu)$	C^V_{RL}	$F^L_{D^*}$	C^S_{RL}	C^S_{LL}	C^V_{LL}	C^T_{LL}	$R_{J/\psi}$
0.400	0.300	0.1	-0.3	0.510	0.330	0.152	1.012	0.092	0.340
0.400	0.300	0.1	-0.5	0.532	0.481	0.321	0.890	0.118	0.347
0.400	0.300	0.1	-0.7	0.552	0.614	0.471	0.764	0.143	0.355
0.400	0.300	0.1	-1	0.580	0.785	0.665	0.567	0.180	0.365

- We need at least all the operators with a given neutrino chirality to explain $R_{D(*)}$ and $F^L_{D^*}$ together.
- One may wonder if the observed $F^L_{D^*}$ is merely a fluctuation too. We should be skeptical of the current experimental result.

Generating C^V_{RL}

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$R_2 = (3,2,7/6)$ and $\tilde{R}_2 = (3,2,1/6)$	✓
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- The vector LQs much more stringently constrained.*
- $R_2 + \tilde{R}_2$ is the least constrained way to generate C^V_{RL}.
- Still, further model-building gymnastic is required to keep the model alive.

Proposals for τ's Asymmetry Observables

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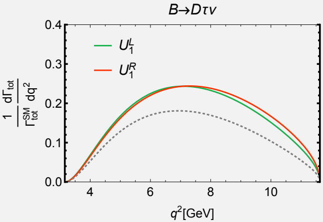

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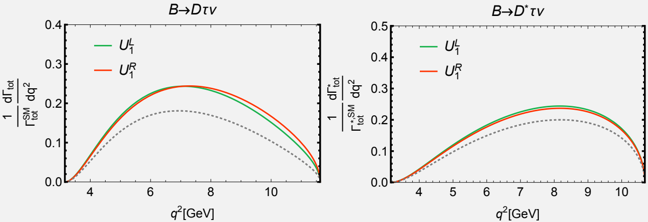

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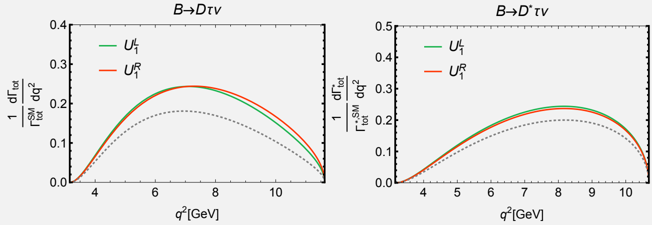

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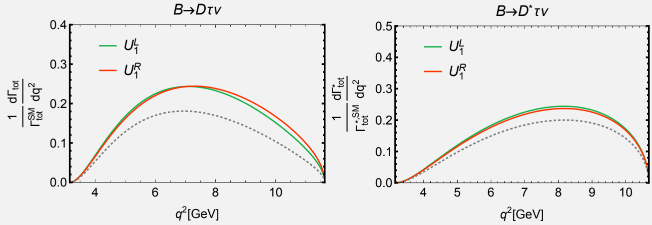

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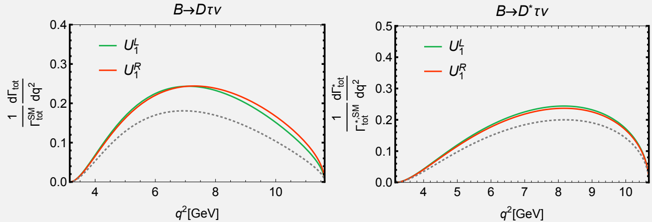

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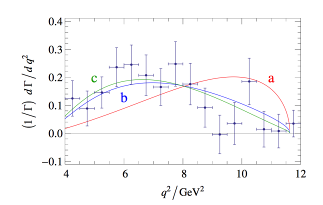

Global Maximum of $F^L_{D^*}$ and $R_{J/\psi}$

$$\mathcal{O} = z_5^\dagger M_{\mathcal{O}} z_5 = x_5^T M_{\mathcal{O}} x_5 + y_5^T M_{\mathcal{O}} y_5,$$

$$z_5 = x_5 + i y_5 = (C^V_{-L}, C^V_{+L}, C^S_{-L}, C^S_{+L}, C^T_{LL}),$$

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We can only find one zero eigenvalue, thus $x_5 \sim y_5$.

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We can only find one zero eigenvalue, thus $x_5 \sim y_5$. We can then rotate away the phase using the phase-invariance in $R_{D^{(*)}}$.

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- In the limit of small correlation, we can again treat $\left[\mathcal{I}_X(\vec{\theta})\right]_{ij}$ entries as a lower bound on the variance of each observable.

More RVs and Chain Rule for Fisher Information

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- In our proposal, $X = N_\pm$ (number of events with $c_{\theta_d} > 0$ or $c_{\theta_d} < 0$) and $Y = s_d$.
- We actually estimate $P(q^2)$ or $A(q^2)$ observables and only translate it into a total error on the inclusive observables (integrated over q^2) weighted by $d\Gamma/dq^2$, i.e. we assume the observables in different q^2 bins are independent.

Fisher Information for Our Proposal

RVs : i : $sign\left(\hat{\theta}_d\right)$, s_d : daughter meson energy.

$$\mathcal{I}_{N_i,s_d}(\theta_i,\theta_j) \;=\; -\sum_{i=\pm}\int ds_d f(N_i,s_d|\vec{\theta})\partial^2_{i,j}\log f(N_i,s_d|\vec{\theta})$$

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$$

$i=\pm1$, $F_X=\int ds_d f_X$